# TAILORING THE EMITTANCE OF A CHARGED PARTICLE BEAM WITH A TUNNEL EMITTANCE METER 

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## Abstract

Based on the "tunnel" emittance used for electron beam focusing, a similar procedure is proposed to evaluate fractional emittances for ion beams using two pairs of slits with variable widths. A measurement starts with closing both slits (one after the other) until a specific fraction of the transmitted beam current is cut off. The emittance and brilliance can then be well defined for the transmitted beam. Formulae are given for the emittance as well as for the brilliance as a function of the slit widths and beam current. This emittance measurement technique is free from the background subtraction problem found in the classical density measurement of phase space(s). The measurement device is also at the same time an adjustable emittance/brilliance filter for the transmitted beam. The emittance and/or brilliance of a beam can thus be tailored to any value within the limits of the beam quality at the expense of transmitted beam current.

## INTRODUCTION

The emittance of ion sources is generally considered an important characteristic value, especially when the ion source will be used in combination with a beam line or the low energy transport (LEBT) section of an accelerator having a well-defined acceptance. Acceptances of beam lines and accelerators usually have the shape of ellipses with uniform density, characterized by the twiss parameters of these elliptical bounded phase spaces. While the matching conditions appear to be well defined by the accelerator expert, the ion source developer has great difficulty in satisfying these requirements for the following reasons:

- Ion source emittances, either measured or calculated, usually are not elliptically bounded but show wings of aberrations caused by the extraction system and the lenses.
- The emittance areas are not uniformly filled.
- Defining an exact value of the emittance is a matter of sophisticated background subtraction [1].
As a solution, we propose here a procedure to not only measure the emittance and brilliance of a beam in a simple and unambiguous way but to also define the beam quality by analysing its fractional emittance. This experimental procedure is based on the well-established experience of electron tube designers in the 1960's who characterized the quality of an electron gun by a "tunnel" emittance device. Recently this procedure has been modified $[2,3]$ by replacing the tunnel with two pairs of slits for the x - and the y -direction.


## TUNNEL EMITTANCE

About 50 years ago the microwave industry developed GHz oscillators and rf-amplifiers called travelling wave tubes (TWT). A medium energy electron beam (similar to that of an EBIS) was focused in a strong solenoid (or an alternating permanent magnet structure) of some kilogauss and inside of a helical rf-structure, amplifying the velocity modulation on the beam by bunching. In order to measure the quality (=emittance) of such a gun and beam forming system a very practical experimental procedure has been developed. The beam is injected into a "tunnel" which is a cooled piece of copper with a hole drilled in it for the electron beam to pass through. Then the experiment shows that the smaller the hole and the higher the transmission at high current, the better the gun quality! [4]


Figure 1: Definition of the tunnel emittance, showing how angles are defined by slit widths and the distance of the two apertures (left panel) and how this translates to the 2D phase space (right panel).

Referring to Fig. 1, assuming that the second aperture with width $d_{2}$ is fully illuminated by the beam passing through the first aperture with width $d_{1}$ we find the relations for the angles shown on the left characterizing the shaded phase space on the right. By focusing with a thin lens in the position of the second aperture with focal length $\boldsymbol{f}=\boldsymbol{\lambda}$, the shaded parallelogram can be turned into the dashed rectangle. This produces the dashed rays found at the right side of the left panel. By definition the full emittance $\varepsilon_{\text {full }}$ then is calculated as the area of this rectangle with side lengths of $d_{2}$ and $d_{1} / \lambda$ :

$$
\begin{equation*}
\varepsilon_{\text {full }}=\frac{d_{1} d_{2}}{\lambda} \quad[m] \tag{1}
\end{equation*}
$$

Quite naturally this emittance has the unit of length (m, $\mathrm{cm}, \mathrm{mm}$, yards, feet, inches, mils; however microns without any $\pi$ or mrad are best!) [5]. It is seen from the dependence on the distance $\lambda$ between the slits that $1 / \lambda^{2}$ enters into the error; hence a minimum distance is required for a given accuracy of the emittance determination. Sophisticated background subtraction, which is essential for emittance obtained by measuring the density distribution of the phase space(s) [1], is not needed for the tunnel emittance method. The measurement starts with both apertures fully open (see Fig. 2) and with a maximum signal on a Faraday cup located behind the slits. The $100 \%$ emittance is determined by reducing the width of both slits until a decrease of the beam intensity is just able to be observed. Then the slits are closed one after the other in succeeding steps to achieve a given transmitted beam fraction. This procedure can be continued until the slits are small enough to cut off most of the ion current. This will define the total and fractional emittances of the beam as a function of the current passing through both slits.


Figure 2: Measurement of full and fractional tunnel emittance by a pair of adjustable slits. For nonaxisymmetric beams 2 pairs of slits in $x$ and $y$-directions are needed. The transmitted beam then has a well defined, tailored emittance.

The tunnel emittances for the $x$ - and $y$ - directions are evaluated by 2 sets of perpendicular slits with widths $d_{x l}$, $d_{x 2}$ and $d_{y l}, d_{y 2}$ placed along the beam path at distances $\lambda_{x}$ and $\lambda_{y}$ apart:

$$
\begin{equation*}
\varepsilon_{x, \text { full }}=\frac{d_{x 1} d_{x 2}}{\lambda_{x}}, \quad \varepsilon_{y, \text { full }}=\frac{d_{y 1} d_{y 2}}{\lambda_{y}} \tag{2}
\end{equation*}
$$

In the case of a round beam the pair of slits cutting out the beam in direction $y$ is not needed - all values may be taken from the emittance measurement in direction $x$. The brilliance then is defined as the current density in the right aperture of Fig. 1 divided by the solid-angle of the transmitted beam:

$$
\begin{equation*}
B=\frac{I}{d_{x 2} d_{y 2}} \times \frac{1}{\frac{d_{x 1}}{\lambda_{x}}} \times \frac{1}{\frac{d_{y 1}}{\lambda_{y}}} \quad\left[\frac{A}{m^{2}}\right] \tag{3}
\end{equation*}
$$

Using the expressions for $\varepsilon_{x, f \text { full }}$ and $\varepsilon_{y \cdot f \text { full }}$ (Eqs. 2) this reproduces the well known expression for the brilliance:

$$
\begin{equation*}
B=\frac{I}{\varepsilon_{x, \text { foll }} \varepsilon_{y, \text { full }}} \tag{4}
\end{equation*}
$$

This formula demonstrates that the use of full emittances is more elegant than the common use of half-axis products. This argument is further enhanced by emittances, which have no point symmetry to the origin in phase space, such as those from ECR ion sources and all ion sources with a coma by misalignment.

## FRACTIONAL EMITTANCES

## Beam with Rectangular Cross Section

In order to analyse the quality of a given beam, we investigate the variation of emittances with the fraction of transmitted current through 2 slits. For a uniform beam with constant current density $j$ we use the relation between current density and beam current, assuming slit 1 is open and slit 2 is partially closed in the $x$ direction while both slits are open in the y-direction:

$$
\begin{align*}
& I^{\text {fract }}\left(d_{x 2}\right)=j_{2} D_{y 2} d_{x 2} \text { and } \\
& I^{\text {total }}\left(D_{x 2}\right)=j_{2} D_{y 2} D_{x 2} \tag{5}
\end{align*}
$$

This together with Eq. 2 gives the proportional relationship between the fractional emittance and current:

$$
\begin{equation*}
\frac{\varepsilon_{x, y}^{\text {fract }}}{\varepsilon_{x, y}^{\text {total }}}=\frac{I^{\text {fract }}}{I^{\text {total }}} \tag{6}
\end{equation*}
$$

Hence, for a beam with uniform current density, the fractional emittance is proportional to the current fraction. The same result is obtained if both slits were closed simultaneously.

## Round Beam

When closing a slit in a round beam (either $x$ - or $y$ direction), the transmitted part becomes a function of the slit width $d$ only, as shown in Fig. 2, due to the circular shape of the beam cross section. For a beam of diameter $D$ and with uniform current density this translates directly to the transmitted current $I^{\text {fract }}$ in relation to the total current $I^{\text {total }}$ :

$$
\begin{equation*}
\frac{I^{\text {fract }}}{I^{\text {total }}}=\frac{2}{\pi}\left\{\frac{d}{D} \sqrt{1-\left(\frac{d}{D}\right)^{2}}+\arcsin \frac{d}{D}\right\} \tag{7}
\end{equation*}
$$

According to Eq. 1 the emittance of a beam passing through a constant slit width $d_{l}$ and a variable slit with width $d$ is given by

$$
\begin{equation*}
\varepsilon_{x, \text { full }}^{\text {fract }}=\frac{d_{x 1} D}{\lambda_{x}} \times \frac{d}{D} \tag{8}
\end{equation*}
$$

where the first fraction gives the total emittance if slit 2 is opened to width $D$. From this we get

$$
\begin{equation*}
\frac{\varepsilon_{x, f \text { full }}^{\text {fract }}}{\varepsilon_{x, \text { full }}^{\text {total }}}=\frac{d}{D} \tag{9}
\end{equation*}
$$

Inserting this into Eq. 7 we obtain an implicit relation for the dependence of the fractional emittance on the current fraction.

$$
\begin{equation*}
\frac{I^{\text {fract }}}{I^{\text {total }}}=\frac{2}{\pi}\left\{\frac{\varepsilon_{x, \text { full }}^{\text {fract }}}{\varepsilon_{x, \text { full }}^{\text {total }}} \sqrt{1-\left(\frac{\varepsilon_{x, \text { full }}^{\text {fract }}}{\varepsilon_{x, \text { foll }}^{\text {total }}}\right)^{2}}+\arcsin \left(\frac{\varepsilon_{x, \text { full }}^{\text {fract }}}{\varepsilon_{x, \text { full }}^{\text {total }}}\right)\right\} \tag{10}
\end{equation*}
$$

In the likely case that both slits are closed in the same way, the emittance will be reduced with the square of $d / D$ and the fractional current will see the correction for the round beam (Eq. 7) twice. This gives
$\frac{I^{\text {fract }}}{I^{\text {total }}}=\frac{4}{\pi^{2}}\left\{\sqrt{\frac{\varepsilon_{x, \text { full }}^{\text {fract }}}{\mathcal{E}_{x, \text { full }}^{\text {total }}}} \sqrt{1-\frac{\varepsilon_{x, \text { full }}^{\text {fract }}}{\mathcal{E}_{x, \text { full }}^{\text {total }}}}+\arcsin \sqrt{\frac{\varepsilon_{x, \text { full }}^{\text {fract }}}{\mathcal{E}_{x, \text { full }}^{\text {total }}}}\right\}^{2}$


Figure 3: Theoretical variation of the fractional emittance on the transmitted current fraction for beams of uniform current density and rectangular or round cross section and for the cases of closing 1 slit only or 2 slits simultaneously.

The results of Eq. 6 for a beam with rectangular cross section and of Eq. 10 and Eq. 11 for round beams is shown in Fig. 3. The uniformly filled beam with rectangular cross sections will show a linear decrease of the emittance with the fraction of current, independent of whether only one slit is being closed or both slits are closed simultaneously. This is different for the round beam. If cut out by two slits simultaneously the round beam will exhibit a stronger decrease of the emittance for the associated loss of beam current.

## CONCLUSIONS

The tunnel emittance measured by 2 pairs of slits for the x - and the y -direction provides an easy method to test the "focusability" of ion beams, e.g. the matching into the acceptance of an accelerator or transport channel. In contrast to the classical method of measuring phase space densities, enclosing emittances are obtained. This eliminates the need for a careful background subtraction. In addition, fractional emittances can be determined which give useful information about the current density distribution inside the beam. This is especially important for emittance growth of high current ion beams by nonlinear fields. For beams with uniform density distribution, formulae have been presented for the dependence of the fractional emittance on the current fraction, both for beams with rectangular cross sections and for round beams. Different relations are obtained for round beams if only one slit is closed or if both slits are closed simultaneously.

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