EVALUATION OF BEAM ENERGY FLUCTUATION CAUSED BY PHASE NOISE

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Abstract

The SSB noises of the RF reference signal dominate the short-term instabilities of the RF phase of the carrier RF. This phase modulation finally results in beam energy fluctuation. This presentation gives a quantitative evaluation of the beam energy fluctuations in an electron linear accelerator caused by phase noises, comparing a theoretical analysis and experimental results. A simple model was introduced to understand how phase noises result in the relative phase difference between a beam bunch and accelerating RF fields. In the experiments, we measured the enhanced beam energy fluctuations by modulating the phase of the reference RF signals with an external signal. The interference between the accelerating RF phase modulation and the timing modulation of a beam bunch was found in the model analysis and also in the experimental results.

INTRODUCTION

Recently, it has become easier to introduce a high-precision signal generator as a master oscillator for an accelerator; hence the phase noises of a master oscillator do not dominate beam instability if a low-power RF system is carefully designed.

However, the effect of phase noises may be evaluated in the following cases. The RF sources for XFELs is generally required to satisfy the integrated phase fluctuation of less than several tens of femtoseconds [1]. A programmable arbitrary waveform generator, whose phase noise levels are larger than those of a normal type, may be required to generate a linac's reference signal synchronizing with that of a storage ring [2].

Since the SPring-8 linac has used the latter system to stabilize the beam energy, we performed the preliminary experiment to understand the beam energy variation caused by phase noises. In this experiment, we modulated the phase of the reference RF signal by external sinusoidal waves with various frequencies or noises and then observed that the resulting beam energy modulations depended on the modulation frequencies.

The motivation of this study to determine the mechanism of the beam energy variation caused by phase noise is based on the above preliminary study.

UNDERSTANDING PHASE NOISE

Interference of Phase Noise

The RF phase variations caused by phase noise seem like random phenomena. On the contrary, we can unfold even a random signal by applying the Fourier integral as an integration of an infinite number of sinusoidal waves. When we pay attention to each Fourier frequency component, this sinusoidal wave must have a constant amplitude and phase advance at least during a period of milliseconds, for example. That is, we can conclude that any sinusoidal wave of the Fourier frequency component is so stable as to show the interference because the length of the sinusoidal wave is sufficiently long in actual linacs.

In addition, since actual circuits including waveguide circuits and accelerating structures have limited frequency bandwidths, the randomness of signals is reduced according to the bandwidths.

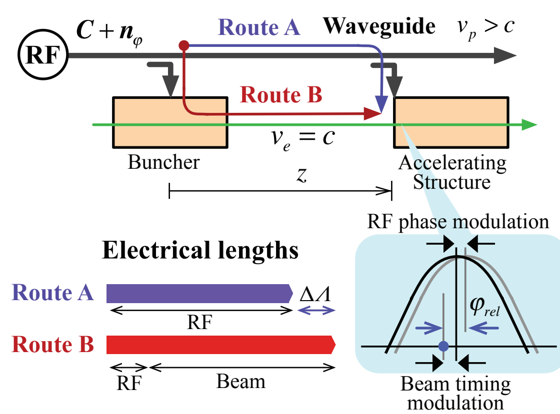

Figure 1: Simplified model of electron linac is proposed to understand the phase noise propagation. Relative phase modulation is caused by the electrical path difference $\Delta\Lambda$, which depends on the RF frequency due to the dispersion of the transmission line.

Figure 1 presents a simplified model of a normal conducting electron linac with a buncher and multiple accelerating structures. This model treats only RF phase, not RF amplitude. In this model, we assume that the phase variation is expressed as the superposition of stable sinusoidal waves as explained above. This figure shows also the electrical lengths of the routes of the accelerating RF and the electron beam bunched in the buncher cavity and a diagram to explain the relative phase deviation that results in the beam energy deviation.

If the phase velocity of the RF wave propagating in the transmission line does not depend on its frequency and is the same as the electron velocity (= c), the length difference $\Delta\Lambda$ in Fig. 1 is almost zero. Assuming that the frequency responses of the buncher and the accelerating structure are the same, the RF phase deviation observed by a beam bunch at the entrance of every accelerating structure is the same as the timing deviation of the beam. Hence the RF phase modulation caused by a master oscillator does not modulate the beam energy.

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In the case where the transmission line has the dispersion and the phase velocity is accordingly a function of the frequency, the difference $\Delta\Lambda$ depends on the frequency and grows at a part farther downstream. This growth of $\Delta\Lambda$ provides the relative phase difference φ_{rel} and consequently the energy deviation depending on the distance from the buncher to the accelerating structure.

It is easily expected that the energy modulation depends on the modulation frequency as well as the distance from the buncher. This means that we have to separate the contributions of the frequency and the distance to clearly investigate the energy modulation caused by the phase modulation. Therefore, we employed a monochromatic frequency modulation method.

Propagation of Phase Noise

We will describe below only how propagating noise vectors can be treated.

An RF carrier signal vector C on the complex plane with minute modulations in amplitude and phase, Δa and $\Delta\varphi$ respectively, can be expressed as follows:

$$\begin{aligned}\boldsymbol{C} &= (1+\Delta a)e^{j(\omega_0 t+\Delta\varphi)} \\ &\cong e^{j\omega_0 t} + \Delta a\, e^{j\omega_0 t} + j\,\Delta\varphi\, e^{j\omega_0 t}\end{aligned} \quad (1)$$

The third term of Eq. (1) contains the imaginary unit j; that is, this term is always perpendicular to the carrier vector $\exp(j\omega_0 t)$ and is consequently the phase modulation vector. The arbitrary phase modulation $\Delta\varphi$ can be expressed as an integral of the Fourier components, and each component $\delta\varphi_m$ is a trigonometric function with a modulation frequency ω_m.

In the following analysis, we will concentrate on the monochromatic phase modulation of a modulation frequency φ_m. The phase modulation vector $\delta\boldsymbol{n}_\varphi$ caused by a modulation φ_m is actually a summation of two vectors $\omega_0+\omega_m$ and $\omega_0-\omega_m$ as follows:

$$\begin{aligned}\delta\varphi_m &= \varphi_m \sin(\omega_m t+\theta_m) \\ \delta\boldsymbol{n}_\varphi &= j\,\varphi_m \sin(\omega_m t+\theta_m)\,e^{j\omega_0 t} \\ &= \frac{1}{2}\varphi_m\left(e^{j((\omega_0+\omega_m)t+\theta_m)} - e^{j((\omega_0-\omega_m)t-\theta_m)}\right)\end{aligned} \quad (2)$$

Here, θ_m is an initial phase of the modulation $\delta\varphi_m$. We assume that random numbers in the world of the phase noise, which will be treated in the Monte Carlo simulation in the latter section, are only the initial phase values θ_m.

A carrier signal and its phase noise are generated by an oscillator and propagate in transmission lines via several RF components such as phase shifters or amplifiers, and finally reach a buncher or an accelerating structure.

When the phase noise vector expressed as Eq. (2) propagates to z-direction in a transmission line, the noise vector including propagation terms is presented with the wave numbers k_m^+ and k_m^- as Eq. (3). In case the phase velocity varies during the propagation, the propagation terms have to be replaced with integrals.

$$\delta\boldsymbol{n}_\varphi(\omega_m, z) = \frac{1}{2}\varphi_m\left(e^{j((\omega+\omega_m)t-k_m^+ z+\theta_m)} - e^{j((\omega-\omega_m)t-k_m^- z-\theta_m)}\right) \quad (3)$$

The relative phase difference φ_{rel}, which determines the beam energy deviation, can be introduced by subtracting the timing deviation, which is given by the phase noise vector in a buncher cavity, from the RF phase deviation, which is experienced by a beam bunch passing through an accelerating structure.

STUDY WITH MODEL LINAC

We investigated the relative phase deviation of accelerating RF waves experienced by a beam bunch using the simple model explained in the above section. The important calculating conditions are as follows.

- Carrier RF: CW (2.856 GHz)
- Phase modulation: 50 kHz, 500 kHz, 2 MHz
- Transmission line: Waveguide (minimum length)
- Phase tuning: An RF phase shifter (coaxial line stretcher) is installed just before every accelerating structure, and it is always tuned to accelerate a beam bunch at the carrier RF crest.

Case 1: No frequency response of RF cavities

It is assumed that the buncher has no bandwidth and the accelerating structure has no dispersion. However, the transmission line has the dispersion and this results in the phase shift depending on the frequency.

Figure 2 shows the relative phase modulation as a function of the distance from the buncher when the initial modulation of 1 deg. in rms is given to the oscillator.

The left graph clearly shows that the RF phase deviation in the accelerating structures upstream is cancelled by the timing deviation of the beam bunch caused by the RF phase deviation in the buncher. In the downstream part and at higher modulation frequency, we observe the clear growth of the relative phase shift due to the dispersion of the transmission line.

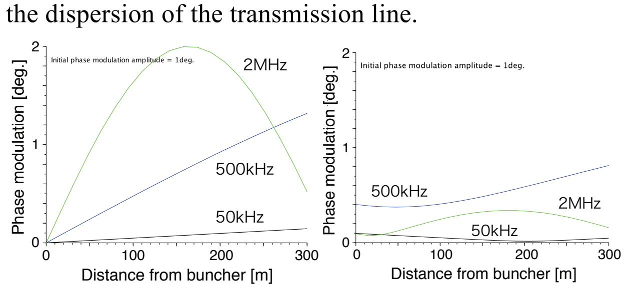

Figure 2: Relative phase modulations experienced by a beam bunch are presented in case the initial modulations are 50 kHz, 500 kHz, and 2 MHz. Left: the case 1, the RF cavity responses are neglected. Right: the case 2, the practical model.

Case 2: Practical model

The buncher is represented as a simple LC circuit with Q_L=5300 and the accelerating structure is assumed to have the typical dispersion. Figure 2 (right) clearly expresses the effects of the RF cavities: At higher modulation frequency, the narrow bandwidth of the buncher degrades the cancelling effect appearing in the relative phase deviation, and the dispersion of the accelerating structure attenuates the integrated RF phase modulation in the accelerating structure.

EXAMPLE: SPring-8 LINAC

We modified the linac model and optimized its parameters as accurately as possible to express the real SPring-8 linac. The expected relative phase modulations are shown in Fig. 3. The histograms of the beam energy variation were calculated by the Monte Carlo simulation, which randomizes the initial phase θ_m.

We performed the experiment to verify the model. The important characteristics of the experiment are as follows.

- The external phase modulation signals are sinusoidal waves (500 kHz, 1 MHz, 2 MHz) and band-limited artificial noises (400 - 600 kHz).
- The relative phase variation at some specific accelerating structure (M2, M10, and M16) was linearly transformed to the beam energy variation by accelerating beams at the zero-cross phase.
- The actual RF signal is pulsed (2 µs), not CW.

Figure 3 also presents the relative phase modulations in rms (solid diamonds) calculated from the obtained histograms of the energy variations at M2, M10, M16 accelerator sections.

The SSB phase noises and the histograms of the energy variations are shown in Fig. 4 including the simulated results obtained by the Monte Carlo method.

We understand the results in Fig. 3 and 4 as follows.

- The relative phase modulations at 500 kHz and 1 MHz apparently show their growth at the part farther downstream. These facts seem to support the model mentioned above which explains the interference between the RF phase modulation and the beam timing modulation.
- The relative phase modulations resulting from the experiment using the band-limited noises (400-600 kHz) are almost the same as those caused by the 500 kHz sinusoidal modulations. This agreement may supports our fundamental assumption that the random phase noise can be approximately treated as the superposition of coherent sinusoidal waves.
- The experimental values in Figs. 3 and 4 are two- or three-times larger than the expected values provided by the model. The reason for this disagreement is still under investigation.

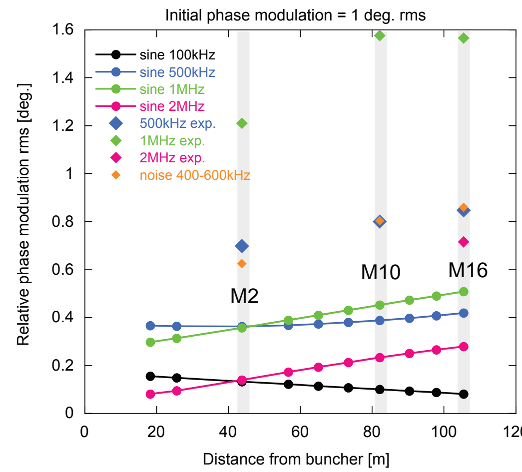

Figure 3: Relative phase modulation along the SPring-8 linac when the initial modulation is 1 deg. in rms. The solid circles are the simulated results and the solid diamonds express the experimental values. M2, M10 etc. are the names of the accelerator sections of the linac.

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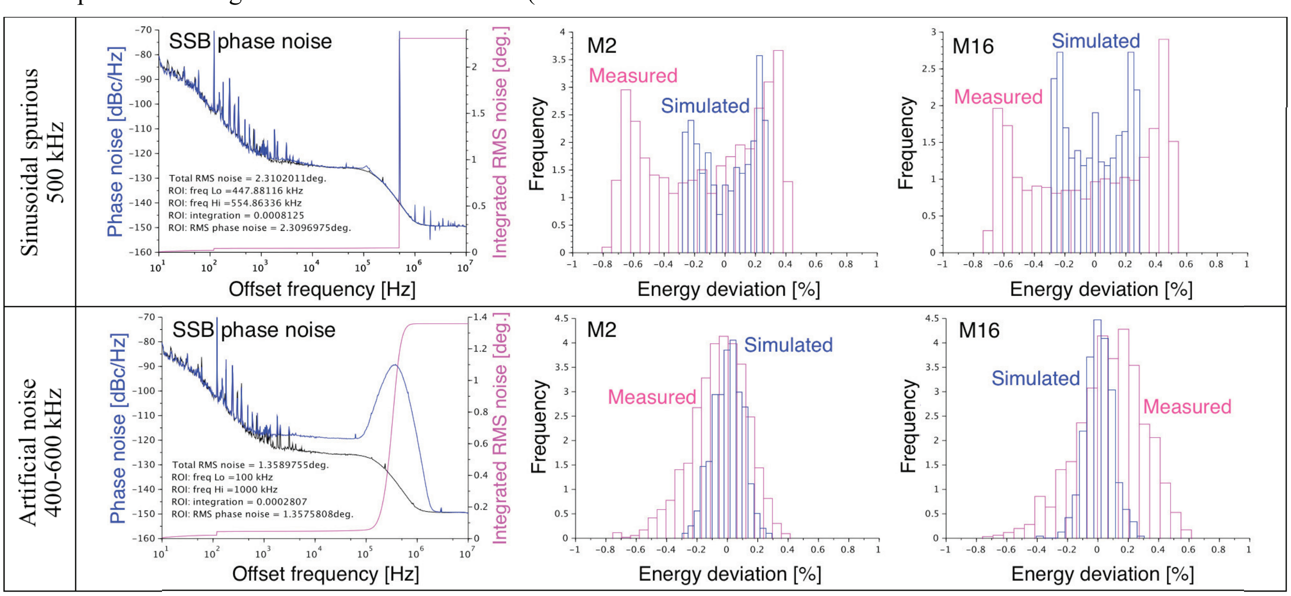

Figure 4:Measured and simulated energy deviations at M2 and M16 caused by forced phase modulations. The energy fluctuations observed at M16 (downstream) are clearly larger than those observed at M2 (upstream).

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