# CYCLOTRON-UNDULATOR COOLING OF ELECTRON BEAMS

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*Abstract*

We propose methods of fast cooling of an electron beam, which are based on wiggling of particles in an undulator in the presence of an axial magnetic field. We use a strong dependence of the axial electron velocity on the oscillatory velocity, when the electron cyclotron frequency is close to the frequency of electron wiggling in the undulator field. The abnormal character of this dependence (when the oscillatory velocity increases with the increase of the input axial velocity) can be a basis of various methods for fast cooling of moderately-relativistic (several MeV) electron beams.

## NON-RADIATIVE UNDULATOR COOLING

Fast development of the technique of photo-cathode electron photoinjectors has resulted in creation of compact and accessible sources of moderately-relativistic (several MeV) dense (~1 nC in a ps pulse) bunches [1-3]. Methods for decrease of the energy spread (cooling) are actual from the point of view of various applications of such beams, including free-electron lasers (FEL) [4-6]. However, cooling methods are developed now basically for electron beams of significantly higher energies [7,8]. As for a moderately-relativistic high-dense short electron bunches, the strong Coulomb interaction of the particles results in a requirement for a short (~ 1 m and even less) length of a cooling system. Therefore, in this situation, the cooling system should possess resonant properties, namely, a strong dependence of parameters of the particles inside the cooling system on their input energies.

In this letter, we propose to provide cooling by the use of electron wiggling in a circular polarized “cooling” undulator in the presence of an axial magnetostatic field $zB_0$ (Fig. 1). If the bounce-frequency of electron oscillations in the undulator, $\Omega_u = V_\parallel h_u$ is comparable with the electron cyclotron frequency, $\Omega_c = eB_0/\gamma mc$ (here $V_\parallel$ is the electron axial velocity, $h_u$ is the undulator wavenumber, and $\gamma = 1/\sqrt{1-(V/c)^2}$ is the relativistic electron mass factor). In this situation, the velocity of undulator oscillations $V_u$ depends strongly on the initial axial velocity.

Non-radiative “axial” cooling is based on the fact that the axial velocity spread is the only factor important for the FEL operation. This spread can be decreased due to its “transformation” into the spread in the velocity of electron rotation in the cooling system. Electrons move along axial magnetic field and enter the cooling undulator with the adiabatically growing field in the input section, where each electron gets its own rotatory velocity (Fig. 1a). If at the input of the system every particle possesses only the axial velocity $V_0 = \bar{V} + \delta V$, then the axial velocity in the regular region of the undulator is determined by the energy conservation law: $V_\parallel^2 \approx V_0^2 - V_u^2$. Thus,

$$V_\parallel^2 \approx \bar{V}^2 + 2\bar{V}\delta V - V_u^2(\bar{V}) - \alpha\delta V . \tag{1}$$

If $\alpha = \partial V_u^2 / \partial V_\parallel = 2\bar{V}$, then the spread in $V_\parallel$ disappears. This condition is independent of the initial spread, $\delta V$. Evidently, we should use the range of parameters, where $\partial|V_u|/\partial V_\parallel > 0$ (Fig. 1 d), so that the initial axial velocity excess, $\delta V$, is compensated by the greater rotatory velocity, $V_u$.

If such a cooling system is used in a FEL, then the operating FEL undulator designed to produce optical radiation can be placed inside the regular section of the cooling undulator (Fig. 1 a). Another way is to “switch off” the field of the cooling “undulator” sharply (Fig. 1 b). Then, forced undulator oscillations of the particles are just transformed into free cyclotron oscillations with the same rotatory velocities, $V_\perp = V_u$ (and the same axial velocities).

Let us consider the equation for the transverse momentum of a particle moving in the axial uniform magnetic field, $\mathbf{B}_0 = \mathbf{z}B_0$, and in the quasi-periodical field of the undulator, $B_+ = B_x + iB_y = B_u(z)\exp(ih_u z)$:

$$\frac{dp_+}{dz} = i\frac{\Omega_c}{c}\frac{p_+}{V_\parallel} + ih_u K(z)\exp(ih_u z) . \tag{2}$$

Here, $p_+ = p_x + ip_y$, $\mathbf{p} = \gamma\boldsymbol{\beta}$, $\boldsymbol{\beta} = \mathbf{V}/c$, and $K(z) = -eB_u / h_u mc^2$ is the undulator factor (the normalized transverse electron momentum, $K = \gamma V_u / c$); its dependence on the coordinate describes smooth entrance of the particles into the undulator. At the beginning, where $K = 0$, there exists a “parasitic” transverse electron motion due to free cyclotron oscillations, $p_+(0) = \gamma_0\beta_{\perp 0}\exp(i\theta_c)$. The initial spread in the transverse velocity is described by the uniform distribution of $\beta_{\perp 0}$ over the interval $0 < \beta_{\perp 0} < \bar{\beta}_{\perp 0}$. As for the axial velocity spread, it is described by uniform distribution of $\beta_{\parallel 0}$ in the interval

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$\bar{\beta}_{\|0} - \delta\beta < \beta_{\|0} < \bar{\beta}_{\|0} + \delta\beta$. If the field taper of the undulator is smooth enough, then the growing forced undulator oscillations do not perturb the free electron cyclotron oscillations. Therefore, the transverse velocity is described in the regular region as follows:

$$\beta_+ = p_+ / \gamma_0 = \beta_u \exp(ih_u z) + \beta_{\perp 0} \exp(i\theta_c). \quad (3)$$

where the velocity of electron rotation in the undulator field, $\beta_u = K / \gamma_0 \Delta$, depends on the mismatch between the electron cyclotron frequency and the bounce-frequency of electron oscillations in the undulator, $\Delta = 1 - \Omega_c / \Omega_u$. Thus, the initial axial velocity is related to the axial velocity in the regular region of the undulator as follows:

$$\beta_{\|,0}^2 + \beta_{\perp 0}^2 = \beta_\|^2 + \beta_{\perp 0}^2 + \beta_u^2 + 2\beta_u \beta_{\perp 0} \cos(\theta_c - h_u z). \quad (4)$$

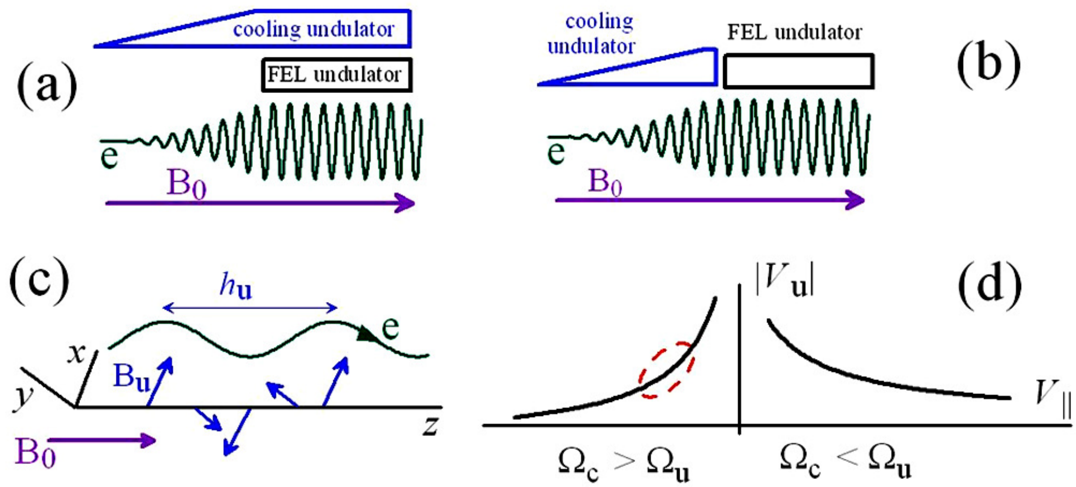


Figure 1: Schematics of non-radiative cyclotron-undulator cooling systems with the operating FEL undulator placed inside (a) and outside (b) of the cooling system; electron motion in the circular polarized undulator with the uniform axial field (c); dependence of the undulator velocity on the axial electron velocity (the optimal range is shown schematically) (d).

Let us introduce the function $\beta_\| = f(\beta_{\|,0})$ determined by Eqs. (3) and (4). Evidently,

$$\beta_\| \approx f(\bar{\beta}_{\|,0}) + (\beta_{\|,0} - \bar{\beta}_{\|,0}) f' + (\beta_{\|,0} - \bar{\beta}_{\|,0})^2 f''/2. \quad (5)$$

Having differentiated Eq. (5), one obtains that spread in $\beta_{\|}$ is minized, when $f' = 0$. This leads to the following simple condition:

$$K^2 \approx -\Delta^3. \quad (6)$$

In order to describe this spread, it is convenient to introduce the axial gamma-factor, $\gamma_\| = 1/\sqrt{1-\beta_\|^2}$, and use the spread $D(\gamma_\|) \approx \gamma^2 D(\beta_\|)$, where $D(\xi) = \left(\overline{(\xi - \bar{\xi})^2}\right)^{1/2} / \bar{\xi}$ denotes the relative dispersion. The uncompensated axial spread is determined by three factors. First, there is the initial spread in transverse velocity (4):

$$D_\perp(\gamma_\|) \approx \gamma_0^2 \beta_u \bar{\beta}_{\perp 0}. \quad (7)$$

Second, there is the spread in the transverse electron position. As the undulator field is not uniform, this spread induces the corresponding spread in the undulator velocity so that:

$$D_r(\gamma_\|) \approx \gamma_0^2 \beta_u^2 (r_e / \lambda_u)^2. \quad (8)$$

The third source of the uncompensated spread is the non-ideal transformation of the axial spread described by the non-zero $f''$ in Eq. (5):

$$D_\|(\gamma_\|) \approx [D_0(\gamma_\|)]^2 (1 + \Delta^{-1}). \quad (9)$$

Let us notice that an increase in the undulator parameter leads to the reduction in the Doppler frequency up-conversion factor $\sim \gamma_\|^2$ due to the increase in the transverse electron velocity. However, $\beta_u$ depends on $K$ weakly in the optimal cooling regime, $\beta_u \approx K^{1/3} / \gamma_0$. At the same time, the undulator factor is related by Eq. (6) to the mismatch between the cyclotron undulator frequencies, $|\Delta| \approx K^{2/3}$. Evidently, $K$ should be great enough to avoid the close-to-resonance situation, when it is difficult to provide the adiabatically smooth entrance into the undulator [6, 9]. In addition, in the case of $|\Delta| << 1$ the system is very critical to the initial spread.

Let us consider a 5 MeV electron bunch with the parameters typical for modern photo-injectors: energy spread $D_0(\gamma) \approx D_0(\gamma_\parallel) \sim 1\%$, normalized emittance $\varepsilon \approx \pi \bar{\beta}_{\perp 0} r_e \gamma \sim \pi$ mm mrad, and the bunch radius $r_e \sim 1$ mm. In the case of a cooling undulator with $\lambda_u = 5$ cm and $K = 0.2$, the condition (6) leads to $\Delta \approx 0.3$ and $\beta_u \approx 0.06$. In this case, the estimations (7)-(9) result in the similar uncompensated spreads in axial velocity so that $D_\perp(\gamma_\parallel) \approx 6\times10^{-4}$, $D_r(\gamma_\parallel) \approx 2\times10^{-4}$, $D_\parallel(\gamma_\parallel) \approx 4\times10^{-4}$.

Figures 2 and 3 illustrate the results of simulations of Eq. (1) for the cooling system with the adiabatic non-regular section $K(z) = 0.2(z/20\lambda_u)^2$. In these simulations, $\gamma = 10$, and there exist initial spreads in the axial velocity, and in the rotatory velocity. If $D_0(\gamma_\parallel)$ is great enough, then the output spread $D(\gamma_\parallel)$ is almost independent on $\bar{\beta}_{\perp 0}$ and determined by estimation (10) (Fig. 2). A decrease in $D_0(\gamma_\parallel)$ leads to a decrease in the output spread $D(\gamma_\parallel)$ down to a saturation value determined by the spread in the rotatory velocity $\bar{\beta}_{\perp 0}$ [estimations (8) and (9)]. In the ideal case, when $\bar{\beta}_{\perp 0} = 0$, $D(\gamma_\parallel)$ decreases with the decrease in $D_0(\gamma_\parallel)$ with no saturations (the lowest curve in Fig. 2).

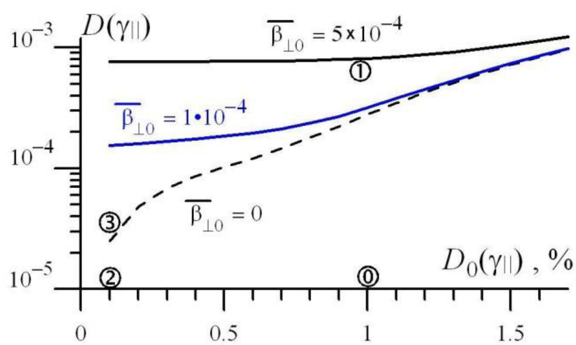


Figure 2: Output axial spread versus the initial spread at various initial spreads in the rotatory velocity $\bar{\beta}_{\perp 0}$.

Figure 3 illustrates the electron motion in the non-regular region of the cooling system. The increase in $\beta_\perp(z)$ leads to a decrease in the averaged axial velocity, so that $\delta\gamma_\parallel/\gamma \sim 10\%$. This is accompanied with the decrease in the axial velocity spread down to $D(\gamma_\parallel) = 3\times10^{-4}$. In this example, $D_0(\gamma_\parallel) = 1\%$ and $\bar{\beta}_{\perp 0} = 10^{-4}$ ($\varepsilon \sim \pi$ mm mrad). In the case of $\lambda_u = 5$ cm, the cooling system length is $L \approx 20\lambda_u \approx 1$ m. The non-relativistic cyclotron wavelength $\lambda_c = 5$ mm corresponds to the axial magnetic field $B_0 \approx 2$ T.

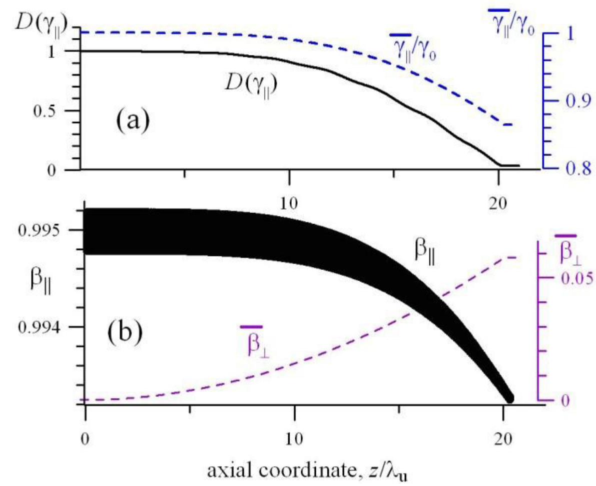


Figure 3: Averaged axial gamma-factor and spread in the axial gamma-factors versus the axial coordinate in the non-regular section of the undulator (a), evolution of the averaged transverse velocity and axial velocities of different electrons (b).

## CONCLUSION

In the suggested non-radiative cooling scheme ~1% velocity spread of 5 MeV electron beam (typical for photoinjectors) can be reduced to ~0.01% at distance as long as 20 undulator periods. The described principle works also for high energy beams (100 MeV and more), where RF undulator instead of DC-magnet undulator is necessary.

## REFERENCES

[1] J.G. Power, Proc. of 14th Advanced Accelerator Concept Workshop, Annapolis, USA, 2010, p.163.
[2] B. Dunham, J. Barley, A. Bartnik et al., Appl. Phys. Lett. 102, 034105 (2013).
[3] F. Stephan, C. H. Boulware, M. Krasilnikov et al., Phys. Rev. ST - AB 13, 020704 (2010).
[4] H.P.Freund, T.M.Antonsen, Principles of free-electron lasers. Chapman & Hall, London, 1996.
[5] Z. Huang and K.-J. Kim, Phys. Rev. ST AB 10, 034801 (2007).
[6] C. Bostedt, J.D. Bozek, P.H. Bucksbaum et al., J. of Phys. B 46 , 164003 (2013).
[7] J.L. Hirshfield and G.S. Park, Phys. Rev. Lett. 66, 2312 – Published 6 May 1991
[8] H. Deng and C. Feng, Phys. Rev. Lett. 111, 084801 (2013).
[9] N.S.Ginzburg, N.Yu.Peskov, Phys. Rev. ST – AB vol.16, 090701 (2013).




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