ROUGHNESS TOLERANCES IN THE UNDULATOR VACUUM CHAMBER OF LCLS-II*

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INTRODUCTION

When a bunch passes through the LCLS-II undulator, the wakefields induced in the vacuum chamber will add energy variation along the bunch, which can negatively impact the FEL performance. The wakefield of the vacuum chamber is primarily due to the resistance of the walls and the roughness of the surface. To minimize the impact of the wakes, one would like a wall surface smooth enough so that the roughness component of the wake is a small fraction of the total wake. In LCLS-I, with an undulator chamber of the same material (aluminum) and of roughly the same aperture, the roughness tolerance specified as an rms slope of the surface, $(y')_{rms}$ = 10–15 mr, was difficult to achieve [1]. The goal of the present study is to understand the consequences to LCLS-II of loosening the roughness specification by a factor of two to 30 mr.

The vacuum chamber within the undulator of LCLS-II will be primarily extruded aluminum with a racetrack cross-section: the aperture can be described as a rectangle of 12 mm by 5 mm (x by y), with added semicircular sides. (In addition, there are short breaks at the quads with a different shape and a larger aperture.) The impedance is essentially the same as for two parallel plates separated vertically by a distance of 5 mm. In this note we calculate the total wake effect of the resistive wall plus a model of roughness. The roughness model we use consists of small, shallow, sinusoidal corrugations [2]. We choose this model because measurements of samples of polished aluminum, similar to that to be used for the undulator chamber, find that the typical roughness is shallow [3]. Note that this model does not generate a so-called "synchronous mode" wake, which applies in the case of small *steep*, periodic corrugations [4, 5].

The calculation of the short-range wake of a resistive pipe has been done before [6], as has the case of a pipe with small, shallow corrugations [7]; in this report we properly combine the two effects. We first consider a round model of the chamber aperture, and then move to the flat geometry. Selected beam and machine properties in the undulator region of LCLS-II, that are used in our calculations, are given in Table I. The bunch charge, Q = 300 pC, is the largest charge envisioned. For the nominal case, Q = 100 pC, the current will also be uniform with I = 1 kA. More details can be found in Ref. [8].

ROUND VACUUM CHAMBER

Consider a round chamber of radius a, with wall resistance and small (in amplitude), shallow sinusoidal corrugations that represent the wall roughness. While in some cases the beam impedance can be calculated as a sum of the impedances due to resistance and that due to wall roughness, in general such summation of impedances is not correct. A more general approach is based on the concept of surface impedance [9] defined as the ratio of the longitudinal electric field and the azimuthal magnetic field at the wall, $\zeta = -(E_z/Z_0 H_\phi)|_{r=a}$, where Z_0 = 377 Ω. Denoting $\zeta_{rw}(k)$ as the wall resistive surface impedance and $\zeta_{ro}(k)$ as the surface impedance due to roughness we can write the beam impedance $Z(k)$ as

$$Z(k) = \frac{Z_0}{2\pi a}\left(\frac{1}{\zeta_{rw}(k)+\zeta_{ro}(k)} - \frac{ika}{2}\right)^{-1} , \quad (1)$$

with wave number $k = \omega/c$, where ω is frequency and c is speed of light. The resistive wall surface (ac) impedance $\zeta_{rw}(k)$, is given by [10]

$$\zeta_{rw}(k) = (1-i)\sqrt{\frac{k(1-ikc\tau_c)}{2Z_0\sigma_c}} , \quad (2)$$

with σ_c the dc conductivity and τ_c the relaxation time of the metallic walls. The roughness surface impedance [7]

$$\zeta_{ro}(k) = \frac{1}{4}kh^2\kappa^{3/2}\left(\frac{\sqrt{2k+\kappa} - i\sqrt{2k-\kappa}}{\sqrt{4k^2-\kappa^2}}\right) ; \quad (3)$$

here the wall profile radius r is assumed to vary sinusoidally with longitudinal position z: $r = h\cos\kappa z$. For the model to be valid we require the oscillations to be small and shallow, *i.e.* $\kappa a \ll 1$ and $h\kappa \ll 1$. Note that Eq. 1 implies that at low frequencies the two contributions to the impedance simply add: $Z(k) \approx (Z_0/2\pi a)[\zeta_{rw}(k) + \zeta_{ro}(k)]$; however, in general this is not true. Once the impedance is known, then the wake is obtained by the inverse Fourier transform:

$$W_\delta(s) = \frac{c}{2\pi}\int_{-\infty}^{\infty} Z(k)e^{-iks}dk , \quad (4)$$

with s the distance the test particle is behind the driving particle, and a positive value of W_δ indicates energy loss.

Table 1: Selected beam and machine properties in the undulator region of LCLS-II that are used in our calculations.

Parameter name	Value	Unit
Charge per bunch, Q	300	pC
Beam current, I	1	kA
Rms bunch length, σ_z	25	μm
Beam energy, E	4	GeV
Maximum beam power, P_b	120	kW
Vacuum chamber half aperture, a	2.5	mm
Vacuum chamber length, L	130	m

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Note that in Ref. [7] further practical considerations for such a calculation as a contour integral are discussed.

For the LCLS-II undulator vacuum chamber the dominant effect is expected to be the resistive wall wake, with the roughness corrugations contributing to a lesser degree. The strength of the resistive wall wake for a short bunch depends on the characteristic distance, $s_0 = (2a^2/Z_0\sigma_c)^{1/3}$, which represents a location near the first zero crossing of the point charge wake. For Al, $\sigma_c = 3.5\times10^7\ \Omega^{-1}\mathrm{m}^{-1}$ and $\tau_c = 8$ fs; with $a = 2.5$ mm, $s_0 = 9.8\ \mu$m. For very short bunches it is s_0 rather than $\sigma_c^{-1/2}$ that gives the scale of the strength of the wake in a bunch.

For the roughness model, the wake [2]

$$W_\delta(s) = -\frac{Z_0 c}{16\pi^{3/2}a}\frac{h^2\kappa^{3/2}}{s^{3/2}} = -\frac{c}{4\pi^{3/2}}\sqrt{\frac{Z_0}{(\sigma_c)_{ro}}}\frac{1}{s^{3/2}}\ , \tag{5}$$

with the overall minus sign in the expression indicating that the test particle gains energy from the leading particle. This is the same s dependence as for the long-range resistive wall wake, and in the second expression on the right we write the wake in terms of an equivalent roughness conductivity, $(\sigma_c)_{ro} = 16/(Z_0h^4\kappa^3)$. Then, in the same way as for the resistive wall wake, we obtain a characteristic roughness distance $(s_0)_{ro}$. Choosing $\lambda_{ro} = 2\pi/\kappa = 300\ \mu$m, $(y')_{rms} = h\kappa/\sqrt{2} = 30$ mr, we find that $(\sigma_c)_{ro} = 2.9\times10^8\ \Omega^{-1}\mathrm{m}^{-1}$ and $(s_0)_{ro} = 4.9\ \mu$m. We see that the characteristic distance for this level of roughness is about half that of the resistance.

We numerically performed the integral of Eq. 4, considering the effects of resistivity and roughness, with $(y')_{rms} = 30$ mr and $\lambda_{ro} = 300\ \mu$m. In Fig. 1 we present the point charge wake $W_\delta(s)$ for the case of a pipe with wall resistance (blue), roughness (red), and both resistance and roughness (yellow). We see that the total wake is dominated by the resistive component, but is not simply given by the sum of the two individual wakes. We further note that $W_\delta(0^+) = Z_0c/(\pi a^2) = 5.8$ MV/(nC m). The first zero-crossing of the wakes is near $s_0 = 9.8\ \mu$m, $(s_0)_{ro} = 4.9\ \mu$m, and $(s_0)_{tot} = 12\ \mu$m, respectively, where the combined effect is approximated by $(s_0)_{tot} = [\sigma_c^{-1/2} + (\sigma_c)_{ro}^{-1/2}]^{-2}$.

The bunch wake is given by the convolution

$$W_\lambda(s) = -\int_0^\infty W_\delta(s')\lambda(s-s')\,ds'\ , \tag{6}$$

with $\lambda(s)$ the longitudinal bunch distribution, and a negative value of W_λ indicates energy loss. In the undulator region of LCLS-II the bunch shape is roughly uniform. For a uniform distribution, with peak current I and length $l = 2\sqrt{3}\sigma_z$, the relative induced energy variation at the undulator end is

$$\delta_w(s) = -\frac{eIL}{cE}\int_0^s W_\delta(s')\,ds' \qquad [0 \le s \le \ell]\ , \tag{7}$$

with L the undulator pipe length and E the beam energy.

Numerical Tests

We performed test calculations for the round geometry with the finite difference, wakefield code ECHO [11]. This code can calculate the effects of both geometric and resistive wall (dc only) wakes (provided that the skin depth is small compared to the size of the wall perturbations). However, a sinusoidal wall oscillation as small as *e.g.* $(y')_{rms} = 30$ mr on an $a = 2.5$ mm pipe is difficult to simulate, so we artificially enlarged the oscillations and reduced the wall conductivity. We consider two cases: (1) roughness alone and (2) roughness plus wall resistance. Parameters are: $a = 2.5$ mm, $\lambda_{ro} = 2.5$ mm, $h = 60\ \mu$m, pipe length $L = 25$ cm, wall conductivity $\sigma_c = 6\times10^5\ \Omega^{-1}\mathrm{m}^{-1}$; so $s_0 = 37\ \mu$m and $(y')_{rms} = 110$ mr. The bunch is Gaussian with rms length $\sigma_z = 60\ \mu$m and the skin depth $\delta_s = 0.8\ \mu$m. The mesh size was taken to be 12 μm. For analytical comparison to the ECHO results, we inserted Eq. 1 into Eq. 4 to find the point charge wake. This function was convolved according to Eq. 6 to obtain the bunch wake. The results are shown in Fig. 2. The ECHO results are given by the solid curves, and the analytic results by dashes. We see good agreement.

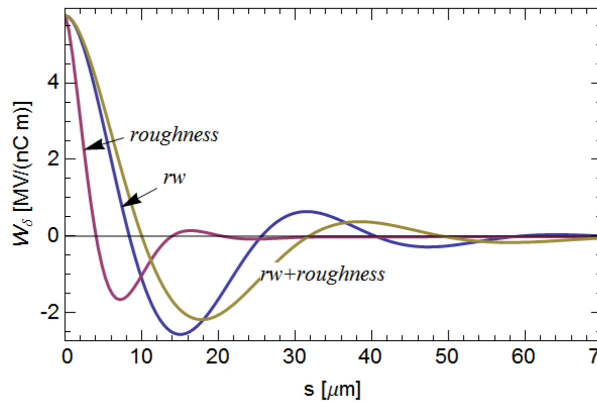

Figure 1: For round geometry, with radius $a = 2.5$ mm: point charge wake $W_\delta(s)$ for a pipe with resistance (blue), roughness (red), and both resistance and roughness (yellow). The roughness model assumes $(y')_{rms} = 30$ mr.

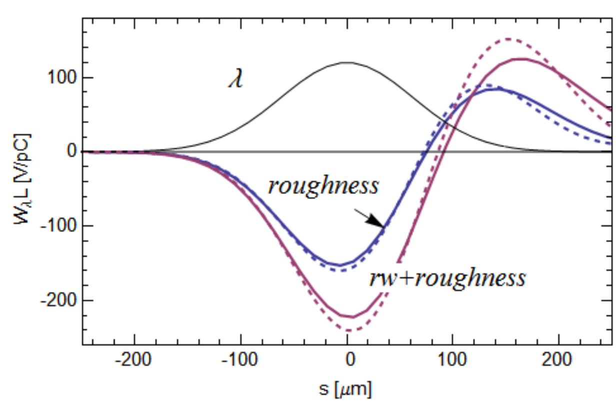

Figure 2: Bunch wake as obtained by ECHO (solid curves) and analytically (dashes) for test examples: (1) a lossless, corrugated pipe (blue), and (2) a lossy, corrugated pipe (red). The bunch shape λ is also shown, with the head to the left.

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FLAT VACUUM CHAMBER

Henke and Napoly give the impedance of a resistive wall in flat geometry in Ref. [12]. With a slight modification we include the effects of both the wall resistance and roughness:

$$Z(k) = \frac{Z_0}{2\pi a}\int_0^\infty dq\, \mathrm{sech}\, q \times \times\left(\frac{\cosh q}{\zeta_{rw}(k)+\zeta_{ro}(k)} - \frac{ika}{q}\sinh q\right)^{-1} . \quad (8)$$

We have repeated the previous calculations for flat geometry, for cases of aluminum with ac conductivity and roughness with $(y')_{rms}$ of: (1) 0 mr, (2) 15 mr, (3) 30 mr, and (4) 45 mr ($\lambda_{ro} = 300$ μm in all cases). The resulting point charge wakes are shown in Fig. 3 (top). We see that, compared to the round case, $W(0^+)$ is reduced by the factor $\pi^2/16$ and the first zero crossing of the wake is increased slightly.

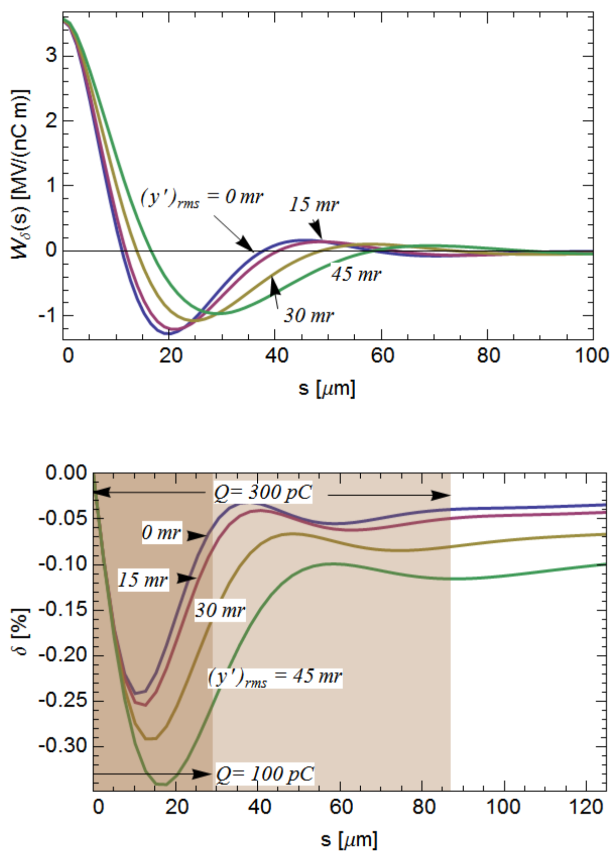

Figure 3: Flat geometry: $W_\delta(s)$ (top) and relative induced voltage for uniform bunch, $\delta_w(s)$ (bottom), considering resistance plus roughness, for $(y')_{rms} = 0$, 15, 30, 45 mr.

In Fig. 3 (bottom) we plot the relative induced energy variation for a uniform bunch distribution as obtained by Eq. 7. Here the peak current $I = 1$ kA; the bunch head is at $s = 0$ and the bunch tail at $s = 30$ (90) μm for the case $Q = 100$ (300) pC (the two cases are indicated by shading in the figure). The length of pipe is assumed to be $L = 130$ m, and the beam energy $E = 4$ GeV.

The wake effect can be quantified by the total induced relative energy variation, $\Delta\delta_w \equiv \max(\delta_w) - \min(\delta_w)$. For a resistive pipe with no roughness (for both 100 pC and 300 pC cases), $\Delta\delta_w = 0.25\%$. Adding roughness increases this value by 5%, 19%, 38%, when $(y')_{rms} = 15$, 30, 45 mr, respectively. Since $W_\delta(s)$ drops nearly linearly to zero near the effective s_0, we can make the simple approximation

$$\Delta\delta_w = \frac{\pi^2}{16}\frac{Z_0\bar{s}_0}{2\pi a^2}\frac{eIL}{E} , \quad (9)$$

where $\bar{s}_0 = s_0$ ($(s_0)_{tot}$) in the case of resistance only (resistance plus roughness). This approximation yields $\Delta\delta_w = 0.23\%$ for no roughness, and an increase in this value by 22% when roughness with $(y')_{rms} = 30$ mr is included, showing good agreement with the more accurate results.

The roughness effect depends on $(y')_{rms}$ and also on λ_{ro}, though the latter dependence is expected to be much weaker. Repeating the (more accurate) calculation for wall resistance plus roughness with $(y')_{rms} = 30$ mr, but taking $\lambda_{ro} = 900$ μm we find that the roughness increases $\Delta\delta_w$ by 27.5% compared to the effect of wall resistance alone. This confirms that the dependence of $\Delta\delta_w$ on λ_{ro} is weak. We have also repeated $\Delta\delta_w$ calculations for numerically obtained bunch shapes, for $Q = 100$ pC, 300 pC, [13] and obtained results very similar to those presented here.

Finally, for completeness, we calculate the wakefield-induced power loss in the undulator beam pipe: $P = -\langle W_\lambda\rangle Q^2 f_{rep}/L$, where $\langle\rangle$ indicates averaging over the bunch. We find that $P = 2.1$ (1.0) W/m for $Q = 100$ (300) pC, using the maximum planned repetition rate in the undulators, $f_{rep} = 300$ (100) kHz; in these cases 0.5 (0.25) W/m is the extra contribution due to the roughness.

In conclusion, we have shown that loosening the roughness tolerance by a factor of two, from an rms slope at the surface of $(y')_{rms} = 15$ mr to 30 mr, will increase the roughness contribution to the total induced voltage, $\Delta\delta_w$, from 5% to 20%, which is still small and should be acceptable.

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