LINEAR OPTICS MODELING IN THE CORNELL ERL INJECTOR

C. Gulliford, I. Bazarov, A. Kim, M. Forster, CLASSE, Cornell University, NY 14853 USA

Abstract

Commissioning of a new high brightness electron source for the Energy Recovery Linac (ERL) at Cornell University is currently underway. Despite the fact that the beam dynamics in this machine is space-charge dominated, a fundamental understanding of the linear optics is crucial in that it determines the effectiveness of space-charge emittance compensation methods, as well as provides the means for achieving various desired beam parameters such as bunch length and energy spread. Here we introduce a new numerical tool being used in commissioning of the injector that provides linear optics matrix calculation using field maps for various optical elements.

INTRODUCTION: THE ERL PHOTO-INJECTOR

In the photo-injector electron bunches created from a gallium arsenide photocathode are accelerated in the high voltage DC gun. The electron bunches then pass through a bunching RF cavity sandwiched between two solenoids before entering the cryomodule housing five superconducting two-cell RF cavities. These cavities are used to accelerate to beam energies between 5 and 15 MeV. The bunch then travels through a straight section consisting of four quadrupoles before reaching a dipole switch yard where the beam can be sent down three different diagnostic beam lines before being dumped. These diagnostic sections allow the measurement of several important beam characteristics such as beam energy, transverse phase space and emittance, and temporal profile of the beam. See [1] for a detailed description of the accelerator.

MODELING OF THE PHOTO-INJECTOR

The ongoing goal of this work is to develop a light weight computer simulation code that is easy to use in the control room, that is sufficiently modular so as to allow easy inclusion of new beam physics, has a working interface to the EPICS control system used to run the injector, and provides useful data to accelerator physicists. The approach taken so far has been to develop an object oriented code in C++ that provides a high level physics description of transport in the photo-injector. This code, called Numerical Transfer Matrix (NTMAT), provides the user with the ability to model optical elements such as axial symmetric electrostatic and standing wave RF cavities, solenoids, quadrupoles, and dipoles. NTMAT also allows the user to specify the position of other types of beam line elements such as beam position monitors (BPMs), view screens, and

steering correctors. The the full 6x6 transfer matrix starting from the photocathode to the positions of these elements is then calculated. The phase space variables used to describe particles in the accelerator are similar to those used in [2]:

$$u = \begin{pmatrix} x & x' & y & y' & \tilde{L} & \tilde{\gamma} \end{pmatrix},$$
$$\tilde{L} = c(t - t_{\text{ref}}), \qquad \tilde{\gamma} = \gamma - \gamma_{\text{ref}}.$$

Here c is the speed of light, and γ is the normalized energy $\gamma = E/mc^2$ of an electron.

The Numerical Transfer Matrix Code

The layout of the ERL photo-injector can naturally be divided into two sections. The first section consists of the high voltage DC gun, two solenoids, a buncher cavity, and five two-cell superconducting RF cavities. In several cases the electric and magnetic fields of these elements overlap each other. In order to properly model this effect, elements in this section of the beam line are described using on-axis field maps. This portion of the accelerator is thus described as the field map region (FMAP) in the code. The second section of the accelerator consists of several diagnostic beam lines. The relevant optical elements in this portion of the accelerator are comprised of dipoles and quadrupoles. For this portion of the accelerator elements can be described using essentially the conventional transfer matrices as given in [2]. This later portion of the injector is thus referred to as the transfer matrix (TMAT).

To represent elements such as solenoids, electrostatic cavities, and standing wave RF structures, NTMAT uses one dimensional on axis field maps. The transverse electric and magnetic field components are calculated using the first order off axis expansion of the fields. For static fields this is given by

$$\begin{aligned} E_{x,y} &= -\frac{x,y}{2}\frac{dE_z}{dz}(r=0), \\ B_{x,y} &= -\frac{x,y}{2}\frac{dB_z}{dz}(r=0). \end{aligned}$$

For purely magnetic or electric field map elements the user must provide an evenly spaced field map of $B_z(r=0)$ or $E_z(r=0)$ respectively. For RF elements the electric field profile along z, $\varepsilon_z(r=0)$, must be provided. NTMAT assumes a time dependence for RF cavities of the form

$$E_z(r=0,t) = \varepsilon_z(r=0)\cos\phi,$$

where $\phi = \omega t + \phi_0$. The off axis expansion in this case is given by

$$E_{x,y} = -\frac{x,y}{2}\frac{d\varepsilon_z}{dz}\cos\phi,$$

$$B_x = \frac{y\omega}{2c^2}\varepsilon_z \sin\phi, \qquad B_y = -\frac{x\omega}{2c^2}\varepsilon_z \sin\phi.$$

Currently the derivatives of these field maps are calculated using linear interpolation between field map points. Other methods for calculating these derivatives can be easily incorporated into the code as desired.

With these field maps the first order relativistically correct equations of motion for the six phase space variables are [3]:

$$x'' + \frac{\gamma_0' x'}{\beta_0^2 \gamma_0} + \frac{2\omega_L y'}{\beta_0 c} + \left(\frac{\omega_L'}{\beta_0 c} + \frac{\gamma_0' \omega_L}{\beta_0 \gamma_0 c}\right) y$$
$$+ \left(\frac{\gamma_0''}{2\beta_0^2 \gamma_0} - \frac{\omega}{2\gamma_0 \beta_0 c}\frac{e\varepsilon_z}{mc^2}\sin\phi\right) x = 0,$$

$$y'' + \frac{\gamma_0' y'}{\beta_0^2 \gamma_0} - \frac{2\omega_L x'}{\beta_0 c} - \left(\frac{\omega_L'}{\beta_0 c} + \frac{\gamma_0' \omega_L}{\beta_0 \gamma_0 c}\right) x$$
$$+ \left(\frac{\gamma_0''}{2\beta_0^2 \gamma_0} - \frac{\omega}{2\gamma_0 \beta_0 c}\frac{e\varepsilon_z}{mc^2}\sin\phi\right) y = 0,$$

$$\tilde{\gamma}' + \frac{\omega}{c}\frac{e\varepsilon_z}{mc^2}\sin\phi\tilde{L} = 0, \qquad \tilde{L}' + \frac{\tilde{\gamma}}{p_0^3} = 0.$$

In this expression the quantity ω_L is the Larmor frequency. Primes denote differentiation with respect to position along the beam line, while the zero subscript denotes that this quantity is evaluated for the reference particle. The quantity $p_0 = \beta_0 \gamma_0$ is the normalized momentum of the reference particle. It is important to distinguish between the quantities ω_L and ω, the former being the Larmor frequency and the later the angular frequency of the standing wave RF cells. The term γ_0', the Larmour frequency ω_L, and their derivatives are calculated from the electric and magnetic fields directly. It is noted that although not taken into consideration here, the addition of other terms such as quadrupole fields can be readily included in these equations. The transfer matrix for a small step $z \to z + dz$ is then calculated by first solving these equations of motion for a small step dz and then computing $M_{ij}(z \to z+dz) \cong \partial u_i(z+dz)/\partial u_j(z)$. Figure-1 shows the results of tracking using the transfer matrices calculated with this method. Here a particle starting with an offset in x of 1 mm and an offset in y of 0.5 mm is tracked through the injector using the transfer matrices computed by NTMAT. The results are compared with those of Astra 3.0 [4].

Replacing the longitudinal phase space variables $L = l(z) - l_0(z)$ and $\delta = \Delta p/p_0$ with the canonical variables $\tilde{L} = c(t - t_0)$ and $\tilde{\gamma} = \gamma - \gamma_0$ requires modifying several of the transfer matrix elements given in [2] for dipoles and quadrupoles. Since the energy is constant in this section of the accelerator the replacement of δ with $\tilde{\gamma}$ allows the computation of the sixth column of the transfer matrix using:

$$\frac{\partial u_f}{\partial \tilde{\gamma}} = \frac{\partial u_f}{\partial(\Delta p/p_0)}\frac{\partial(\Delta p/p_0)}{\partial \tilde{\gamma}} = \frac{\gamma_0}{p_0^2} \cdot M_{\text{transport}}, \quad (1)$$
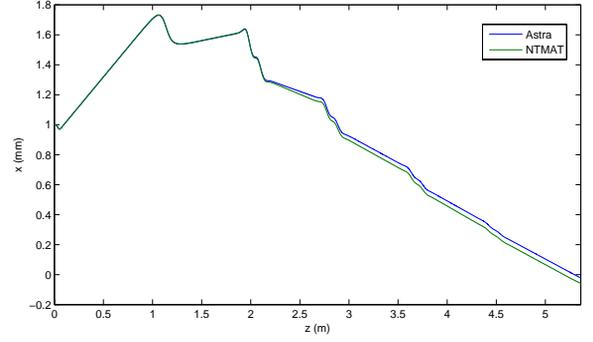

Figure 1: Tracking of an offset particle using Astra 3.0 and NTMAT. The initial kinetic energy is 10 eV, and the integration step size is 1 mm.

where $M_{\text{transport}}$ is the corresponding matrix element from [2]. The matrix elements in the fifth row can be found by expanding the off axis path length l to first order. This gives the first order expansion of $\tilde{L}$:

$$\tilde{L} \cong \tilde{L}_i + \sin(\kappa s)\frac{x_i}{\beta_0} + \frac{1}{\kappa}(1 - \cos(\kappa s))\frac{x_i'}{\beta_0}$$
$$+ \left[\frac{\gamma_0}{p_0^2\beta_0\kappa}(\kappa s - \sin(\kappa s)) - \frac{s}{p_0^3}\right]\tilde{\gamma}_i.$$

The term κ is the dipole strength if present. With this equation and (1), the longitudinal portions of the transfer matrices using the modified phase space variables can be found.

It is apparent in the equations of motion and their first order solutions discussed above that a reference particle trajectory must be given in order to calculate the transfer matrices of the machine. The equations of motion for an on-axis particle in the FMAP region are

$$\gamma'(z) = \frac{e}{mc^2}E_z(z,t),$$
$$t'(z) = \frac{1}{c} \cdot \frac{1}{\sqrt{1 - 1/\gamma^2}}.$$

These equations can be integrated using one of several standard techniques [5], or using an approximate analytic method. This method assumes that the field does not vary over one integration step Δz. The analytic solution for the change in time over this step is then given by

$$\Delta t = \frac{1}{c\gamma'}\left[\sinh\left(\cosh^{-1}\gamma(z+\Delta z)\right) - \sinh\left(\cosh^{-1}\gamma(z)\right)\right].$$

NTMAT has been designed to be implemented together with a Matlab toolkit which provides the user with the ability to interface with the actual photo-injector. NTMAT additionally allows phasing of RF cavities, as well as multiple parameter scans. The user can also track an offset particle using the transfer matrices computed by the code. The default output of NTMAT is comprised of the reference trajectory, and the transfer matrices from the photocathode to all user defined output points. The results of the auto-phasing, and parameter scans can also be output as well.

Beam Dynamics and Electromagnetic Fields

Matlab Interface to the Control System

The current version of the Matlab toolkit used in conjunction with NTMAT includes several scripts which provide useful abilities to beam experimenters. These include the capability to load and save different current, voltage, and phase configurations for elements in the accelerator, compute the response matrix from any set of correctors to any set of BPMs using the transfer matrices from NTMAT, and determine the transfer matrix between any two user defined points in the beam line. The user also has the ability to perform parameter scans across a range of values for a given set of desired scan parameters and respective beam line elements. The Matlab scripts are functionally basic enough to allow the user to include them as building blocks for more complicated scripts, and allow for control of the injector through EPICS.

ONGOING WORK

The initial package containing NTMAT and the Matlab toolkit has been installed in the injector control room. A number of verifying measurements is underway, which includes measurement of the response matrix of the injector and comparing it to the simulation output. Subsequent work will consist of beam based alignment in optical elements such as solenoids and RF cavities, orbit correction and feedback, time of flight measurements, and emittance measurements. In addition to this, NTMAT will be upgraded with new physics such as space charge and other non-linear effects necessary to explain the beam dynamics in the accelerator.

Work on the software is proceeding in three directions. As mentioned above, modifications to NTMAT will be developed in the lab with possible integration with the Unified Accelerator Library (UAL) [6]. The primary goal of work in the near future will involve the inclusion of space charge forces as the beam dynamics in the injector is space charge dominated. This will be done by upgrading NTMAT to include the transport of second order moments starting from an initial beam distribution. The effects of the space charge forces on the bunch are then linear if the bunch is assumed to have elliptical symmetry and the boundary and emittance of the bunch are defined by rms values [7]. Modification of the equations of motion to include these space charge forces will then allow the calculation of the transfer matrices and subsequent propagation of the beam envelope under the influence of both external and self forces. With this, characterization of the space charge effects on bunch length and energy spread will be explored. Following this work, beam envelope treatment of space charge in the presence of dispersion will also be added [8]. In addition, there are immediate plans to include ion neutralization effects on the beam which are thought to be of significant operational importance for the injector.

There is also ongoing work to implement the NTMAT method into the XAL [9]. The XAL framework provides a complete object-oriented description based upon the Element-Algorithm-Probe methodology in [6]. New elements are being created for XAL which include the ability to use on-axis field maps. The NTMAT method for calculation of the transfer matrices through these elements is being implemented as an algorithm for propagating existing XAL probe types. When complete this work will take advantage of XAL's EPICS interface and many existing control system applications. The simulation effort undertaken is closely coupled with our experimental program as part of the current commissioning effort, and should allow important insights into the beam physics of the Cornell ERL injector as it operates in a new range of beam parameters.

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