# ORBIT RESPONSE MATRIX MEASUREMENTS IN THE LOS ALAMOS PROTON STORAGE RING* 

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## Abstract

Orbit response matrix (ORM) techniques have been used in numerous electron storage rings to elucidate various optical properties of the machines. Such measurements in a long-pulse accumulator ring have unique complications. Presented here are the techniques and results of such a measurement at the Los Alamos Proton Storage Ring (PSR). Also shown here are the deficiencies in previous models of the ring and a comparison of the beta-functions as fit by the orbit response method to direct measurements by quadrupole magnet variations.

## INTRODUCTION

An orbit response is the change in the closed orbit (CO) due to a dipole error or kick, see eq. 1 , where $s$ is the longitudinal coordinate with a dipole error of strength $\theta$ at $\mathrm{s}_{0}$ and positive phase advance $\mu$ between $\mathrm{s}_{0}$ and s [1].

$$
\begin{align*}
& \Delta x_{c o}(s)=\frac{\sqrt{\beta(s) \beta\left(s_{0}\right)}}{2 \sin \pi \nu} \cos \left(\pi v-\mu_{s_{0} \rightarrow s}\right) \theta\left(s_{0}\right)  \tag{1}\\
& \Delta x_{c o}(s)=G\left(s, s_{0}\right) \theta\left(s_{0}\right)
\end{align*}
$$

The orbit response is linearly related to the dipole kick by the Green's Function solution to Hill's Eq. for a dipole error. By making several dipole corrector kicks and measuring the corresponding CO differences a system of equations may be formed, where because it is a Green's Function, R is the unique ORM for the machine.

$$
\begin{equation*}
\Delta \vec{x}_{B P M}=R \bullet \vec{\theta}_{\text {Corrector }} \tag{2}
\end{equation*}
$$

One can perform the same experiment in a model and compare the resulting measured and model ORM, which is a function of several model parameters. Because the ORM is unique, if the model properly describes the real machine, the model and measured ORM will be the same. If model and measured ORMs do not agree, one can iteratively minimize the difference by changing the model parameters i.e. gradients, rolls, positions, currents... LOCO [2] was used in the analysis to iteratively minimize the $\chi^{2}$ between model and measured ORMs.

## ORBIT RESPONSE MATRIX MEASUREMENT

The horizontal and vertical COs are measured at each of the 18 beam position monitors (BPMs) in PSR, but due

[^0]to the configuration of the electronics, only one BPM can process data at a time. So the closed orbit at each BPM is measured with a different beam pulse. The BPMs are tuned to the linac induced 201.25 MHz micro-pulse structure of the beam bunch. This longitudinal frequency structure is quickly washed out due to the energy spread of the beam, only allowing ~25-30 turns of beam position data to be measured at production injection offsets, 17 mm in the vertical. The turn-by-turn beam position data is fit to a cosine wave, and the $\mathrm{CO}, \mathrm{O}_{\mathrm{ffset}}$, is extracted.
\[

$$
\begin{equation*}
x_{n}=A \cos (2 \pi v n+\phi)+O_{f f s e t} \tag{3}
\end{equation*}
$$

\]

For this experiment, beam was injected nearly on axis, -.7 mm horizontal and 2 mm vertical. This allowed for 40 turns of BPM data to be captured before the 201.25 Mhz washed out. This also kept all beam positions within the linear measurement region of the BPMs.

For the ORM measurement the 11 horizontal bending dipoles and the 9 vertical correctors in PSR (labeled correctors $1-11 \mathrm{x}$ and $12-20 \mathrm{y}$ ) were used to kick the beam and the orbit response was measured at 17 horizontal and 18 vertical BPMs (labeled BPMs 1-17x and 18-35y). (One of the horizontal BPMs was too unreliable to use in the ORM analysis). Kicks to the beam were such that the maximum change in the CO was $\sim 4 \mathrm{~mm}$. This kick was chosen because it was large enough to give a significant change in the CO everywhere, but not large enough to bring the beam into the nonlinear measurement region of the BPMs. Three kicks were made for each corrector, baseline or no kick, plus, and minus. Ten CO measurements were taken for each kick, and the results were averaged. The average baseline CO was then subtracted from the average plus and minus COs to get the orbit response. Then the two orbit responses, plus and minus, were subtracted from each other, cancelling any systematic errors such as magnet drift, to make a column in the ORM.

## LOCO RESULTS

The initial model did not accurately predict the vertical tune, differing from measured by .05 , see table 1 . This led to the belief that the model differed from the real machine in one of the 20 quadrupole gradients which were the only model parameters varied in the LOCO routine. LOCO also varied the corrector strengths and BPM gains in its analysis. Coupling was ignored. A bi-way dispersion measurement was made to help constrain the LOCO fit, but it was found a better $\chi^{2}$ could be achieved if it was not included in the analysis. This is because the dispersion function given by the initial model did not agree at all with the measured dispersion function. The initial $\chi^{2} /$ DOF
fit was 267.28 , see fig. 1 . Notice the large deviations only occur in the $x-x$ and $y$-y quadrants in the orbit response where kickers influence BPMs of the same direction. The coupling quadrants, $x-y$ and $y-x$, differ only in BPM noise showing no coupling, which was not fit in the LOCO analysis. The LOCO routine converged in six iterations, yielding a final model to measured ORM $\chi^{2} /$ DOF fit of 11.2 , see fig. 2. The mean difference in the resulting model and measured orbit responses is about half the BPM CO measurement error.
While no single quadrupole stood out as bad in the ORM analysis, the LOCO fit indicated an average 3\% decrease in the defocusing gradients, $\sim 10 \mathrm{~A}$. Magnet current read back and field measurements were made to find the source of the $3 \%$ difference but none was found. The corrector strengths and BPMs gains were not significantly modified during the LOCO analysis.


Figure 1: Initial model minus measured orbit response, $\chi^{2} /$ DOF 267.28.


Figure 2: LOCO fit model minus measured orbit response, $\chi^{2} /$ DOF 11.2. Aside from a few outliers, all other $x-x$ and $y$ - $y$ quadrant differences have the same magnitude as the BPM error noise in the $x-y$ and $y-x$ coupling quadrants.

## BETA FUNCTION MEASUREMENT

Immediately following the ORM measurement, a beta function measurement was performed using the quadrupole perturbation method. Twenty sets of BPM measurements were taken at each of four shunt-current
values for each quad. As before, the turn-by-turn BPM data was fit to a cosine wave, but this time the fractional tune instead of the CO was extracted from the fit, see eq. 3. The average beta function in the quad being shunted is related to the slope of the tune with respect to the gradient length.

$$
\begin{equation*}
\langle\beta\rangle=4 \pi \frac{\Delta v}{\Delta K l} \tag{4}
\end{equation*}
$$

Measuring the tune by fitting the cosine wave to the turn-by-turn BPM data yields a very good tune measurement with rms spreads of $4 \times 10^{-4}$ in the horizontal and $3 \times 10^{-4}$ for the vertical. The four shunted gradient lengths and their corresponding fitted tune values were fit to a line yielding a $\chi^{2} / \mathrm{DOF}$ of a few $10^{-8}$ or better and the slope of the fitted line was used for the beta function calculation. The resulting measurement error was $2.5 \%$ for the large beta functions.

## MODEL COMPARISON

After the LOCO analysis was performed on the ORM data, the new model with the LOCO fitted parameters needed to be verified. As a first test the model should give the proper tunes, see table 1. The LOCO fitted model has better tune predictions than the initial model.
Table 1: The model was not able to predict the vertical tune, but after the ORM analysis, the model using the LOCO fitted parameters gives the correct vertical tune.

| Comparison of Fractional Tunes |  |  |  |
| :--- | :--- | :--- | :--- |
|  | Measured | Model | LOCO Fit |
| Horizontal | .19143 | .2123 | .1825 |
| Vertical | .19794 | .2553 | .1971 |



Figure 3: The LOCO fit only slightly modifies the vertical beta functions from the initial model. However, the horizontal beta functions are worse than those given by the initial model when compared to the measurement.

The second test used to verify the LOCO fitted model was the beta function measurement. Figure 3 shows the comparison of the beta functions given by the initial
model, the LOCO fitted model, and the measured beta functions. The horizontal beta functions given by the LOCO fitted model greatly deviate from the measured horizontal beta function. Overall the initial model gives beta functions closer to those measured. This suggests that the model given by the LOCO fit is not an improvement. It also suggests that the initial model, which is the starting place in the LOCO analysis, is incomplete.

## COMPLICATIONS OF CLOSED ORBIT AND ORM MEASUREMENT

The Los Alamos PSR is an accumulator ring where beam is accumulated for $625 \mu$ s and then extracted. The short storage time is a complication in the ORM analysis because many different beam pulses must be used during the ORM measurement compared to the single beam pulse that is stored for several hours in the electron storage rings where ORM and LOCO have been very successful. The measurement is further hindered because data at only one of the 18 BPMs in PSR can be processed per pulse. Thus 18 different pulses are needed for a single closed orbit measurement.
Each beam pulse has a slightly different central momentum. The rms spread of the pulse-to-pulse central momentum variation is $\sim .004 \%$ which is $\sim 10$ times smaller than the momentum spread of a single beam pulse, $\sim .03 \%$. However, the pulse-to-pulse central momentum variation has a great effect on the rms spread of the horizontal CO measurement. The intrinsic error of the BPM CO measurement as calculated from a Maximum Likelihood Analysis of the cosine wave fit to the turn-by-turn BPM data and observed in the rms spread of the vertical CO measurement indicate a measurement error of $\sim .02 \mathrm{~mm}$. This is contrasted with the rms CO spread of the horizontal measurement, $\sim .15 \mathrm{~mm}$. Thus the horizontal CO measurement is made with 18 different beam bunches of varying central momentum with different horizontal COs which causes the measurement spread to be $\sim 10$ worse than the vertical.
The $\chi^{2} / \mathrm{DOF}$ is used as a figure of merit in the LOCO fit to describe the difference between the measured and LOCO fitted orbit responses. However, the $\chi^{2}$ is greatly influenced by the BPM CO measurement error which is about $\sim 10$ times greater in the horizontal due to the pulse-to-pulse central momentum variation. This heavily weights the vertical part of the ORM in the figure of merit.

More in depth analysis of the BPM data has revealed several data acquisition errors where the turn-by-turn position data does not resemble a sinusoidal wave. These errors yield bad CO and tune measurements, which if not removed from the data set can greatly affect an ORM analysis. In a CO measurement study, it was seen that $48 \%$ of the measurements had an error. One horizontal BPM was so inconsistent that it was not used in the ORM analysis.

The BPMs are sensitive to the linac frequency, 201.25 MHz , which is quickly washed out in PSR due to
energy spread in the beam, limiting CO measurements to only 40 turns of BPM data. The measurement would improve if more turns of BPM data could be obtained.

When the CO is changed due to a dipole kick in the ORM measurement, the injection offset is also changed because the CO at the injection is changed, a problem not experienced in electron storage rings because they only use one beam bunch for their ORM experiment. This can lead to large amplitude betatron oscillations about the CO, which will wash out the 201.25 MHz frequency structure more quickly. It has also been shown that the larger amplitude oscillation does not fit a cosine wave as well as the smaller amplitude data even though the relative error on the BPM position measurement is less with a larger amplitude oscillation; however the absolute error for large amplitude oscillations is greater.

## CONCLUSIONS

An ORM measurement was performed for PSR, and the result was analyzed using LOCO. The LOCO fitted model was tested against measured tunes and beta functions.

The ORM data was complicated because 18 beam pulses are needed to measure one CO. It was also seen that about half of the measured COs had data acquisition errors. The LOCO fitted model suggested a $3 \%$ decrease in the defocusing quadrupole gradients. This discrepancy was sought by measuring magnet current read backs and magnetic fields in the quadrupole but was not found. Although the LOCO fitted model better reproduced the measured tunes compared to the initial model, it did not fit the measured beta functions as well. For these reasons, the LOCO fitted model was not verified as an improved model. Instead, it has led to the belief that the initial model is incomplete.

The circulating beam is influenced by the fringe fields of the two extraction septum magnets. These fringe fields should be included as multipoles in the model before the LOCO analysis is preformed again. Beam based multipole measurements of the extraction septa fringe fields were made but compromised due to bad BPM data acquisition.

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