ERRORS IN BEAM EMITTANCE MEASUREMENT IN A TRANSPORT CHANNEL*

Y. K. Batygin, NSCL, Michigan State University, East Lansing, MI 48824, USA
M. Woodley, SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA

Abstract

Determination of exact values of beam emittance is important for practical applications including future linear collider. Beam emittance measurements technique is based on measurement of beam sizes at several beam profile stations in a quadrupole channel shifted between each other by a specific value of phase advance of betatron oscillations. Four-dimensional beam emittance measurements require determination of ten values of the beam sigma-matrix, while two-dimensional beam emittance measurements scheme requires determination of six values of sigma-matrix. Measurement procedure is sensitive to variation of beam sizes at the beam profile stations, which might result in unstable determination of beam emittance [1]. Paper discusses errors of beam emittance measurements as a function of errors in beam size measurement. Regions of stable and unstable beam emittance measurements are determined.

ANALYTICAL TREATMENT OF BEAM EMITTANCE ERROR

Consider 2D beam emittance measurement model for the beam propagating in FODO channel. Single particle transformation matrix is

$$\begin{vmatrix} x \\ x' \end{vmatrix} = \begin{vmatrix} C & S \\ C' & S' \end{vmatrix} \begin{vmatrix} x_o \\ x'_o \end{vmatrix} , \qquad (1)$$

where $C = \cos\mu + \alpha \sin\mu$, $S = \beta \sin\mu$, $C' = -\gamma \sin\mu$, $S' = \cos\mu - \alpha \sin\mu$ are matrix elements, α, β, γ are Twiss parameters, and μ is the phase advance of transverse particle oscillations between x_0 and x. Transformation of Twiss parameters is given by:

$$\begin{vmatrix} \beta \\ \alpha \\ \gamma \end{vmatrix} = \begin{vmatrix} C^2 & -2CS & S^2 \\ -CC' & SC'+S'C & -SS' \\ C'^2 & -2S'C' & S'^2 \end{vmatrix} \begin{vmatrix} \beta_o \\ \alpha_o \\ \gamma_o \end{vmatrix} . \qquad (2)$$

Measurements are provided for beam sizes at different locations. Beam size R is connected with beam emittance ε via equation $\beta\varepsilon = R^2$. We select the equation for Twiss parameter β in Eq. (2) at three different locations to determine unknown beam parameters α_o, β_o, γ_o. After multiplying them by the value of beam emittance, ε, we have

$$\begin{vmatrix} R_1^2 \\ R_2^2 \\ R_3^2 \end{vmatrix} = \begin{vmatrix} C_1^2 & -2C_1S_1 & S_1^2 \\ C_2^2 & -2C_2S_2 & S_2^2 \\ C_3^2 & -2C_3S_3 & S_3^2 \end{vmatrix} \begin{vmatrix} \beta_o\varepsilon \\ \alpha_o\varepsilon \\ \gamma_o\varepsilon \end{vmatrix} . \qquad (3)$$

Solution of Eq. (3) can be written as

$$\alpha_o\varepsilon = [C_3^2(R_2^2S_1^2 - R_1^2S_2^2) + C_1^2(R_3^2S_2^2 - R_2^2S_3^2) + C_2^2(R_1^2S_3^2 - R_3^2S_1^2)]/2d , \qquad (4)$$

$$\beta_o\varepsilon = [-R_3^2S_1S_2(C_2S_1 - C_1S_2) + C_3S_3(R_2^2S_1^2 - R_1^2S_2^2) - C_3^2(C_1S_1R_2^2 - C_2S_2R_1^2)]/d , \qquad (5)$$

$$\gamma_o\varepsilon = [R_1^2C_2S_3(C_2S_3 - C_3S_2) + C_1S_1(R_2^2C_3^2 - R_3^2C_2^2) + C_1^2(C_2S_2R_3^2 - C_3S_3R_2^2)]/d , \qquad (6)$$

$$d = (C_2S_1 - C_1S_2)(C_3S_1 - C_1S_3)(C_2S_3 - C_3S_2) . \qquad (7)$$

Beam emittance is determined by

$$\varepsilon = \sqrt{(\beta_o\varepsilon)(\gamma_o\varepsilon) - (\alpha_o\varepsilon)^2} . \qquad (8)$$

Beam size measurements usually are performed with certain error. Differentiation of the equation (8) over beam radiuses gives an error in beam emittance determination as a function of errors in beam sizes:

$$\frac{d\varepsilon}{\varepsilon} = -2(C_2S_1 - C_1S_2)^2(C_3S_1 - C_1S_3)^2(C_3S_2 - C_2S_3)^2$$
$$[-\frac{(C_1^2R_2^2 - C_2^2R_1^2)(C_1^2R_2^2\frac{dR_2}{R_2} - C_2^2R_1^2\frac{dR_1}{R_1})}{C_1^2C_2^2(C_2S_1 - C_1S_2)^2} - \frac{(C_1^2R_3^2 - C_3^2R_1^2)(C_1^2R_3^2\frac{dR_3}{R_3} - C_3^2R_1^2\frac{dR_1}{R_1})}{C_1^2C_3^2(C_3S_1 - C_1S_3)^2} +$$
$$+\frac{(C_2^2R_3^2 - C_3^2R_2^2)(C_2^2R_3^2\frac{dR_3}{R_3} - C_3^2R_2^2\frac{dR_2}{R_2})}{C_2^2C_3^2(C_3S_2 - C_2S_3)^2} + \frac{2C_2(C_3^4R_1^4\frac{dR_1}{R_1} - C_1^4R_3^4\frac{dR_3}{R_3})}{C_1^2C_3^3(C_2S_1 - C_1S_2)(C_3S_1 - C_1S_3)} +$$
$$+\frac{2C_1(C_3^4R_2^4\frac{dR_2}{R_2} - C_2^4R_3^4\frac{dR_3}{R_3})}{C_2^2C_3^3(C_2S_1 - C_1S_2)(C_2S_3 - C_3S_2)}]/D , \qquad (9)$$

where denominator of equation (9) is:

*Work supported by DOE Contract DE-AC02-76SF00515
#batygin@nscl.msu.edu

$$D = G^4[(\frac{A+B}{G})^2 - 1]\,[(\frac{B-A}{G})^2 - 1]\,, \qquad (10)$$

and the following notations are used:

$$A = R_1(C_2S_3 - C_3S_2)\,,\; B = R_2(C_3S_1 - C_1S_3)\,, \qquad (11)$$

$$G = R_3(C_2S_1 - C_1S_2)\,. \qquad (12)$$

Large error in beam emittance is expected if the denominator, Eq. (10), is close to zero, or

$$a = \frac{A+B}{G} \approx 1\,, \qquad b = \frac{B-A}{G} \approx 1\,. \qquad (13)$$

Consider FODO structure with parameters presented in Table 1. Assume that measured beam sizes are spread as:

$$R_1 = R_1^{(o)}(1+f)\,,\; R_2 = R_2^{(o)}(1+g)\,,\;\; R_3 = R_3^{(o)}(1+h)\,, \qquad (14)$$

where $R_1^{(0)}$, $R_2^{(0)}$, $R_3^{(0)}$ are unperturbed values of measured beam sizes, and *f, g, h* are errors uniformly distributed within interval [-δ, δ].

Table1: Parameters of FODO Structure

Beam energy	5 GeV
Quadrupole length	0.3 m
FODO period	8.8 m
Lens gradient	17.8 T/m

Figures 1, 2 illustrate possible stable and unstable beam emittance measurement setups in the FODO structure with phase advance per period of $\mu = 90^{o}$. In the first case, measurement stations are separated by phase shifts of 45°, 45°, 45°. In the second case, they are separated by phase shifts of 45°, 45°, 135°. In both cases, measured beam radiuses were supposed to be varied randomly within ± 5% of their absolute values (σ_R/R = 2.88%). In stable measurement setup, the values of parameters a = 0.17, b = 5.78 are far from unity, and the error in determined beam emittance is approximately ± 10% ($\sigma_\varepsilon/\varepsilon$= 4.3%). In unstable measurement setup, the value of parameter a = 0.85 is close to unity, parameter b = 2.63, and the error in beam emittance is significantly larger.

EMITANCE ERROR AS A FUNCTION OF LATTICE PHASE ADVANCE

Most of measurement setups utilize beam measurement stations placed in FODO channel shifted by a fixed value of phase advance of transverse oscillations per period. Error in beam emittance determination depends on the value of phase advance. To study this dependence, simulations were performed in FODO structure with parameters presented in Table 2 (see Fig. 3).

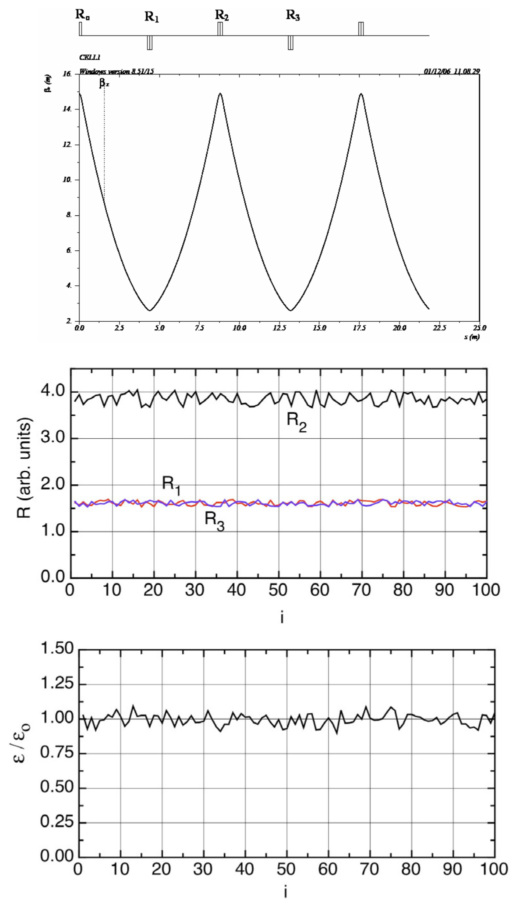

Figure 1: Stable beam emittance measurement setup.

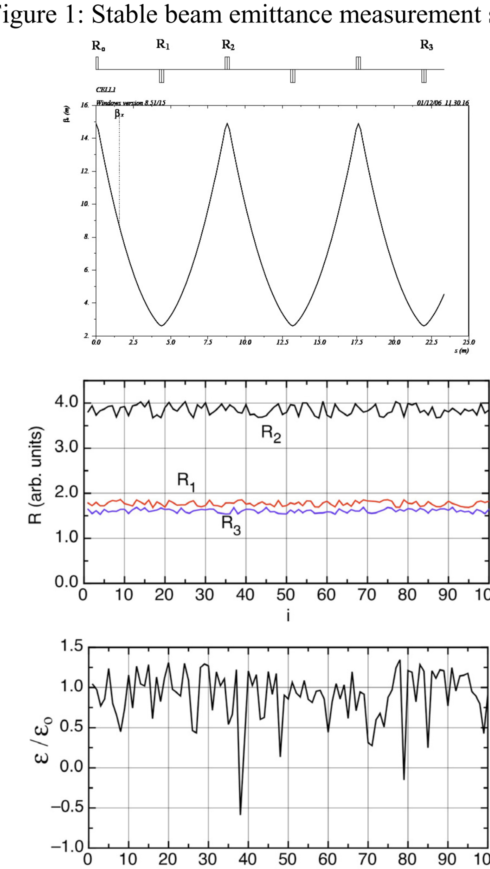

Figure 2: Unstable beam emittance measurement setup.

Table 2: Parameters of FODO Structure for Study Minimization of Beam Emittance Error vs. Phase Advance

Beam energy	250 GeV
Quadrupole length	1.0 m
FODO period	27.0 m
Lens gradient (36 degree / cell)	39.2 T/m
Lens gradient (45 degree / cell)	48.5 T/m
Lens gradient (60 degree / cell	63.4 T/m

Simulations were done for three different setups with different number of wire scanners $N_w = 3, 4, 5$. Each wire scanner was placed 0.5 m upstream of defocusing quadrupole in every FODO cell. For each combination of number of scanners N_w and FODO phase advance per cell value, 1000 emittance measurements were done with 5% (Gaussian) beam size measurement error. Input beam was supposed to be perfectly matched with the structure, and perfect knowledge of the wire-to-wire matrices was provided. In each case the following linear system was solved:

$$R_i^2 = C_i^2 \beta_o \varepsilon - 2C_i S_i \alpha_o \varepsilon + S_i^2 \gamma_o \varepsilon, \; i = 1, N_w, \quad (15)$$

where R_i are measured beam sizes, and C_i, S_i, are the transfer matrix elements between the evaluation point and the n-th wire scanner. For $N_w = 3$, the system was solved by inversion of the matrix, Eq. (15), for $N_w > 3$ the system was solved by linear least squares method.

Fig. 4 illustrates dependence of the root-mean-square error in beam emittance measurements

$$\frac{\sigma_\varepsilon}{\varepsilon} = \frac{1}{\varepsilon}\sqrt{\frac{1}{(N-1)}\sum_{i=1}^{N}(\varepsilon_i - \bar{\varepsilon})^2}\,, \quad (16)$$

as a function of FODO phase advance per cell for each of three setups. It is clear that for $N_w = 3$ the minimum in beam emittance is achieved at the value of $\mu = 60^o$ FODO phase advance. For $N_w = 4$ the minimum is achieved at $\mu = 45^o$ FODO phase advance, and for $N_w = 5$ at $\mu = 36^o$. Therefore, error is minimized at the value of $\mu = \pi / N_w$ phase advance.

REFERENCES

[1] P. Emma, M. Woodley, Proceedings of the XX International Linac Conference, Monterey (2000), SLAC-R-561, p. 196.

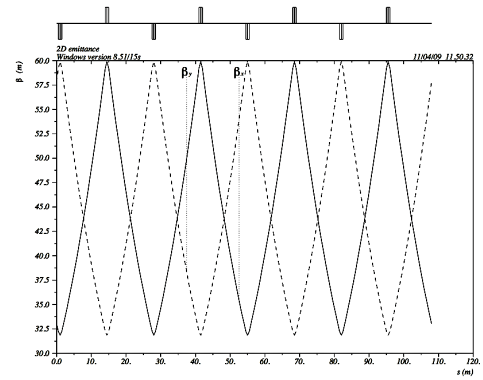

Figure 3: Measurement setup in 2D beam emittance determination as a function of phase advance.

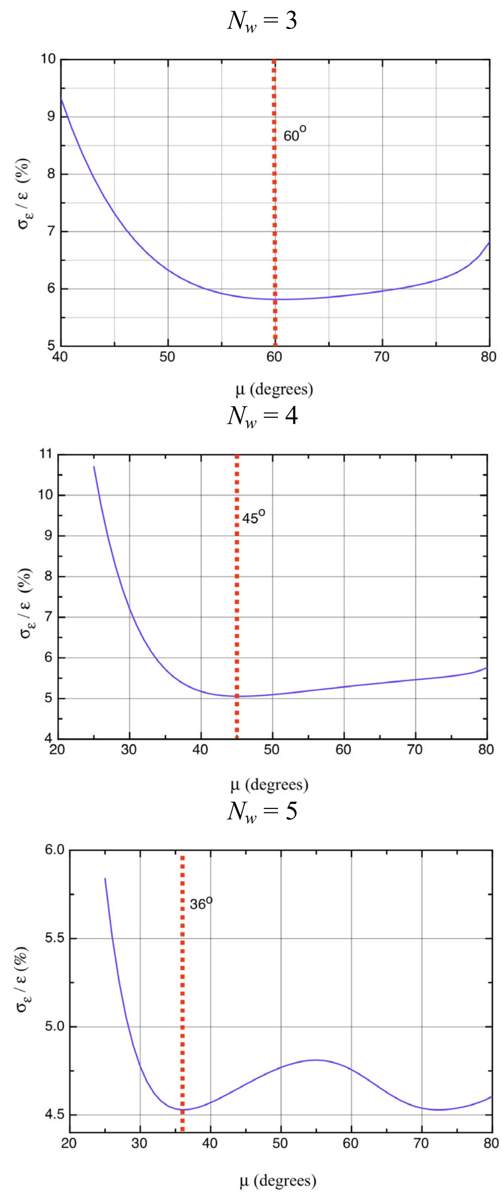

Figure 4: Error in 2D beam emittance determination as a function of phase advance for different numbers of wire scanners.