CYCLOTRON MATCHING INJECTION OPTICS OPTIMIZATION*

R. Baartman, TRIUMF, Vancouver, Canada

Abstract

Injection from an external ion source into a cyclotron results in unavoidable emittance growth when the cyclotron's first turn radius is small compared with the pole gap. In such a congested geometry, the injected beam first has the two transverse directions coupled on entering the axial magnetic field of the cyclotron, then transverse and longitudinal phase spaces are coupled by the inflector. Generally, to avoid loss, the beam is focused tightly through the inflector. It thus arrives at the first turn strongly mismatched because the vertical focusing in such a cyclotron is rather weak (vertical tune < 0.3). Space charge exacerbates the mismatch because it depresses the vertical tune further. Emittance growth from all these effects can be calculated using the full Sacherer 6D envelope formalism commonly used in transport lines. We develop the technique to include cyclotrons and in particular the transverse optics of the rf gaps, and apply it to the re-design of the TRIUMF 300 keV vertical injection line.

INTRODUCTION

Among large cyclotrons, the TRIUMF 500 MeV cyclotron is unusual in having a very low injection energy. The consequence is that injection is similar to that into a compact cyclotron, where there is little to no magnetic focusing on the first few turns. Nevertheless, the TRIUMF cyclotron is a high intensity machine, with cw output intensity limit of 0.5 mA[1]. The feature making this possible is the use of H^- ions rather than protons as accelerated particles. Since extraction is by stripping to protons, separating a turn from the circulating beam in the extraction region is not necessary, and so the phase width of the beam can be very large; on the order of 45°.

As is well-known, longitudinal focusing in rf gaps results in transverse defocusing. Conversely, in an isochronous cyclotron, where longitudinal focusing is irrelevant, the acceleration gaps can impart vertical focusing if the particles are phased such that leading particles gain more energy than the reference particle, and lagging particles less. This is the technique used in the TRIUMF cyclotron and in many compact cyclotrons as well.

A drawback is that since the transverse focusing depends on the slope of the energy gain, which is sinusoidal, the matching condition varies along the phase acceptance window and a perfect match cannot be obtained. Nevertheless, as will be shown below, the emittance growth can be made sufficiently small.

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The phase acceptance window limits arise from the following two effects: at the early limit the vertical focusing is too small so the beam is lost vertically, and at the late limit there is insufficient energy gain to clear the centre post so the beam is lost radially. Since space charge reduces vertical focusing, the early limit moves to later times as the intensity is raised. This results in a hard upper limit on charge per bunch, which can only be raised by raising the rf voltage. Experimentally, 420 μA extracted has been achieved (in a pulsed mode) with dee voltage of 105 kV (420 keV peak energy gain per turn).

The TRIUMF injection line beam energy is 300 keV. The rf frequency is 23 MHz, making $\beta\lambda = 0.33$ m. Typically, the β-function is 1.3 m, so the beam radius (2rms) averages 3 mm. There are a 23 MHz and a 46 MHz buncher; these together transform the DC beam into bunches of half length about 2 cm at the injection gap. At the intensity limit, the local beam current within the bunch at the point of injection is about 5 mA.

The normalized 4rms source emittance is 0.15 μm[1] at 0.5 mA, but we leave open the option of running unbunched at 5 mA, in which case the normalized emittance is $\sim$ 0.3 μm.

ENVELOPE TECHNIQUE

Bunches of beam can be described by their 6×6 σ-matrix. In the case considered here, there is first a coupling of the transverse directions due to the varying axial field in the vertical section, then the electrostatic inflector also couples the transverse to longitudinal. As a result, the bunches' axes lie along none of the beam axes. Only linear space charge is calculated in this model. This is done by first calculating the appropriate elliptic integrals to determine the linear part of the forces in the ellipsoid's natural coordinate system, then transforming by a 3D rotation to the beam axes. This technique is described by deJong and Heighway [3]. I have modified the deJong/Heighway code `TRANSOPTR` to include varying axial fields, inflector, bunching and acceleration by rf gaps.

Bunching

There is no linear way to take the beam from DC to bunched, so an approximate technique is used. Bunches are launched at the location of the main buncher. As the bunches in this model are ellipsoidal and not cylindrical, they are longer than the bunch spacing, because in this way the overlapping ends better approximate the initial DC

[1] I use the convention where emittance is the "half-axis product" as defined by Becker[2].

beam than bunches which just touch one another. The correct bunch length is $(3/2)\beta\lambda$, so $\sigma_{55} = (3/4)\beta\lambda$. The buncher voltage is used to calculate an appropriate σ_{66}, with correlation parameter r_{56} set to -1.

An example of such a calculation is shown in Fig. 1. Here the coherent phase advance per cell is $45°$. The beam current is $0.5\,\text{mA}$ and this results in the incoherent phase advance being depressed by space charge to $41°$ at the buncher, and to $16°$ at the end where bunch length is minimum. The increasing space charge due to the bunching beam causes the transverse size to double, but it happens slowly enough that the beam stays quite well matched.

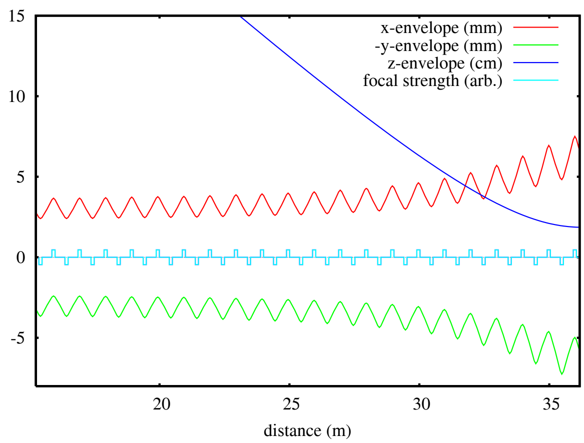

Figure 1: Periodic section, as beam bunches.

Along the vertical injection line, the magnetic field varies from zero to the final $0.3\,\text{T}$. The technique used to track the envelopes in this region is described in ref. [4].

Inflector

The TRIUMF inflector has a height, or electric radius of 30 cm, and a tilt at exit of $47°$. The Baartman/Kleeven Hamiltonian[4] is built into `TRANSOPTR` to track the envelopes (with space charge as needed) through the inflector. The transfer matrix resulting from single particle tracking is found to agree closely with that found by Root[5] by tracking through the calculated field maps, in spite of the fact that the magnetic field varies sharply at the entrance.

The inflector is followed by an electrostatic deflector which is used to properly centre the injected beam. Traditionally, this deflector has been cylindrical. However, I also considered the possibility of using plates curved vertically to augment the vertical focusing. It is as well possible to improve the focal properties of the inflector by curving its plates in this way[6]. This is under investigation.

Cyclotron – Acceleration

In the TRIUMF cyclotron, as with most compact cyclotrons, the injection gap is symmetric, but thereafter the gap extends radially, providing vertical focusing but no radial. This is essential in the TRIUMF cyclotron to achieve vertical stability in the first few turns where there is no magnetic focusing. In `TRANSOPTR`, this is handled with a thin lens whose focal strength is proportional to the derivative of the accelerating voltage. The longitudinal direction is similarly simple ($R_{55} = 1$) provided the longitudinal coordinate is time. However, if as in `TRANSOPTR` and commonly in other codes as well, the longitudinal coordinate is $z = t/(\beta c)$, then $R_{55} = \beta_\text{f}/\beta_\text{i}$. `TRACE3D` documentation erroneously has $R_{55} = 1$, though the code has recently been corrected[7].

The TRIUMF cyclotron has a built in isochronism error to generate a phase slip to both gradually move the beam from the falling edge of the waveform back onto the peak, and to partially compensate for radial-longitudinal coupling[8]. This is built into the calculation by storing the isochronism versus E in a function.

Cyclotron – Magnetic Focusing

Between gaps, the calculation takes the beam through a flat dipole interspersed with thin vertical focusing lenses whose stength is adjusted to give the correct overall magnetic ν_z as a function of E.

Cyclotron – Space Charge

It is well-known that the space charge effect on bunches circulating in a cyclotron is very different than in a synchrotron. This is because the cyclotron is constantly on transition. The result is that bunches rotate in the median plane, the rate of rotation depending upon the charge density. The stationary distribution is circular (bunch length = radial width). This was first derived by Kleeven[9]. In general, injected bunches are much longer azimuthally than their radial width. In this case, the only way for the bunch evolution to stabilize is to split into many small droplets. This was experimentally demonstrated in the SIR (Small Isochronous Ring) at MSU[10]. Perhaps surprisingly, the circular shape of the stationary distribution does not depend upon intensity; higher intensity means the evolution is quicker. The rate at which the vortices rotate is simply twice the Laslett radial tune shift[11].

In the linear force envelope model, bunches will rotate, propeller-like, as a whole. This would show up as an out of phase modulation of bunch length and radial width. An example is shown in Fig. 2 for the TRIUMF case ($0.5\,\text{mA}$ or $22\,p\text{C}$ charge per bunch, $\beta\gamma\epsilon = 0.3\,\mu\text{m}$). The lower plot is for a bunch sufficiently short that space charge effects are strong enough to start the "propeller-like" effect. In the upper plot, the bunch is launched at twice the length and in that case it continues to lengthen linearly with R as one would expect, because in the negligible space charge case, the phase width of the bunch is invariant. Launching bunches that are shorter still ($\sim 4\,\text{mm}$ radius) eventually leads to matched circular bunches. So counter-intuitively, long bunches continue to grow linearly keeping their phase width constant, but very short bunches **of the same total**

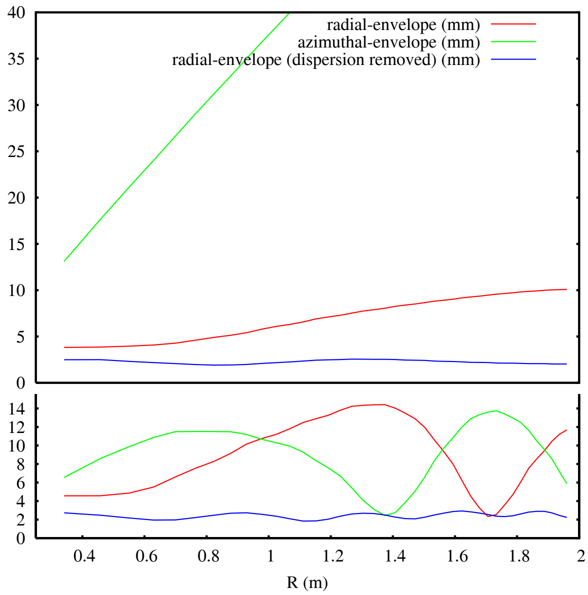

Figure 2: Beam being accelerated in a cyclotron, envelopes as function of R; initial bunch length = 10 mm (upper), and 5 mm (lower). The parameters correspond to the TRIUMF 500 MeV cyclotron: $R_\infty = 10$ m, so the energy range on this plot is from 0.3 to 20 MeV; 50 turns since $\Delta E = 0.4$ MeV/turn.

charge remain very short with their phase width decreasing monotonically.

In H$^-$ cyclotrons, however, the fluctuating radial width of the circulating bunches is of little import. This is because when extracting by stripping, it does not matter if turns overlap; the only important determining factor of extracted beam quality is the correlation between a particle's R and E. Since space charge moves particles to higher radius by giving them extra kinetic energy, the effect is not important. This is indicated by the blue curves in Fig. 2. These are the radial envelopes with local dispersion removed. They indicate that space charge would not affect the radial beam quality.

OPTIMIZATION

The locations, orientations, and strengths of the final 5 quadrupoles, as well as the bunching parameters are adjusted to minimize the maximum vertical beam size, and the maximum dispersion-corrected radial beam size in the cyclotron. The minimization is performed using a simulated annealing algorithm. The result is shown in Fig. 3. In this figure, zero is the location of the inflector. Upstream of this point one can see the bunch coming to a minimum (blue). Downstream of the inflector, the radial beam size grows rapidly; this is due both to space charge and $R-\phi$ coupling. However, as noted above, the widening turns are not a problem; the dispersion-corrected widths (black) look very good. The vertical beam envelope (green) is mismatched in the cyclotron. This is due to the unavoidably-too-small beam at the inflector exit.

The maximum envelope sizes, are indicative of the eventual circulating emittances, as precessional mixing will occur. These are $\beta\gamma\epsilon_r = 1.5\,\mu$m, $\beta\gamma\epsilon_z = 1.7\,\mu$m, to be compared with the injected emittances $\beta\gamma\epsilon_x = \beta\gamma\epsilon_y = 0.32\,\mu$m.

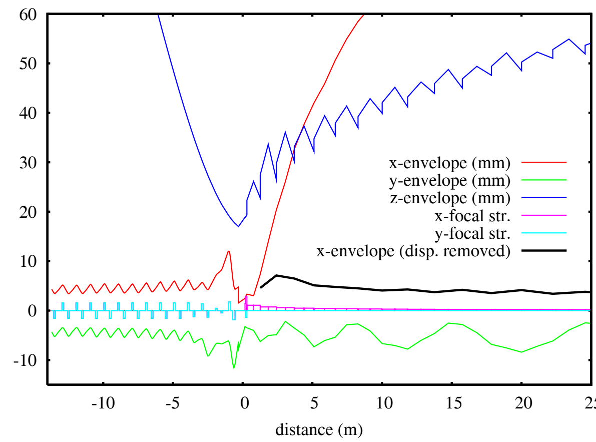

Figure 3: Beam envelopes through the injection line and into the cyclotron. Charge per bunch is 22 pC for a time average current 0.50 mA.

This injection line is currently under construction. Commissioning is scheduled for Spring 2010.

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