# POSSIBLE LIMITATIONS IN COUPLING CORRECTION USING ORBIT RESPONSE MATRIX ANALYSIS\*

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#### Abstract

The specified vertical emittance for the ILC damping rings is 2 pm. A major goal for the ATF is to demonstrate a reliable tuning technique for operation in this low emittance regime. Information on the optics of a storage ring can be obtained by analysis of the closed orbit response matrix (ORM); the information can be used to determine the changes in quadrupole gradients that best restore the design optics, and to determine the settings for skew quadrupoles to correct betatron coupling [1]. This technique has been applied successfully in a number of machines worldwide. In this paper, we present the results of simulations exploring the applicability of ORM analysis to ATF for low emittance tuning. We discuss possible limitations in the technique, arising from degeneracies between errors identified by ORM analysis.

## INTRODUCTION

Past attempts to minimise the vertical emittance using orbit response matrix (ORM) analysis in the KEK-ATF have met with limited success [2]. Furthermore, the procedure for collection of ORM data is time-consuming and can limit the correction to one or two iterations during a regular shift of eight hours. We report the results of a simulation study aiming to address both these issues: this study is being performed in the context of a renewed effort to achieve 2 pm vertical emittance in the KEK-ATF [3]. First, we try to optimise the conditions for fitting a machine model to the ORM data, with the goal of determining, as accurately as possible, the correction required to minimise the emittance. As part of this process, we also explore possible degeneracies between errors (such as sextupole misalignments) that lead to real coupling, and errors (such as bpm tilts) that give the appearance of coupling in the data; such degeneracies may place a lower limit on the vertical emittance that can be achieved with this technique. Second, we investigate how far it is possible to reduce the number of orbit corrector magnets used in the data collection, without having a significant adverse effect on the quality of the coupling correction that can be achieved. The time taken for data collection is directly proportional to the number of orbit corrector magnets used; reduction by a factor of two would be a significant benefit to application of this technique at ATF.

We use the code LOCO [4] for ORM analysis at ATF. Orbit data are collected from 96 bpms in each plane, in response to changes in 50 horizontal and 51 vertical steering

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magnet strengths. Coupling correction can be achieved using 68 skew quadrupoles, each of which is superposed onto a sextupole magnet. LOCO fits a range of parameters in a machine model to match the measured data; the parameters generally include normal and skew quadrupole strengths, bpm gains and couplings, and corrector strengths and tilts. Achieving an accurate and reliable fit requires optimisation of a range of fit conditions, including selection of machine parameters, weights applied to the dispersion data, and rejection of singular values in the fitting algorithm.

# **OPTIMISATION OF FIT CONDITIONS**

For our simulations we first introduce random errors on the skew quadrupoles superposed on the focusing sextupoles, and then simulate the ORM data. The next step is to use LOCO to fit the simulated ORM data, to try to identify the skew quadrupole strengths that were applied. Fig. 1 shows that a good fit to the applied skew quadrupole



Figure 1: Comparison between applied and fitted skew quadrupole strengths.

strengths can be achieved, if the conditions of the fit are optimised (Fit Conditions 1 and 2 refer to different cut-off thresholds,  $2 \times 10^{-5}$  and  $2 \times 10^{-4}$  respectively, for the singular values to be used in the fit). Figure 2 shows the distribution of the final vertical emittance after correction for different sets of random skew quadrupole errors. Since there are no other errors included at this stage, this indicates the best possible emittance that could be achieved using this technique; the average is just under 2 pm (starting from an average initial emittance of around 30 pm).

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Figure 2: Distribution of final vertical emittance after correction for skew quadrupole errors (250 seeds).

# **POSSIBLE DEGENERACIES**

Degeneracies in the fit can be identified by applying one kind of error in simulation, and fitting for another. Of particular concern are degeneracies between orbit corrector magnet and bpm parameters, and skew quadrupole strengths. Corrector magnet or bpm tilts will give the appearance of coupling in the measured ORM and dispersion, without actually generating coupling in the machine. If these errors are degenerate with the strengths of the skew quadrupoles, then an attempted correction using skew quadrupole strengths determined from ORM analysis could *increase* the coupling in the machine.

LOCO was used to fit skew quadrupole strengths to data generated from a model of the lattice where the only errors present were corrector magnet rotations around the beam axis. The vertical emittance was then calculated after a correction based on the fitted skew quadrupole strengths. Fig. 3 shows the distribution of the final vertical emittance for a number of sets of corrector tilts of 50 mrad rms; the average final emittance is below 0.1 pm. This is much smaller than the limit of just below 2 pm, that comes from the accuracy with which real skew quadrupole errors can be determined. Since the actual corrector tilts in ATF are believed to be much smaller than 50 mrad, we conclude that there is no significant degeneracy between the corrector magnet tilts and the skew quadrupole strengths.



Figure 3: Distribution of "corrected" vertical emittance with tilts on the orbit correctors (250 seeds).

Next, we applied bpm coupling errors, and again fitted for skew quadrupole errors. The bpm coupling errors were modelled using off-diagonal terms in the gain matrix:

$$\begin{pmatrix} x_m \\ y_m \end{pmatrix} = \begin{pmatrix} g_{xx} & g_{xy} \\ g_{yx} & g_{yy} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$
(1)

where  $x_m, y_m$  and x, y are the measured and real positions of the beam, respectively. Independent values for  $g_{xy}$  and  $g_{yx}$  were used for each bpm, with rms 0.02. The distribution of vertical emittances after a "correction" based on skew quadrupole strengths is shown in Fig. 4. The average final vertical emittance is 5.6 pm. This is significantly larger than the target value of 2 pm, and may imply a limitation in the technique. Values of order 0.02 for the coupling components of the bpm gain matrix are believed to be realistic for the bpms in ATF.



Figure 4: Distribution of "corrected" vertical emittance with bpm coupling (250 seeds).

#### **REDUCING CORRECTORS**

Collection of a full set of ORM data, using all the orbit correctors in the ATF, takes between two and four hours. However, even with a large set of parameters (normal and skew quadrupole gradients, bpm gains and couplings, corrector magnet strengths and tilts) the fit is highly overconstrained. This raises the possibility of reducing the number of correctors used in the data collection: however, the question then arises as to how many correctors, and which correctors, are best used for the analysis?

To determine the most effective orbit correctors to use for determining coupling errors, we simulate ORM data for a model of the ATF in which only one skew quadrupole error has been applied at a time. Using only the skew quadrupoles on one family of sextupoles, we construct 34 orbit response matrices. From these matrices, we take the components corresponding to the horizontal response to a vertical kick, constructing a 3-D array of dimension  $96 \times 51 \times 34$  (96 horizontal bpms, 51 vertical correctors, and 34 skew quadrupoles). This array can be restructured into a set of 51 matrices  $B_k$ , each with dimension  $34 \times 96$ : each of the matrices  $B_k$  gives (for a particular orbit corrector) the responses of the bpms to changes in strength of the skew quadrupoles. Finally, we construct a matrix C where each column corresponds to an orbit corrector k, and the

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elements within a column are the elements of  $B_k$ :

$$\begin{pmatrix} \frac{\partial X_i}{\partial S_j} \\ \vdots \end{pmatrix} = C \cdot \begin{pmatrix} Y_k \\ \vdots \end{pmatrix}$$
(2)

where  $X_i$  is the reading on (horizontal) bpms i,  $S_j$  is the strength of skew quadrupole j, and  $Y_k$  is the strength of (vertical) orbit corrector k.

Now we perform a singular value decomposition of C which gives a factorization:

$$C = U \cdot W \cdot V^{\mathrm{T}}.$$
 (3)

where W is a diagonal matrix of the singular values of C, and V and U are unitary matrices. The rows of  $V^{T}$  corresponding to the smallest singular values indicate those correctors to which the various  $\partial X_i/\partial S_j$  are least sensitive; in other words, those correctors that are least effective in telling us the skew quadrupole strengths based on analysis of the ORM. If we can identify a set of correctors residing almost entirely in the lowest rows of  $V^{T}$ , then we should be able to exclude those correctors from the ORM data, without affecting our ability to fit the skew quadrupole errors.

Figure 5 shows the elements in the rows of  $V^{T}$  corresponding to the smallest singular values of C. We find that these rows are populated almost entirely by orbit correctors in the two long straight sections of the ATF lattice. This is perhaps not surprising, given that the skew quadrupoles are located entirely in the arcs.



Figure 5: Elements in the rows of  $V^{T}$  corresponding to the smallest singular values of C.

Using only correctors in the arcs would reduce the time taken to collect ORM data by a factor of two. To test whether this would impact the quality of the fit, we simulated a correction procedure using ORM data including different sets of correctors:

- correctors located in the straight sections only;
- correctors located in the arc sections only;

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- half of the correctors, evenly spaced around the ring;
- all available correctors.

The distributions of final vertical emittances for each set are shown in Fig. 6.



Figure 6: Distribution of final vertical emittance after correction based on ORM data using different sets of orbit correctors.

## **CONCLUSIONS**

For the simple case of a lattice with only skew quadrupole errors, the simulations suggest that we can achieve a vertical emittance of just below 2 pm in the ATF damping ring. We also established that there is no significant degeneracy between skew quadrupole strengths and corrector magnet tilts. However, there does appear to be a degeneracy between the skew quadrupole strengths and the bpm couplings, that may put a lower limit on the vertical emittance, that can be achieved using this technique, of around 6 pm.

Simulations also suggest that the ORM data can be collected using about half the total number of orbit correctors, without compromising the final outcome of the correction. This would have significant practical benefit, in reducing by half the time taken for collection of ORM data.

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