ON THE TIME SCALE OF HALO FORMATION IN HOMOGENEOUS MISMATCHED BEAMS*

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Abstract

Experiments and numerical simulations show that high-intensity beams composed by charged particles usually reach their final stationary state with a progressive populating of a spatial region external to its original border. This populating process occurs in such terms that beam spatial limits at equilibrium increase by an amount of two or three times its initial nominal size. This is known as halo in Beam Physics. In this way, this work intends to better understand the time scale of halo formation. The carried out investigation has shown that the time scale of halo formation in fact can be segmented in two different quantities, each one associated to distinct physical mechanisms. One is related with the initial non-homogeneity naturally present in such systems, and the other is a result of the initial beam envelope mismatch. This investigation seems to be useful to design more efficient collimation systems and/or non-linear control systems for the next generation high-power accelerators.

INTRODUCTION

It is well-known that beams of charged particles with mismatched envelopes achieve its equilibrium state with a not negligible growth of its emittance [1][2]. If the beam total energy is conserved, any increase of the beam emittance will be unconditionally related with an associated decrease of the beam envelope. This occurs just because energy is a constraint of beam dynamics. While emittance indirectly indicates how much kinetic energy beam earns in a certain time, the envelope indicates the same amount of potential energy beam should lose in this same interval of time, once beam overall energy must be a conserved quantity along the time. If beam is initially cold (all constituent particles have initial velocity equal to zero), it could be said that beam suffers a progressively heating while travels inside the accelerator structure [3].

In previous works [3][4], it has been spent efforts in to better understand beam characteristics at equilibrium. Analytical models have been developed to predict beam quantities of interest at equilibrium as the envelope r_{beq} and the emittance ϵ_{eq} in relation to the beam initial conditions. Beam initial characteristics could be conveniently represented in a condensed way by just the initial beam envelope mismatch $r_b(s=0) \equiv r_o$, once azimuthal symmetry is a condition attained by beam during its overall dynamics. In this way, in fact, the expressions obtained through the models have propitiated that beam quantities at equilibrium could be calculated by just specifying r_o, mathematically $r_{beq} = r_{beq}(r_o)$ and $\epsilon_{eq} = \epsilon_{eq}(r_o)$. One must emphasize that energy conservation was a key issue to reach this goal in that opportunity.

In previous works, it has been exposed that beam equilibrium is reached when its phase-space is topologically invariant with the time [3]. Initially, beam state is nonstationary and after some characteristic time it suffers an abruptly changing, directing itself toward a stationary state in which quantities are almost invariant. The time elapsed between the beam transition from an initial state to a final state corresponds to τ, which is expected to be a function of the initial conditions ascribed to the beam. The main difference between the initial beam phase-space and the final stationary beam phase-space is the presence of a curve region in which particles are able to visit. Particles at this specific region are interpreted as halo particles. Thus τ can be identified as the time scale of halo formation [5].

In this sense, the goal of this work is to better understand and compute the time scale τ in which beam halo is formed. Just to clarify, the system here consists of an initially cold, mismatched, azimuthally symmetric, and homogeneous beam, focused by a constant magnetic field, and encapsulated by a conducting pipe. Beam is perfectly aligned with the pipe axis, being the oscillations of its centroid unimportant to approach the problem.

TIME SCALE OF HALO FORMATION

One way of computing the time scale τ is with the aid of the fraction of halo particles f. This quantity has been introduced in previous works [3]. In essence, fraction f is a scalar quantity that accounts the progressive population of the curve region in the beam phase-space, which has been already commented above. Since particles at this region are recognized as halo particles, fraction f is nothing more that the ratio of the number of halo particles N_h by the total number of beam particles N_b, $f \equiv N_h/N_b$. The quantity N_b is a constant in the present case.

Note that fraction f is in fact an intrinsic function of time s. When the population process starts, this occurs fast. However, this is still progressive, which means that for each time s one will have a specific and defined value for N_h. In this sense, the amount of halo particles is a function of time s, $N_h = N_h(s)$, and, due to its definition, the fraction of halo particles also is, $f = f(s)$. For computing the time scale of halo formation τ, it seems clear that analyze the dynamics of fraction $f(s)$ is a good strategy.

Figure 1 presents time behavior of beam phase-space and number of halo particles N_h. The results have been

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computed through full self-consistent N-particle beam numerical simulations. Particle interactions have been calculated by the Gauss Law method [5]. The total number of particles employed was $N = 10{,}000$ and the initial beam mismatched was $r_o = 1.5$. Time quantities are measured in units of $\sqrt{\kappa_{zo}}$, where κ_{zo} is the coefficient of constant magnetic focusing.

While Figure 1a and Figure 1c show respectively the initial beam phase-space, at $s = 0$, and the final beam phase-space, at a time after $s \geq \tau$, Figure 1b shows beam phase-space at some intermediate time. The sequence of beam phase-space portraits Figure 1a-c illustrates the previously commented populating process. It can be also observed that after a characteristic time τ, beam phase-space structure becomes invariant with the time s. This signals that possibly beam equilibrium was reached [3].

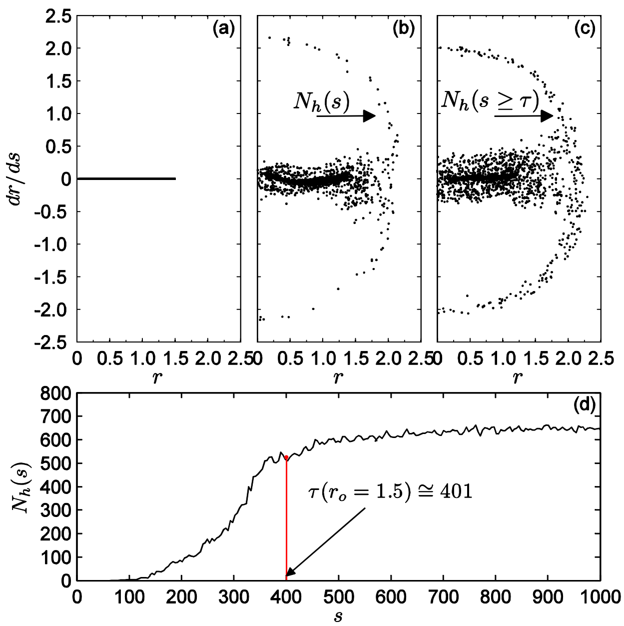

Figure 1: Beam phase-space dynamics inside the focusing channel. Panels (a) and (c) show beam phase-space respectively at $s = 0$ (initial nonstationary state) and at some time $s \geq \tau$ (final stationary state). Panel (b) shows beam phase-space in an intermediate time. Panel (d) shows the time dependence of halo particles N_h. All results have been obtained through numerical simulation with an initial mismatch of $r_o = 1.5$.

But not only beam phase-space becomes geometrically invariant. Particle density in each region of beam phase-space also becomes independent of time. This could be verified analyzing $N_h(s)$ present in Figure 1d. Observe that after some time, to say $\tau \cong 401$, $N_h(s)$ becomes constant, with a value specified by $N_h(s \geq \tau)$. Since dimensions of curve region are almost invariant, then halo particle density in phase-space is approximately constant in this place. In this way, with this condition and considering the previous commented geometric invariance, after the time scale τ, beam is topologically invariant, achieving equilibrium [5]. Established this concepts, one is able to quantitatively determine the time scale of halo formation τ with a simplified model and invoke interpretations about its nature.

THE MODEL

Last section of this work has shown that the complex process of population of the curve region by beam particles can be compactly described accounting $N_h(s)$. Qualitatively, it is this population process that conducts beam to its equilibrium state, being responsible by beam emittance growth and envelope decay. How fast equilibrium approaches can be thus obtained by analyzing at each time how fraction of halo particles f is changing.

It is clear that fraction f depends of time, since the number o halo particles N_h is a function of time. Since beam evolves with an azimuthal symmetry constraint, the only information about its initial state is r_o. Then, functionally fraction f depends

$$f = f(r_o, s). \tag{1}$$

Figure 1d shows that N_h and as a consequence f has well-defined behavior along time s. This suggests that an analytical function could be assigned to fraction f. Once $N_h(s)$ has an inflection point around τ, a sigmoid function, which is widespread used to model transfer functions of neurons in artificial intelligence, seems to be suitable

$$f(s) = \frac{a}{1 + be^{-cs}} + d. \tag{2}$$

In principle, this expression for fraction f has 4 free parameters, which could be obtained through direct fitting of equation (2) to the simulation results of Figure 1d. However much more information could be acquired studying cases limits and making use of previous results already obtained by us about the matter.

Objectively, once beam is initially cold, thus at the $s = 0$ limit

$$\lim_{s \to 0} f(s) = 0, \tag{3}$$

which means no particles at the curve phase-space region at the initial beam state. Also, at equilibrium, the result produced by equation (2) must match with $f(s \geq \tau)$

$$\lim_{s \to +\infty} f(s) = f(s \geq \tau), \tag{4}$$

a quantity obtained with models developed and presented in previous published works [3][4].

Applying the conditions present in equations (3) e (4) to the equation (2), two equations relating two parameters – which initially have been supposed free – are obtained. The equations are in the form $d(a, b)$ and $a(b, f(s \geq \tau))$. With these last equations, equation (2) assumes the format

$$f(s) = \frac{\left(1 + \frac{1}{b}\right) f(s \geq \tau)}{1 + be^{-cs}} - \frac{f(s \geq \tau)}{b}, \tag{5}$$

being just b and c parameters to be still obtained. Note that these parameters in fact are functions of r_o, as f also is. It is important emphasize that fraction $f(s \geq \tau)$ is a quantity determinable with models described in references [3][4]. Nevertheless, experience show that b is not an impacting parameter in $f(s)$. Without much discussion, parameter b can assume $b = 750$, for a wide range of beam initial mismatches. Therefore, the only one free parameter in essence is c, which can be promptly determined by fitting equation (5) to the numerical simulation results, for each initial mismatch r_o. The performance of the model represented by equation (5) in to describe results of numerical simulations is shown in Figure 2. The according is reasonable for both semicircular and semi-elliptical approximations [4] to the fraction of halo particles $f(s \geq \tau)$. The value of c for this mismatch is $c(r_o = 1.5) = 0.02125$.

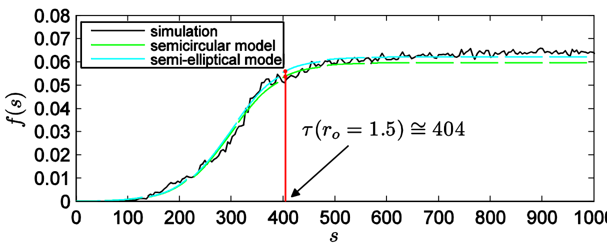

Figure 2: The fraction f computed through numerical simulation (black line) and the developed model, using semicircular and semi-elliptical approximations (respectively green and cyan lines). Beam initial mismatch of $r_o = 1.5$ and $c(r_o) = 0.02125$.

To determine τ, one has to establish a critical value as

$$f(s = \tau) = c_{cri} \cdot f(s \geq \tau), \quad (6)$$

in which c_{cri} is a critical coefficient here chosen as $c_{cri} = 0.9$. Note that if equation (2) was only an exponential, the critical value would be $1 - 1/e$.

Inserting equation (6) into equation (5) and solving for the time scale τ one obtains

$$\tau = -\frac{1}{c}\ln\left(\frac{1 - c_{cri}}{1 + bc_{cri}}\right). \quad (7)$$

Considering the previous numeric values for b, c and c_{cri}, equation (7) returns $\tau(r_o = 1.5) \cong 404$, which is much similar with the value calculated in Figure 1d, being the slight difference due to the fluctuations normally present in the results provided by numerical simulations. Figure 3 presents τ as a function of other values of r_o, which are usually of interest in beam physics.

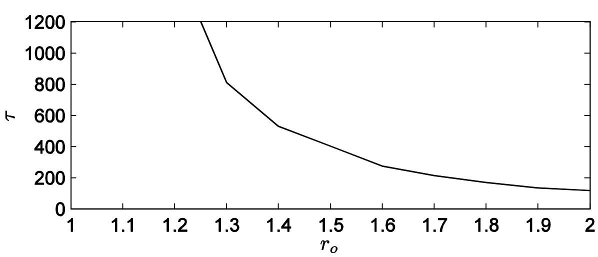

Figure 3: Dependence of the time scale of halo formation τ as a function of the initial beam mismatch r_o. Note the exponential behaviour of τ in relation to r_o.

Expression (7) can be still segmented in two distinct quantities

$$\tau = -\frac{1}{c}\ln(1 - c_{cri}) - \frac{1}{c}\ln\left(\frac{1}{1 + bc_{cri}}\right) = \tau_\eta + \tau_\lambda, \quad (8)$$

in which explicitly

$$\tau_\eta \equiv -\frac{1}{c}\ln(1 - c_{cri}) \text{ e } \tau_\lambda \equiv -\frac{1}{c}\ln\left(\frac{1}{1 + bc_{cri}}\right) \quad (9)$$

Numerically, $\tau_\eta(r_o = 1.5) \cong 108.37$ and $\tau_\lambda(r_o = 1.5) \cong 295.85$. Numerical results for b, c, τ, τ_η and τ_λ for many values of r_o are presented in Table 1. Results for $r_o = 1.1$ and $r_o = 1.2$ are not calculated once computing time of numerical simulations is too long.

Table 1: The time scale of halo formation τ for many values of initial beam mismatch r_o.

r_o	b	c	$\tau = \tau_\eta + \tau_\lambda$	τ_η	τ_λ
$= 1.0$	$= 750$	$= 0$	$\to \infty$	$\to \infty$	$= 0$
$= 1.3$	$= 750$	$= 0.01089$	$\cong 809.80$	$\cong 211.44$	$\cong 598.36$
$= 1.4$	$= 750$	$= 0.01665$	$\cong 529.65$	$\cong 138.29$	$\cong 391.36$
$= 1.5$	$= 750$	$= 0.02198$	$\cong 401.21$	$\cong 104.75$	$\cong 296.46$
$= 1.6$	$= 750$	$= 0.03212$	$\cong 274.55$	$\cong 71.68$	$\cong 202.87$
$= 1.7$	$= 750$	$= 0.04142$	$\cong 213.01$	$\cong 55.59$	$\cong 157.32$
$= 1.8$	$= 750$	$= 0.05244$	$\cong 168.16$	$\cong 43.90$	$\cong 124.26$
$= 1.9$	$= 750$	$= 0.06613$	$\cong 133.35$	$\cong 34.81$	$\cong 98.53$
$= 2.0$	$= 750$	$= 0.07516$	$\cong 117.33$	$\cong 30.63$	$\cong 86.69$

CONCLUSIONS

The fraction o halo particles f has in a condensed way much information about beam phase-space dynamics. Specially beam route to equilibrium. Fraction f not only allows to determine beam macroscopic equilibrium quantities, such as emittance $\epsilon(s \geq \tau)$ and envelope $r_b(s \geq \tau)$, but also to acquire information about the time scale of halo formation τ. The model here presented has propitiated to compute τ in a systematic way. Also, the model has identified that in fact τ is composed by two distinct quantities, τ_η and τ_λ. The physical meaning of these quantities will be addressed in an extended version of this paper, but it is possible to advance that the first one is a result of the initial spurious inhomogeneity naturally associated to homogeneous beams and the second is a result of large resonant islands induced by the initial beam mismatch r_o.

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