NEW DIFFUSION ANALYSIS TOOLS FOR BEAM BEAM SIMULATIONS*

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Abstract

A new set of tools for BBSIM[1] has recently been developed to analyze the nature of the diffusion in multiparticle simulations. The diffusion subroutines are currently used to accelerate beam lifetime calculations by estimating the diffusion coefficient at various actions and integrating the diffusion equation. However it is possible that there may be regimes where anomalous diffusion dominates and normal diffusion estimates are incorrect. The tools we have developed estimate the deviation from normal diffusion and can fit the coefficients of a jump diffusion model in the event that this type of diffusion dominates.

INTRODUCTION

One approach to accelerate the evolution of the beam distribution, has been to generate diffusion parameters with can later be used to evolve beam distributions. In the past years, a body of research has developed around the characterization various types of diffusion. This in turn can be a guide to the correct diffusion-like equation necessary to evolve a given distribution. To date, this technology has been sparingly used to its full potential in order to accelerate such computationally intensive projects as evolving a particle distribution on the scale of a beam lifetime.

BBSIM is one of the few codes employing this approach and is equipped with existing diffusion subroutines. These routines are used to quickly calculate lifetimes and emittance growth by integration of the diffusion equation. However, until recently, the existing treatment was incomplete since the validity of the normal diffusion approximation was not verified. To address this problem we created new a library of routines to verify the validity of the diffusion model in action space.

VERIFYING THE DIFFUSION APPROACH

An approach that assumes normal diffusion in action space and develops an estimate of the diffusion coefficient has been considered in the past for analytical [2, 3, 4] and computational [5] beam-beam simulations. In this approach, a diffusion coefficient is calculated and can then be used to evolve a distribution in order to calculate emittance growth and beam lifetimes. However, in the application of this approach one needs to be careful concerning its regions of validity. Thus, it is critical in this approach that there is a characterization of the dynamics around the action where the diffusion coefficient is calculated. Here we have applied two methods to distinguish between the normal and anomalous diffusion based on an analysis of the dynamics and computation of the scaling exponents.

• **Simulated path approach**

One of the more successful approaches we implemented was based on [6]. In this method a C algorithm was developed which tested whether the sample estimates lie inside the confidence bands for all initial actions (J) within J_{max} to J_{min}. To accomplish this, the action moments where calculated using

$$M_j(J) = \frac{\sum_{t=1}^{n} K(\frac{J_t - J}{h})(J_{t+1} - J_t)^j}{\Delta \sum_{t=1}^{n} K(\frac{J_t - J}{h})}. \qquad (1)$$

Here a Gaussian kernel function K was used with the parameter h acting as the bandwidth determining the smoothing behavior of the kernel function. For a continuous diffusion process, M_1 and M_2 are estimates of the drift and squared diffusion coefficient, respectively. Moments are calculated through 4th order and then used to simulate m paths of a continuous diffusion process. Then for each of the m paths, m moments $M_j^{(m)}$ are calculated and then used to generate the median, 10th and 90th percentiles for each moment. These now serve as the confidence bands for some values of initial action J. If the moments lie within the confidence bands then normal diffusion hypothesis is confirmed. However if the sample estimates lie outside the confidence bands for some values of J the normal diffusion hypothesis is rejected.

The results from the application of this algorithm are shown in Fig. 1. Here we see that below the dynamic aperture (roughly 7σ) normal diffusion dominates, however at actions close or above the dynamic aperture normal diffusion is rejected.

While the above approach is straightforward and direct, it still requires simulation of over 100 random diffusion paths to build up a set of valid statistics. The computational time needed to simulate these paths are not excessive (it is still a very small fraction of the time required for any normal lifetime time simulation) and thus this approach appears to meet the basic requirements for any normal diffusion verification algorithm.

• **Variance based approaches**

Several papers[7, 8, 9] advocate the use of variance based approaches where the time functional relationship of a calculated variance is measured,

$$< \Delta J^2 > \sim t^{2H}. \qquad (2)$$

Here H is the associated scaling exponent. If $H = 1/2$ then regular diffusion model is accepted. We also tested

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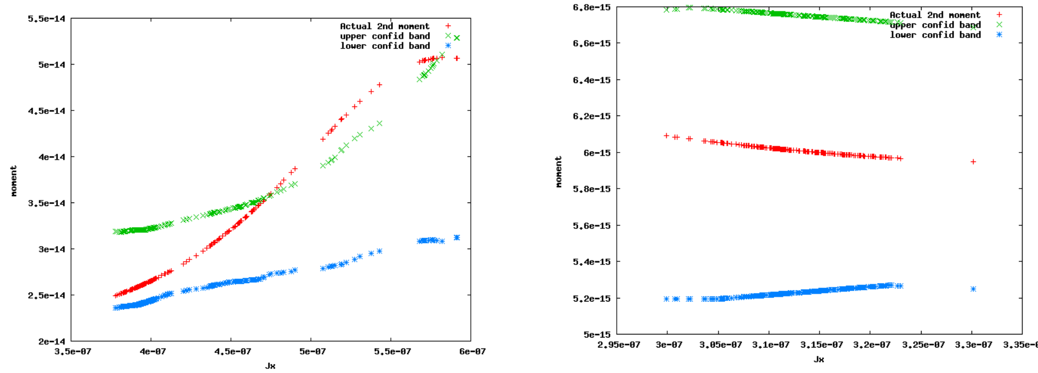

Figure 1: (left)The evolution of the second moment as the action nears the dynamic aperture at 6 σ initial transverse horizontal (X) action in a RHIC simulation with both long-range and head-on kicks. The diffusion ceases to be regular beyond the dynamic aperture. (right) The evolution of the second moment below the dynamic aperture at 5 σ initial transverse horizontal (X) action in a RHIC simulation with both long-range and head-on kicks. The diffusion appears regular.

this approach. The results are shown in Fig. 2 which confirm the results obtained using the simulated path approach shown in Fig. 1.

Extraction of Jump-Diffusion Parameters

When the normal diffusion hypothesis is rejected, the diffusion in action space may be modeled as a mixed-jump diffusion process. Following the approach of [6], we can extract the jump diffusion parameters for the following model:

$$\begin{aligned} dJ(t) &= \mu(J(t))dt + \sigma(J(t))dW(t) + dI(t), \\ I(t) &= \sum_{i=1}^{N(t)} Y_i. \end{aligned} \qquad (3)$$

Here I is a compound jump process with intensity $\lambda(J)$ and Y_i, the random jump size is independent and identically distributed, whose common distribution p_Y is assumed to be independent of the Brownian noise W and the counting process N. In this case the common expected moments become,

$$\begin{aligned} \mu(J) &= \lim_{\Delta t -> 0} \frac{1}{\Delta t} E[J(t+\Delta t) \\ &\quad -J(t)|J(t)=x], \\ \sigma(J)^2 + \lambda(J)E_Y[Y^2] &= \lim_{\Delta t -> 0} \frac{1}{\Delta t} E[(J(t+\Delta t) \\ &\quad -J(t))^2|J(t)=x], \\ \lambda(J)E_Y[Y^j] &= \lim_{\Delta t -> 0} \frac{1}{\Delta t} E[(J(t+\Delta t) \\ &\quad -J(t))^j|J(t)=x]. \end{aligned} \qquad (4)$$

Here $E_Y[]$ is the expected values base on the p_Y probability distribution function for Y. If we further assume that the jump size Y is normally distributed with zero mean and variance $\sigma_Y^2 > 0$, then we can obtain the following approximate relations, using the calculated 2nd, 4th and 6th moments,

$$\begin{aligned} M_2(J) &\approx \sigma^2(J) + \lambda(J)\sigma_Y^2, \\ M_4(J) &\approx 3\lambda(J)\sigma_Y^4, \\ M_6(J) &\approx 15\lambda(J)\sigma_Y^6. \end{aligned} \qquad (5)$$

In this case the mixed jump-diffusion parameters can be extracted,

$$\begin{aligned} \mu(J) &= M_1(J) \\ \sigma_Y^2 &= \frac{1}{n}\sum_{t=1}^{n} \frac{M_6(J_t)}{5M_4(J_t)} \\ \lambda(J) &= \frac{M_4(J_t)}{3 * \sigma_Y^4} \\ \sigma^2(J) &= M_2(J) - \lambda(J)\sigma_Y^2. \end{aligned} \qquad (6)$$

CONCLUSION

We have applied two approaches to validate the diffusion approximation under beam-beam particle tracking simulations. In the case when normal diffusion is rejected we can calculate jump diffusion parameters. Later we hope to use these parameters we can then proceed to solve the mixed-jump diffusion equation, employing a Monte-Carlo approach. In this approach the lifetime can be estimated by simulating many random paths to obtain an expected action at each time and thus the estimated emittance growth and lifetime for the beam. This code is general enough that it could be applied to variety of other systems out side of beam-beam simulations.

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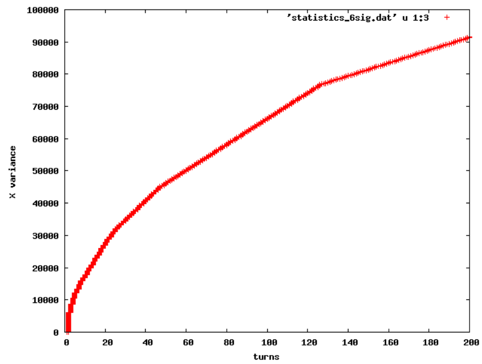

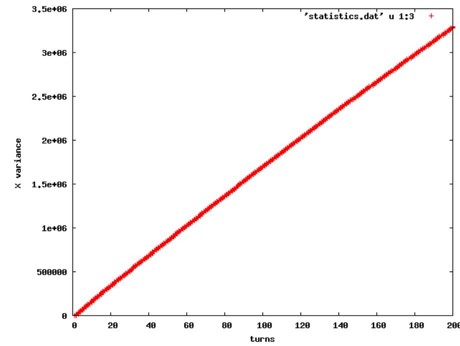

Figure 2: (left) The time dependence of the variance at 6 σ initial action. The diffusion ceases to be regular since the time dependence is not linear $H \neq 1/2$ (agrees with results of simulated paths). (right) The time dependence of the variance at 5 σ initial action. The diffusion should be regular since the time dependence is clearly linear $H = 1/2$ (agrees with results of simulated paths).

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