HANDLING OVERLAPPING FIELDS WITHIN THE V-CODE BEAM DYNAMICS SIMULATION TOOL*

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Abstract

The V-Code simulation tool implemented at TEMF is a fast tracking code applying the VLASOV equation. Instead of directly solving the partial differential equation, the considered particle distribution function is represented by a discrete set of characteristic moments. To describe the time evolution of these moments ordinary differential equations can be deduced, wherein the external forces caused by the electric and magnetic fields within the accelerator components are considered. The entire beam line is represented in the code as a successive alignment of separate independent beam line elements. The proximity of some beam forming elements may lead to overlapping fringe fields between consecutive elements. In order to simulate even such beam lines with the V-Code its database of disjunctive beam line elements has to be enhanced to deal also with superposed fields. In this paper a summary of issues regarding the implementation complemented with simulation results is provided.

INTRODUCTION

For the new 100 keV polarized electron source at the Superconducting DArmstadt LINear ACcelerator (S-DALINAC) a new low energy injection concept has to be designed. In order to handle the limited space in the accelerator hall, a very compact setup was elaborated. The proximity of various beam forming elements leads to overlapping fringe fields between the elements. For the same purpose a space saving quadrupole triplet was designed. Another important component within the new injector beam line is a Wien filter using a magnetic field to rotate the spin of the electrons and a superposed perpendicular electric field to compensate the deflection caused by the magnetic field. Within the V-Code simulation tool the beam line is represented as a consecutive alignment of separate independent beam line elements. This procedure allows to simulate even very long beam lines with minimum requirements to the computer memory. In order to accurately reproduce the whole injector with the V-Code without neglecting slowly decreasing fringe fields its database of disjunctive beam line elements has to be enhanced to deal with superposed fields. In the past, elements like the Wien filter and the compact quadrupole triplet were realized as dedicated beam line elements with high implementation efforts [1, 2]. Such purpose-built developments are avoidable if superposed fields can be treated in a more general approach.

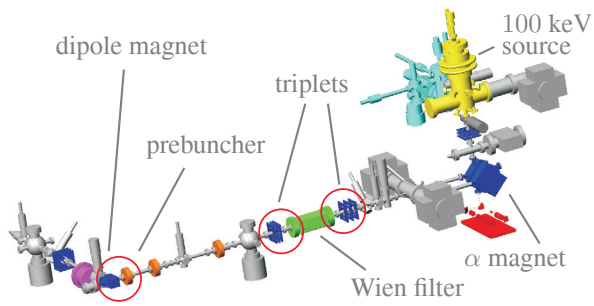

Figure 1: Schematic computational model of the compact injector design for the new polarized electron source at the S-DALINAC. Selected areas with overlapping fringe fields are marked by circles.

NUMERICAL MODEL

The main idea behind the moment approach to beam dynamics [3] is to speed up the calculation time for simulations by considering a discrete set of characteristic moments of the particle distribution instead of the particle distribution itself. When particle-particle collisions can be neglected, the VLASOV equation is suitable [4]. The forces mentioned in this equation can be deduced from the LORENTZ equation if the distributions of electric and magnetic fields are known. By means of proper multipole expansions for the individual beam line elements the three-dimensional field distributions in the immediate vicinity of the particle trajectory can be reconstructed from extracted one-dimensional field parameters [5]. Finally, an explicit RUNGE-KUTTA type time integration method is used in order to solve the moment evolution equations in time.

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Moment Evolution

Starting from the VLASOV equation, the time evolution of the moment parameters can be expressed as follows:

$$\begin{aligned}\frac{\partial <\mu>}{c\,\partial t} &= <\mathrm{grad}_{<\vec{r}>}(\mu)> \cdot <\frac{\vec{p}}{\gamma}> \\ &+ <\mathrm{grad}_{<\vec{p}>}(\mu)> \cdot <\frac{\vec{F}}{m_0 c^2}> \\ &+ <\mathrm{grad}_{\vec{r}}(\mu) \cdot \frac{\vec{p}}{\gamma}> + <\mathrm{grad}_{\vec{p}}(\mu) \cdot \frac{\vec{F}}{m_0 c^2}>\end{aligned} \quad (1)$$

where $<\mu>$ is any moment obtained from the distribution function f by integrating over the whole phase space Ω

$$<\mu> = \int_{\Omega} \mu\, f(\vec{r},\vec{p}\,)\, \mathrm{d}r^3\mathrm{d}p^3.$$

An initial set of moments may be obtained via a statistical analysis from a given particle distribution.

The contribution to the moment equation time derivatives can be separated into drift and transportation terms. The drift terms represent the fundamental internal feedback and depend on the momentum $\vec{p}$. The transportation terms consider the interaction with the external forces $\vec{F}$ expanded in terms of the present fields inside the various beam line elements. The specified Lorentz force $\vec{F} = q \cdot (\vec{E} + \vec{v} \times \vec{B})$ can be evaluated as a sum of forces resulting from various superposed fields if linear materials can be assumed. The drift and transportation terms can be evaluated independently of each other witch results in an efficient implementation. Here, it is permitted to apply one transportation term for each overlapping field component.

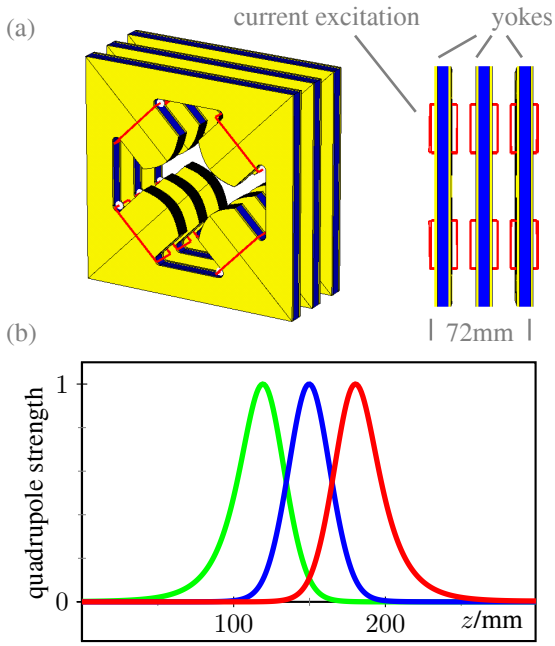

Figure 2: (a) Different views on a schematic computational model of a compact quadrupole triplet as used at the S-DALINAC. In (b) the extracted normalized quadrupole strength of each individual magnet is displayed.

Field Calculation

A detailed knowledge of the distribution of the various electric and magnetic field components along the beam line is necessary in order to calculate their impact on a particle bunch. To obtain this data we use the 3D magnetostatic, elctrostatic and eigenwert solvers integrated in the CST DESIGN ENVIRONMENT [6]. In case of the nearness of consecutive beam line elements all relevant materials influencing the fields of the considered element have to be taken into account. In Fig. 2 the computational model together with the extracted quadrupole strength of each individual magnet of a compact quadrupole triplet is shown. Each field component of each beam line element can be weighted individually.

IMPLEMENTATION

One goal for the implementation was to enable the flexible reuse of the existing beam line element database. For that purpose a new enveloping macro element has been created. It enables to combine all beam line elements with overlapping fields. The calculation of the drift terms in (1) is done within the new element once on every time step only. The transportation terms are calculated and superimposed for each inner element on every time step.

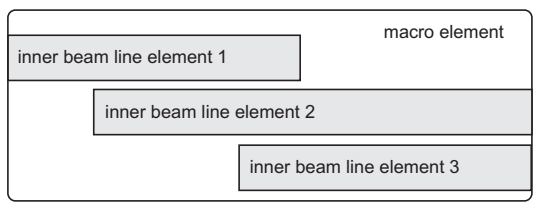

Figure 3: Scheme of an enveloping macro element with three overlapping inner beam line elements.

Any existing beam line element except those disposing curved reference paths can immediately be used as inner elements. For the bended trajectories inside the dipole- and the alpha magnet it is advisable to handle a global unrotated coordinate system and a local coordinate system successively rotated along the curved reference path [7, 8] and additional coordinate systems oriented along the longitudinal axes of the inner beam line elements. Furthermore, it is necessary to align the global coordinate system along the successive beam line once the considered beam line elements have been passed.

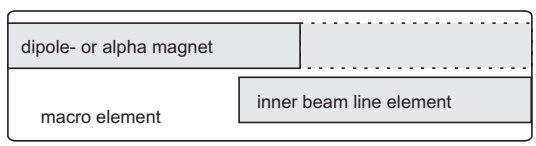

Figure 4: Scheme of a dipole- or alpha magnet enveloping macro element together with an overlapping inner beam line element.

SIMULATION

In order to study the applicability of the proposed implementation the simulation results from V-Code are compared to simulation results obtained with the particle tracker integrated in the CST DESIGN ENVIRONMENT. In addition results from a previously proposed hard coded implementation for compact quadrupole triplets are matched. Before being able to compare the simulation results between the different codes a convergence study was performed in order to determine an appropriate mesh that satisfactorily balances accuracy and computing resources for the CST magnetostatic and tracking solvers. Finally, a mesh cell size of 0.5mm was chosen.

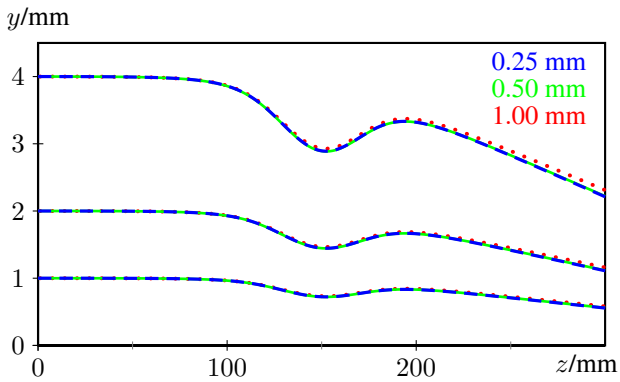

Figure 5: Trajectories of single particles with vertical offset of 1, 2 and 4 mm within the compact quadrupole triplet shown in Fig. 2. The results of simulations with the CST particle tracker on different mesh densities (0.25, 0.5 and 1mm) are displayed.

In Fig. 6 and Fig. 7 the transversal focusing behavior inside the compact quadrupole triplet are illustrated. From the CST Design Environment the bunch dimensions were obtained by tracking a bunch of particles and analyzing their trajectories via the build-in particle monitors. In V-Code the bunch dimensions at any time can directly be deduced from the moments. In all simulations space charge effects were neglected.

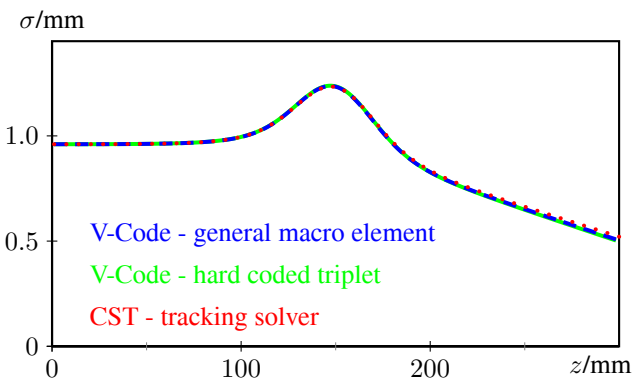

Figure 6: Simulation of the horizontal focusing effect within the compact quadrupole triplet shown in Fig. 2

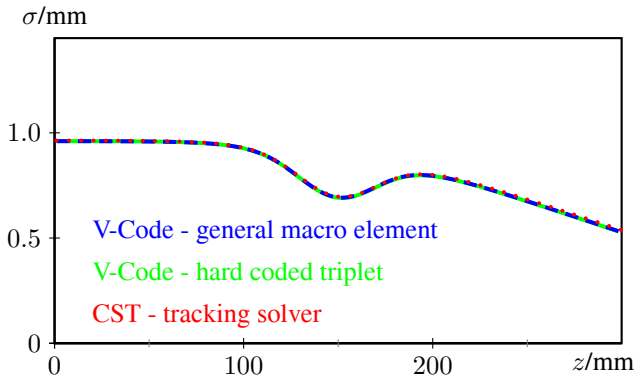

Figure 7: Simulation of the vertical focusing effect within the compact quadrupole triplet shown in Fig. 2

The simulations with the two V-Code implementations yield the same results witch underlines a proper implementation. The simulated focusing behavior obtained with the V-Code and with the CST particle tracker are in good agreement. The small deviation minimizes further with a refined mesh utilized by the CST solvers.

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