SELF-CONSISTENT NON-STATIONARY MODEL FOR MULTIPACTOR ANALYSIS IN DIELECTRIC-LOADED ACCELERATOR STRUCTURES*

O. V. Sinitsyn, G. S. Nusinovich, T. M. Antonsen, Jr. and R. A. Kishek, IREAP, University of Maryland, College Park, MD 20742, U.S.A.

Abstract

In this paper, a self-consistent non-stationary 2-D model of multipactor in dielectric-loaded accelerating structures is proposed. In comparison with existing models, this model calculates dc electric field produced by secondary electrons self-consistently and also it includes effects of cylindricity. Results of our calculations are compared with the ones obtained for the 11.424-GHz alumina-based dielectric-loaded accelerating structure designed and experimentally tested by Argonne National and Naval Research Laboratories.

INTRODUCTION

Multipactor (MP) is known as the avalanche growth of the number of secondary electrons emitted from a solid surface exposed to an rf electric field under vacuum conditions. The secondary electrons appear as the result of the surface impacts of energetic primary electrons accelerated by the rf field. MP may occur in various microwave and rf systems such as microwave tubes, rf windows and launchers, accelerating structures and rf satellite payloads. In different systems, it occurs in different situations: single- and two-surface MP, resonant and poly-phase MP, on the surface of metals and dielectrics where the rf electric field is, correspondingly, normal or tangential etc. MP is a severe problem in modern microwave systems: it generates rf noise, reduces rf power flow, changes device impedance, stimulates rf breakdown etc. Therefore, theoretical and experimental studies of MP are of great interests to the physicists working in various areas of physics and engineering.

In this work, MP in dielectric loaded accelerator (DLA) structures is studied. The starting point for our work is experimental and theoretical studies of such structures jointly done by Argonne National Lab and Naval Research Lab [1, 2]. In the theoretical model developed during those studies, the space charge field due to the total number of particles is taken into account as a parameter. We offer a simple non-stationary 2-D model in which the dc field is taken into account self-consistently. The paper is organized as follows: the next section contains description of the analytical model and its numerical implementation, then we show some results of our simulations and, finally, we give a summary.

GENERAL FORMALISM

The cross-section of a cylindrical DLA structure is shown in Fig. 1. In this figure indices 1 and 2 designate vacuum and dielectric regions, respectively, r_w is the waveguide wall radius and r_d is the radius of dielectric.

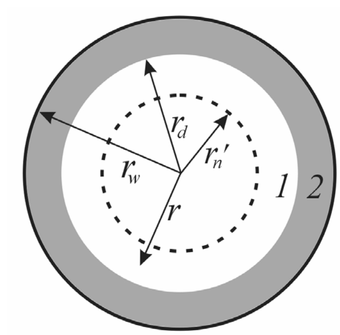

Figure 1: Cross-section of a cylindrical DLA structure. 1 – vacuum region, 2 – dielectric, r_d – radius of dielectric, r_w – radius of waveguide wall, r_n' - location of the charged layer.

Equations of electron motion in the cylindrical coordinates in the presence of both rf and radial dc fields can be written as

$$\frac{dv_r}{dt} = \frac{v_\varphi^2}{r} - \frac{e}{m}\left(E_{dc} + E_{r1} - v_z B_{\varphi 1}\right) \quad (1)$$

$$\frac{dv_\varphi}{dt} = -\frac{v_r v_\varphi}{r} \quad (2)$$

$$\frac{dv_z}{dt} = -\frac{e}{m}\left(E_{z1} + v_r B_{\varphi 1}\right) \quad (3)$$

Here v_r, v_φ and v_z are the radial, azimuthal and axial components of the particle velocity, r is the radial coordinate of the particle, e and m are electron charge and mass, respectively. Also, in these equations, E_{dc} is the radial dc electric field created by the space charge, E_{r1} and E_{z1} are radial and axial electric components and $B_{\varphi 1}$ is the azimuthal magnetic component of the rf field in the vacuum region. We limit our consideration by TM_{01}-wave only, which is commonly used in DLA structures. The rf field components of this wave in the vacuum region are

$$E_{r1} = \mathrm{Re}\left(i\frac{k_z}{|k_{\perp 1}|} A I_1(|k_{\perp 1}| r) e^{i(\omega t - k_z z)} \right) \quad (4)$$

$$E_{z1} = \mathrm{Re}\left(A I_0(|k_{\perp 1}| r) e^{i(\omega t - k_z z)} \right) \quad (5)$$

$$B_{\varphi 1} = \mathrm{Re}\left(i\frac{\omega}{c^2 |k_{\perp 1}|} A I_1(|k_{\perp 1}| r) e^{i(\omega t - k_z z)} \right) \quad (6)$$

Here A is the rf amplitude, $\omega = 2\pi f$ is the frequency of rf signal, k_z is the axial wave-number, $k_{\perp 1}$ is the transverse wave-number in the vacuum region, c is the speed of light and I_0, I_1 are the modified Bessel functions of the first

*This work was supported by the Office of High Energy Physics of the US Department of Energy.

kind. We assume that the problem has azimuthal symmetry, i.e. that the charge density is uniform along φ-direction. Then we may calculate E_{dc} as follows. Consider a cylindrical layer of charges (see Fig. 1) of radius r'_n and length L. The charges in this layer start their motion from the surface of the dielectric where their initial surface charge density is σ_{n0}. As the layer is moving away from the dielectric, the total charge it contains, $Q = 2\pi r_d L\sigma_{n0}$, remains the same but the charge density increases, so at $r = r'_n$ it becomes $\sigma_n = \sigma_{n0} r_d / r'_n$. Considering a cylindrical surface of radius r such that $r'_n < r < r_d$ and applying Gauss law for this surface gives $E_{dc}2\pi rL = Q_{enc}/\varepsilon_0$, where Q_{enc} is the total charge enclosed by the surface. If we have N cylindrical layers of charge enclosed by this surface, then

$$Q_{enc} = \sum_{n=1}^{N} 2\pi r'_n L\sigma_n . \tag{7}$$

Using expressions obtained above, one may easily get

$$E_{dc}(r) = \frac{r_d}{r\varepsilon_0}\sum_{n=1}^{N}\sigma_{n0} = \frac{r_d\sigma_0}{r\varepsilon_0}\sum_{n=1}^{N} w_n , \tag{8}$$

where we have used $\sigma_{n0} = \sigma_0 w_n$. Here w_n is the relative weight of the n-th layer and σ_0 is the initial (seed) surface charge density which is assumed to be the same for all layers in our model. Initially, the weight of each new layer $w_n = 1$, then, during the calculations, it varies in accordance with the secondary yield model when particles in the layer impact the dielectric wall.

If $v_z << c$, the z-motion particles can be neglected and the set of equations (1) - (3) can be reduced to

$$\frac{d\tilde{v}_r}{d\tilde{t}} = \frac{\tilde{v}_{\varphi 0}^2}{\tilde{r}^3} + \frac{1}{\tilde{r}}\upsilon^2\sum_{n=1}^{N} w_n + \frac{\alpha}{\rho_1} I_1(\rho_1\tilde{r})\sin(\tilde{t}) \tag{9}$$

$$\frac{d\tilde{r}}{d\tilde{t}} = \tilde{v}_r \tag{10}$$

$$\frac{d\varphi}{d\tilde{t}} = \frac{\tilde{v}_{\varphi 0}}{\tilde{r}^2}. \tag{11}$$

Here we have introduced the following normalized variables: $\tilde{r} = r / r_d$, $\tilde{t} = \omega t$, $\tilde{v}_r = v_r / r_d\omega$ and $\tilde{v}_{\varphi 0} = v_{\varphi 0} / r_d\omega$. Also, in (8), $\alpha = eA/mc\omega$ is the normalized rf field amplitude and $\upsilon^2 = e|\sigma_0|/m\varepsilon_0 r_d\omega^2$ is the so-called plasma parameter. The initial surface charge density, σ_0 can be expressed either through the surface particle density ρ_S, $\sigma_0 = e\rho_S$, or through the linear particle density ρ_L, $\sigma_0 = e\rho_L/2\pi r_d$.

One may solve the system of equations (9) - (11) numerically by using the Monte-Carlo algorithm and computing particle trajectories with random initial conditions. Assume that seed macro-particles with certain charge density appear on the surface of dielectric with some periodicity (instead of such periodic seeding one may choose random times for generating seed particles). They have random initial energies $E_0 = mv_0^2/2$ and emission angles θ_e obeying the following probability distribution functions [3]:

$$f(E_0) = \frac{E_0}{E_{0m}^2} e^{-E_0/E_{0m}} \tag{12}$$

$$g(\theta_e) = \frac{1}{2}\sin(\theta_e), \quad 0 < \theta_e < \pi \tag{13}$$

Here E_{0m} is the energy corresponding to the peak of the energy distribution function, which is of the order of the work function of the material. The particles leave the surface of dielectric due to the effect of the rf field (if it is in the accelerating phase) and start their motion in the vacuum region. During the motion, each particle produces dc field which affects motion of other particles. On each time-step, every particle gets checked whether it impacted the dielectric wall. If so, the secondary emission yield δ is calculated based on its impact energy V_i and impact angle θ_i relative to the surface normal. We use the well-known Vaughan model for these calculations [4]. After the value of δ is obtained, the particle weight gets multiplied by the one and the result is checked against limiting values. If the new weight $w'_n = \delta w_n$ is greater than some upper limit, w_{max}, then the particle gets split into m new (secondary) particles, where m is equal to $w'_n / w_{\max}$ rounded up to the nearest integer. The weights of the new particles therefore become equal to w'_n / m. Each new particle is then assigned a random emission velocity and angle in accordance with expressions (12) and (13). If w'_n is smaller than some lower limit, w_{min}, the particle gets removed from further calculations. At each time step the particles get sorted by height which simplifies the calculation of dc fields.

In order to calculate the power loss induced by MP, one may start with the instant power of one electron:

$$p = \frac{d\varepsilon_k}{dt} = -ev_r(E_{dc} + E_{r1}) \cdot \tag{14}$$

Then, using the same simplifications and normalized variables that were introduced while deriving (9)-(11), one may obtain

$$p = m\omega^3 r_d^2 (\tilde{r}\tilde{v}_r\alpha\sin(\tilde{t}) + \upsilon^2\frac{\tilde{v}_r}{\tilde{r}} w_\Sigma) \cdot \tag{15}$$

Here $\upsilon^2 w_\Sigma$ is the normalized dc field acting on the particle. The average power of MP over an rf period can be obtained by adding up instant powers of all electrons and integrating the result over the rf period:

$$P_{MP} = \frac{\rho_L}{2\pi}\int_0^{2\pi} d\tilde{t} \sum_{n=1}^{N_{total}} w_n p_n \cdot \tag{16}$$

Here ρ_L is the linear particle density introduced above and, therefore, P_{MP} has dimensions W/m.

RESULTS OF SIMULATIONS

Some simulations results are shown in Figs. 2-4. Fig. 2 shows the normalized amplitude of the dc field as function of time. Results are obtained for the set of experimental parameters listed above and two values of rf amplitude that are indicated in the figure. Other parameters in these calculations are the following: the surface seed particle density $\rho_S = 10^9$ m^{-2}, maximum yield value at normal incidence $\delta_{max0} = 3$, impact energy corresponding to the maximum yield $V_{max0} = 600$ eV and material smoothness factors $k_{sV} = k_{s\delta} = 1$ (see Table I in Ref. 3, values for alumina).

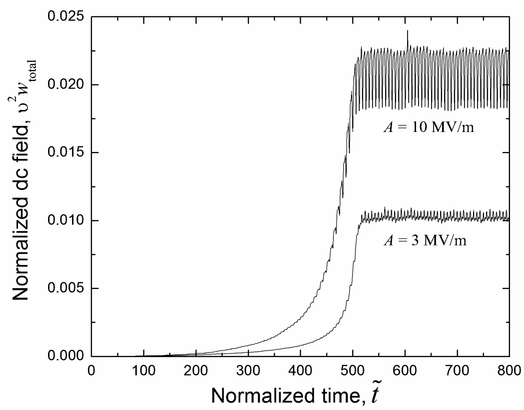

Figure 2: Number of macroparticles (a) and normalized total dc field (b) as functions of time for two values of the rf amplitude.

Figure 3 shows the ratio of MP-induced loss (calculated for 20 cm long structure, cf. Fig. 1 of Ref. 1) to rf power flowing in the structure. These results qualitatively agree with the ones shown in Fig. 3 of that paper. To perform more accurate comparison, the precise secondary yield data for the material used in the experiment should be known. In our calculations we used approximate values from the corresponding range for alumina. One may also notice in Fig. 3 that at some point losses saturate and become insensitive to further increase of the rf amplitude.

The model also can be used for estimation of multipactor onset characteristic time in a given structure. This time depends on the rf amplitude, secondary yield properties of the dielectric material and the seed particle density. Fig. 4 shows the normalized dc field as function of time for two values of seed particle density. One may see that at lower density it takes longer for the process to saturate while the saturation level in both cases is the same. Therefore, 'cleaner' structures with lower seed density are less prone to multipactor occurrence. Also, if the rf pulse is short enough, it is possible to avoid multipactoring in such structure completely.

SUMMARY

We have developed a 2-D model of multipactor which allows studying the effect for a reasonably small set of parameters. It uses a self-consistent time-dependent approach allowing analysis of the process dynamics and obtaining important time constants. In particular, it allows estimating the growth rates and times required for multipactor onset in the given structure. In comparison with available PIC codes, which also use self-consistent approach, our model requires significantly less time for calculations and, therefore, is suitable for rather broad studies. Preliminary results of our calculations qualitatively agree with the ones obtained both experimentally and theoretically by other research teams. To perform a more detailed comparison, we need to verify our results with more experimental data for various DLA structures.

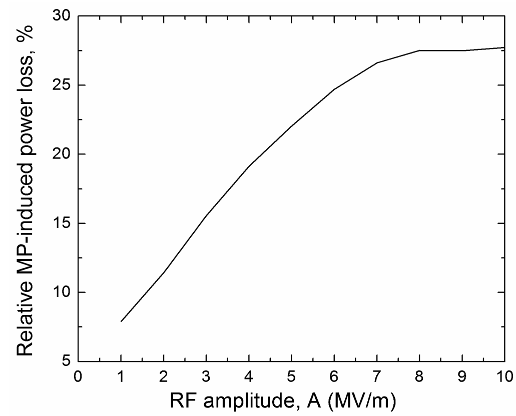

Figure 3: Ratio of MP-induced loss to power flowing in the structure as function of the rf amplitude. Calculations are done for a 20 cm long structure shown in Fig. 1 of Ref. 1.

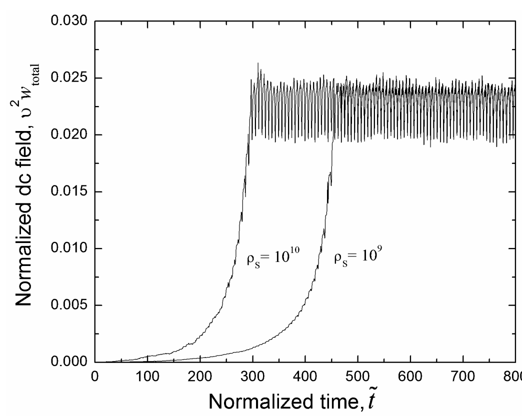

Figure 4: Normalized total dc field as function of time for two values of the seed particle density. The rf amplitude is equal to 10 MV/m in both cases.

REFERENCES

[1] J. G. Power, W. Gai, S. H. Gold et al., Phys. Rev. Lett., 92, 164801 (2004).
[2] J. G. Power and S. H. Gold, AIP Conf. Proc., 877, 362 (2006).
[3] R. A. Kishek and Y. Y. Lau, Phys. Rev. Lett., 80, 193 (1998).
[4] J. R. M. Vaughan, IEEE Trans. Electron Devices, 36, 1963 (1989).