# BEAM OPTICS STUDY ON THE EXTRACTION REGION FOR A HIGH INTENSITY COMPACT CYCLOTRON * 

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## Abstract

At CIAE, the China Institute of Atomic Energy, a high intensity compact $\mathrm{H}^{-}$cyclotron, CYCIAE-100, is being constructed to provide two 75 to 100 MeV proton beams in opposite directions. In the extraction region, the main magnet field and its fringe field, as well as the field of the combination magnet, will all influence the beam optics. The dispersion of the extracted beam can not be ignored and will lead to beam envelope and emittance increases in the radial dimension. A radiation problem may occur that must be controlled. To study the beam optics in this region, orbit tracking and transfer matrix calculations, including the symplectic condition by function extension of the code STRIPUBC and modification of GOBLIN, have been implemented. The characteristics of the extracted beams have been investigated in detail on the basis of the main field from a FEM code, that overlaps with the fields generated by the combination magnets. Results have also been compared with those from CIAE's code CYCTRS. These confirm the validity of the prediction. The transfer matrix from this simulation was analysed and used for the down-stream beam line design.

## INTRODUCTION

CYCIAE-100, a compact high intensity cyclotron, was selected as the driver for the Beijing Radioactive Ion Facility (BRIF) [1]. A $200 \mu \mathrm{~A} \mathrm{H}^{-}$beam will be accelerated and two proton beams between 75 and 100 MeV will be extracted in opposite directions using carbon stripping foils. Each extracted beam will be transported to its beam line by a combination magnet. The extracted particles will pass through the cyclotron field, the fringe field and the field of the combination magnet. Because of the compact configuration of this cyclotron, the influence of the fringe field on the beam optics is more critical in comparison with separated sector machines. To study this optics, three codes: STRIPUBC, GOBLN and CYCTRS were used $[2,4]$. The positions of the stripping foils for different extraction energies were calculated and the transfer matrices including the symplectic condition from the stripper to the exit of the combination magnet was analysed. The extracted beam optics was studied in detail with orbit tracking. The results are compared in detail below.

## TRANSFER MATRIX, TRAJECTORIES AND DISPERSION AT EXTRACTION

[^0]Upstream the stripping foil, the radial equations of motion are:

$$
\begin{align*}
& \frac{d r}{d \theta}=\frac{r P_{r}}{P_{\theta}} \quad ; \quad \frac{d P_{r}}{d \theta}=P_{\theta}+q r B_{z} \\
& \frac{d \phi}{d \theta}=\frac{\gamma m_{0} \omega_{0} r}{P_{\theta}}(1+\varepsilon)-1 \tag{1}
\end{align*}
$$

where (r, $\operatorname{Pr}, \varphi$ ) are the coordinates of the reference particle, $\phi=\omega_{0} t-\theta$, and $\theta$ is the independent variable.

Downstream, the equations of motion are:

$$
\begin{align*}
& \frac{d \theta}{d s}=\frac{P_{\theta}}{r P} \quad ; \quad \frac{d r}{d s}=\frac{P_{r}}{P} \\
& \frac{d P_{r}}{d s}=\frac{d P_{r}}{d \theta} \frac{d \theta}{d s}=\frac{P_{\theta}}{r P}\left(P_{\theta}+q r B_{z}\right) \tag{2}
\end{align*}
$$

where s is the independent variable.
We then transported the equations of motion of the particles to Cartesian coordinates (transverse and longitudinal phase-space): (x, x', y, y’, z, $\delta$ ).

In a linear approximation, we can write:

$$
\begin{equation*}
\left(x, x^{\prime}, z, z^{\prime}, z, \delta\right)^{T}=R\left(x_{0}, x_{0}^{\prime}, z_{0}, z_{0}^{\prime}, z_{0}, \delta_{0}\right)^{T} \tag{3}
\end{equation*}
$$

where R is the $6 \times 6$ transfer matrix between $\left(x_{0}, x_{0}^{\prime}, z_{0}, z^{\prime}{ }_{0}, z_{0}, \delta_{0}\right)$, the positional and longitudinal coordinates at the stripping foil, and ( $x, x^{\prime}, z, z^{\prime}, z, \delta$ ), the conditions along the trajectory. By integrating numerically equations 1 and 2 in the new coordinate system and using 5 particles with initial conditions $(1,0,0,0,0),(0,1,0,0,0) \ldots(0,0,0,0,1)$, and imposing the symplectic condition, we obtained the full matrix with STRIPUBC. Thus the matrix elements will satisfy the following:

$$
\left|\begin{array}{ll}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{array}\right|=1,\left|\begin{array}{ll}
a_{33} & a_{34} \\
a_{43} & a_{44}
\end{array}\right|=1,\left|\begin{array}{ll}
a_{11} & a_{12} \\
a_{51} & a_{52}
\end{array}\right|=a_{16},\left|\begin{array}{ll}
a_{21} & a_{22} \\
a_{51} & a_{52}
\end{array}\right|=a_{26}
$$

where $\mathrm{a}_{16}$ and $\mathrm{a}_{26}$ define the dispersion.

## CODE INTRODUCTION

The stripping points for various energies have been calculated with code CYCTRS[4], and checked separately
with codes GOBLIN and STRIPUBC. Transfer matrices were obtained numerically with STRIPUBC and GOBLIN.

STRIPUBC [2], was developed at TRIUMF in 1970's for the 500 MeV cyclotron and uses LINUX. We translated it from LINUX to WINDOWS for our machine. In the code, we expanded the calculation region beyond the combination magnet. The results from the code were verified with GOBLIN and CYCTRS.

GOBLIN is the well known particle tracking computer code that uses polar coordinates in cyclotrons. The field of the combination magnet was transformed from Cartesian to polar coordinates and a new function was added to calculate $\mathrm{R}_{56}$.

CYCTRS [4] was developed at CIAE in Cartesian coordinates. It tracks particles using RK integration and finds the position of the stripping foils.

## BEAM SIMULATION

## Trajectory and Position of Stripping Foils

The magnetic field on the median plane as seen by the extracted beam is shown in Fig. 1. Two combination magnets bend particles with different energies to be aligned along two external beam line $180^{\circ}$ apart. The magnetic field at extraction will be different for different energies, the highest field being 5.22 kG .


Figure 1: Magnetic field along the extracted beam path.

The positions of stripping foils are given in Table 1 as calculated with STRIPUBC and confirmed by CYCTRS. For the same desired energy, the difference in extraction radius is $0.12 \%$ at 70 MeV and $0.0017 \%$ at 100 MeV .

Beam trajectories for extracted energies between 70 MeV and 100 MeV calculated with STRIPUBC and GOBLIN, are shown in Fig. 2. Programmes are in agreement.

Table 1: Positions of Stripping Foils for Different Extraction Energies

| $\mathbf{E}(\mathbf{M e V})$ | R(cm) | THETA(degree) |
| :--- | :--- | :--- |
| 100 | 187.554 | 59.956 |
| 90 | 179.370 | 59.308 |
| 80 | 170.565 | 58.725 |
| 70 | 160.983 | 58.122 |



Figure 2: Extracted trajectories for $70 \mathrm{MeV}, 80 \mathrm{MeV}$, 90 MeV and 100 MeV particles.

## Transfer Matrix and Dispersion

The transfer matrices calculated with GOBLIN and STRIPUBC for 100 MeV particles are shown in Table 2 and Table 3. The agreement is good.

Table 2: Transfer Matrix Calculated with GOBLIN

$$
\left[\begin{array}{cccccc}
1.040 & 0.210 & 0 & 0 & 0 & 65.9 \\
-0.067 & 0.949 & 0 & 0 & 0 & 414.6 \\
0 & 0 & 0.427 & 0.190 & 0 & 0 \\
0 & 0 & -3.123 & 0.950 & 0 & 0 \\
-0.435 & -0.024 & 0 & 0 & 1 & 167.7 \\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
$$

Table 3: Transfer Matrix Calculated with STRIPUBC

$$
\left[\begin{array}{cccccc}
1.039 & 0.210 & 0 & 0 & 0 & 65.9 \\
-0.051 & 0.952 & 0 & 0 & 0 & 415.7 \\
0 & 0 & 0.428 & 0.190 & 0 & 0 \\
0 & 0 & -3.127 & 0.947 & 0 & 0 \\
-0.435 & -0.024 & 0 & 0 & 1 & 161.5 \\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
$$

Figure 3 shows the dispersion of beams at 70 MeV and 100 MeV . The field of the combination magnet will increase the dispersion at 100 MeV beam, but will reduce it at 70 MeV .

## Low and Medium Energy Accelerators and Rings



Figure 3: Beam dispersions at 70 MeV and 100 MeV .

## PARTICLES DISTRIBUTION

The beam distribution of the 100 MeV beam at the stripping foil, was obtained with COMA [3], and shown in the upper part of Fig. 4. The distribution at the exit of the combination magnet is shown in the lower part of Fig. 4. A simple programme was written to solve $\sigma_{2}=R \sigma_{1}$, where $\sigma 1$ represent ( $\mathrm{x}, \mathrm{x}^{\prime}, \mathrm{y}, \mathrm{y}^{\prime}, \mathrm{z}, \delta$ ) at the stripping foil and $\sigma 2$ at the exit of the combination magnet; R is the transfer matrix.

The beam emittance and envelope in the $x-x$ ' space increase because of dispersion effects. $x-x$ ' and $y-y$ ' distributions will be used as the input for beam line design.

## SUMMARY

The positions of stripping foils for different extraction energies are calculated and the transfer matrix satisfying the symplectic condition from stripper to the exit of the combination magnet was analysed with codes STRIPUBC, GOBLIN and CYCTRS [2,4]. Extraction beam optics for CYCIAE-100 was studied in detail through beam tracking. The dispersion can't be ignored and will lead to beam envelope and emittance increase in the radial direction. Results are used for beam line design.

## REFERENCES

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Figure 4: Distribution of 100 MeV particles at the stripper and the exit of the combination magnet .


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