COUPLING RESONANCE Qx-Qy =0 AND ITS CORRECTION IN AXIAL INJECTION CHANNEL OF THE CYCLOTRON

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Abstract

In axial injection channels of FLNR JINR cyclotrons the axial symmetric ion beam is formed just after the analyzing bending magnet. This gives an opportunity to use for beam focusing at vertical part of the channel solenoidal magnetic lenses only. During the motion of intense axial symmetric beam in the longitudinal magnetic field of solenoids and cyclotron the transverse tunes Qx, Qy coincide. In this case the small disturbance of beam axial symmetry leads to excitation of coupling resonance Qx-Qy=0 due to beam self-fields. The influence of the resonance results in significant asymmetry of the transverse beam emittances. The magnitude of this asymmetry is evaluated within the framework of moments method and is in a good agreement with one obtained in the macro-particles simulation. The correction of resonance by means of the normal quadrupole lens is proposed.

INTRODUCTION

Modernization of cyclotron U400 – project U400R [1] will reduce to 1.8 T the maximum cyclotron magnetic field and to four times the consumption of electric power. Besides the intensity of accelerated 48Ca ions will increase up to 3 pμA. Thereby the intensity of injected Calcium ions has to be increased up to 30 pμA. This can be achieved by using the scheme of axial injection channel of DC350 cyclotron [2] shown in Fig. 1.

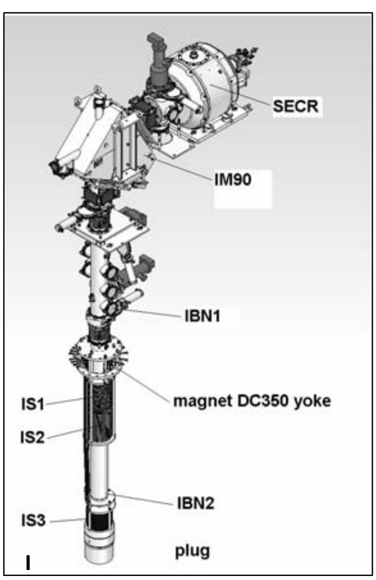

Figure 1: DC350 axial injection channel layout.

The channel consists of Superconducting Electron Cyclotron Resonance ion source (SECR) and analyzing bending magnet IM90 placed at horizontal part of the beam line. The focusing solenoids IS1-IS3, linear IBN1, sinusoidal IBN2 bunchers and spiral inflector I are placed at vertical part of the channel.

The beam of $^{48}Ca^{7+}$ ions with current 210 μA was considered in simulation. The extraction voltage of SECR ion source is equal to 26 kV. The beam emittance defined by SECR magnetic field had value 150 π mm×mrad. The beam envelopes and emittances during transportation in the beam line are shown in Fig.2-3.

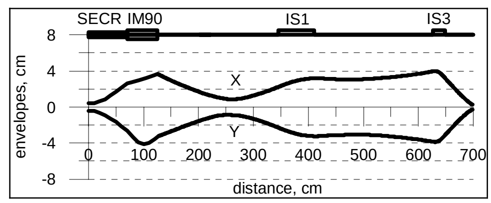

Figure 2: Horizontal (X) and vertical (Y) beam envelope.

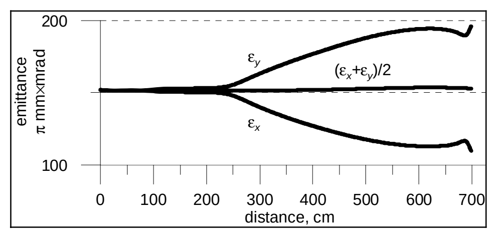

Figure 3: Horizontal (ε_x) and vertical (ε_y) beam emittance.

Just after bending magnet IM90 the beam has almost axial symmetric form (see Fig.2). The final value of vertical emittance is two times greater than horizontal one while the sum of emittances is constant (see Fig.3).

The effect of emittance asymmetry growth may be explained by excitation of coupling resonance Qx-Qy=0 due to beam self-fields. The magnitude of this asymmetry is evaluated within the framework of moments method and is in a good agreement with one obtained in the macro-particles simulation.

The correction of this resonance may be implemented by means of the normal quadrupole lens installed just after bending magnet IM90.

MOMENTS METHOD EQUATIONS

Let us introduce the matrix M of second order moments of the beam distribution function f:

$$M = \overline{ZZ^T} = \frac{1}{N}\int ZZ^T\, fdz \qquad (1)$$

Here N – number of particle per unit beam length, superscript "T" defines transpose vector or matrix,

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$Z^T = (x, y, x', y')$ – phase space vector, integration in (1) is fulfilled over transversal phase space occupied by particles. Matrix M satisfies to equations [3]:

$$M' = AM + MA^T;\, M = \begin{pmatrix} M_{xx} & M_{xv} \\ M_{xv}^T & M_{vv} \end{pmatrix};\, A = \begin{pmatrix} 0 & E \\ b & 0 \end{pmatrix} \qquad (2)$$

Where (2×2) matrix M_{xx} is defined by rms dimensions $r_x = \sqrt{\overline{x^2}}$, $r_y = \sqrt{\overline{y^2}}$, M_{vv} – by angular spreads of the beam, $b = b_{ext} + b_s$ – by quadrupole coefficients of external and self fields, respectively, E is unit matrix.

In linear for correlation term $r_{xy} = \overline{xy} / r_x r_y$ approximation matrix b_s has the following form [3]:

$$b_s = \frac{Q}{(r_x + r_y)^2} \begin{pmatrix} 1 + r_y / r_x & -r_{xy} \\ -r_{xy} & 1 + r_x / r_y \end{pmatrix} \qquad (3)$$

Here $Q = \frac{Z}{A} \frac{I}{I_A} \frac{1}{(\beta\gamma)^3}$, Z/A – charge-to-mass ratio of ion, I – beam current, I_A – Alfven's current, γ, β – ion relativistic factor and velocity, respectively.

It should be noted that for motion in longitudinal magnetic field representation of the matrix A in form (2) is valid in coordinate frame rotating with Larmor's frequency around the longitudinal axis.

In the case of uncoupled motion (matrix b is diagonal) the system (2) has 2D invariant:

$$\overline{xy}\,\overline{x'y'} - \frac{1}{4}\left(\overline{xy'} + \overline{x'y}\right)^2 + \frac{1}{4} j^2 = const\,;\; j = \overline{xy'} - \overline{x'y} \qquad (4)$$

where j is average angular momentum of the beam. In the axial symmetric case ($\overline{xy} \equiv 0$) we have

$$j = const \qquad (5)$$

In the case of motion in external longitudinal magnetic field in the presence of beam self field the conservation law (5) is valid for arbitrary matrix M_{xx}. Indeed anti-symmetric matrix $J = M_{xv} - M_{xv}^T$ ($J_{12} = -J_{21} = j$) in accordance with system (2) satisfies the equation:

$$J' = b_s\, M_{xx} - M_{xx}\, b_s = 0 \quad , \qquad (6)$$

as matrices b_s and M_{xx} are commutative [3].

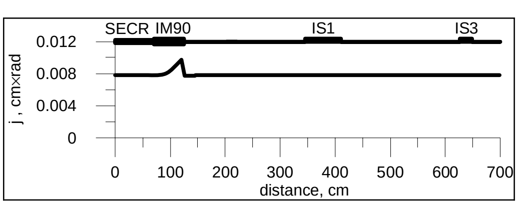

Figure 4: Average angular momentum of $^{48}Ca^{7+}$ beam.

Dependence of the average angular momentum of $^{48}Ca^{7+}$ ion beam on distance along the beam line is shown in Fig.4. Before analyzing magnet IM90 beam moving in the longitudinal field of SECR source and the average angular momentum is constant. Inside the magnet the value of j is changing due to a violation of axial symmetry of the magnetic field and beam. In accordance with (4) at the exit of magnet average angular momentum j recovers the initial value, because of beam become almost axial symmetric. At the vertical part of the beam line value of j does not change since beam moving in the axially symmetric longitudinal magnetic field of solenoids IS1, 3 and cyclotron U400R.

The conservation law of average angular momentum (in the lab frame this is equivalent to Bush theorem) was used in [4] for determination of the ions distribution at the entrance of the spiral inflector.

RESONANCE Qx-Qy=0

For almost axial-symmetric beam ($r_x \cong r_y$) the rms emittances $\varepsilon_{x,y\,rms}$ in accordance with system (2) satisfy the following equation:

$$(\varepsilon_{x\,rms}^2)' - (\varepsilon_{y\,rms}^2)' = \frac{Q}{2} r_{xy}\, j \qquad (7)$$

The second equation we have from general theory of coupling resonances of betatron oscillations:

$$\varepsilon'_{x\,rms} + \varepsilon'_{y\,rms} = 0 \qquad (8)$$

It is verified by results of simulation (see Fig.3). The solutions of system (7, 8) are:

$$\begin{aligned} \varepsilon_{x\,rms} &= \varepsilon_{x\,rms}^0 + \frac{1}{4}\int_0^s Q(s')\, r_{xy}(s')ds' \\ \varepsilon_{y\,rms} &= \varepsilon_{y\,rms}^0 - \frac{1}{4}\int_0^s Q(s')\, r_{xy}(s')ds' \end{aligned} \quad , \qquad (9)$$

where superscript "0" denotes the initial values of parameters and equality $j \cong 2\varepsilon_{x\,rms}^0 = 2\varepsilon_{y\,rms}^0$ [4] was used.

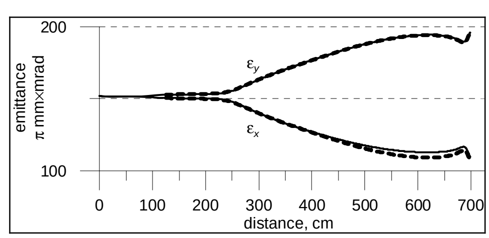

Figure 5: Horizontal (ε_x) and vertical (ε_y) beam emittance.

The dependence of beam emittances on distance along the beam line (9) is shown in Fig.5 (dashed lines). These results are in in a good agreement with ones obtained in the macro-particles simulation (solid line).

RESONANCE CORRECTION

The excitation of the resonance is caused by the violation of axial symmetry of the beam, resulting in the difference from zero of correlation coefficient r_{xy}. It occurs in the bending magnet IM90 as shown in Fig.6.

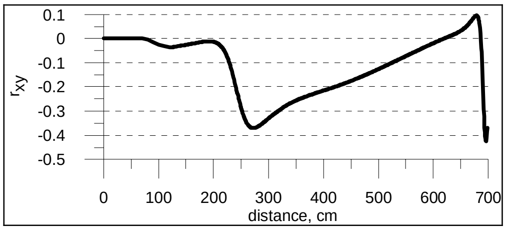

Figure 6: Correlation coefficient r_{xy} (without correction).

In accordance with the formula (9) the beam emittance asymmetry may be reduced by decreasing the average value of correlation coefficient r_{xy} at the vertical part of the channel. This is achieved by using Panofsky lens installed just after bending magnet IM90. The small gradient of quad (-3 Gauss/cm at 20 cm effective length) gives possibility to decrease the r_{xy} average value to 20 times. The correlation coefficient after correction is shown in Fig.7.

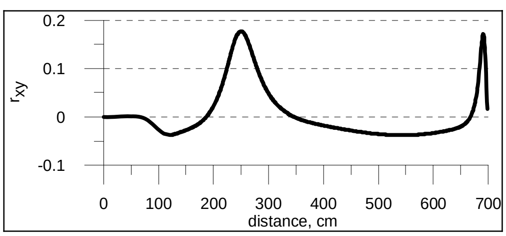

Figure 7: Correlation coefficient (after correction).

The dependence of beam emittances on distance along the beam line after correction of resonance is shown in Fig.8.

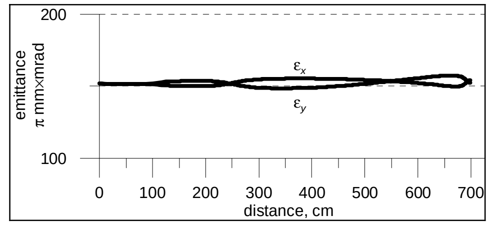

Figure 8: Horizontal (ε_x) and vertical (ε_y) beam emittance (after correction).

As a result of correction the horizontal and vertical beam emittances have practically the same value.

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