FFAGS AND CYCLOTRONS WITH REVERSE BENDS

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Abstract

This paper describes tracking studies of FFAGs and radial-sector cyclotrons with reverse bends using the cyclotron equilibrium orbit code CYCLOPS. The results for FFAGs confirm those obtained with lumped-element codes, and suggest that cyclotron codes will prove to be important tools for evaluating the measured fields of FFAG magnets. The results for radial-sector cyclotrons show that the use of negative valley fields would allow axial focusing to be maintained, and hence allow intense cw beams to be accelerated, to energies ≈10 GeV.

INTRODUCTION

Isochronous cyclotrons and FFAGs are both members of the fixed-magnetic-field or cyclotron family [1]. Thus the former may be regarded as simply a special case of FFAGs where the orbital period is fixed, and the latter as just sector-focused ring synchrocyclotrons. Nevertheless, they have been developed by two different communities, which have sometimes taken different approaches in their work. The studies described here bridge this gap to some extent by exploring the use of techniques developed for one type of accelerator in the other:

1. Applying isochronous cyclotron orbit codes to FFAGs.
2. Using reverse-bend magnets (as in radial-sector FFAGs) to enhance the axial focusing and extend the energy range of radial-sector isochronous cyclotrons.

FFAG TRACKING STUDIES USING THE CYCLOTRON CODE CYCLOPS

In recent years FFAG designs have generally been developed using synchrotron lattice codes – or adaptations of them – perhaps because their designers have mostly come from a synchrotron background. But synchrotron codes are poorly adapted for use in accelerators with fixed magnetic fields, where the central orbit is a spiral rather than a closed ring, so that the magnetic field must be characterized over a wide radial range. Special arrangements must therefore be made to deal with momentum-dependent effects accurately.

Here, we report studies made with the cyclotron orbit code CYCLOPS [2], which tracks particles through magnetic fields specified on a polar grid and determines the equilibrium orbits (E.O.) at each energy and their optical properties. This has the advantages of:

- being designed for multi-sector machines with wide aperture magnets;
- allowing simultaneous computation of orbit properties at all energies;
- having the capability of tracking through measured magnetic fields;
- the availability of its sister code GOBLIN for studies of accelerated orbits.

We have studied three very different FFAG lattices.

F0D0-2

Our first test of CYCLOPS on an FFAG lattice was made with F0D0-2, an 82-cell lattice designed by J.S. Berg [3] for accelerating muons from 10 to 20 GeV. Both the positive-bending D and negative-bending F are sector magnets, in which the field magnitudes decrease outwards with constant gradient. (This is a "linear non-scaling" or "LNS" design, characterized by betatron tunes which fall sharply with energy.). The CYCLOPS results agreed very closely with Berg's for all the parameters examined (orbit radius, beta functions, tunes, and orbit time) over the full energy range, as illustrated in our initial report on these studies [4]. Hard magnet edges were assumed, but it was possible to get good E.O. solutions provided a fine enough (400 × 800) r-θ field grid was used for the lattice cell.

Tune-Stabilized Medical FFAG

Johnstone and Koscielniak have developed an LNS FFAG for cancer therapy with 18-400 MeV/u carbon ions [5]. Like Berg's, this is based on a F0D0 lattice, but uses edge- as well as gradient-focusing to minimize the tune variation. But non-radial hard magnet edges proved tricky to model with a polar grid – and, even with 37 million grid points, led to noisy results from CYCLOPS – typically ±0.3 in the tunes. To smooth the field's hard edges we have introduced a sinusoidal field variation – an approximate but effective procedure (Figure 1).

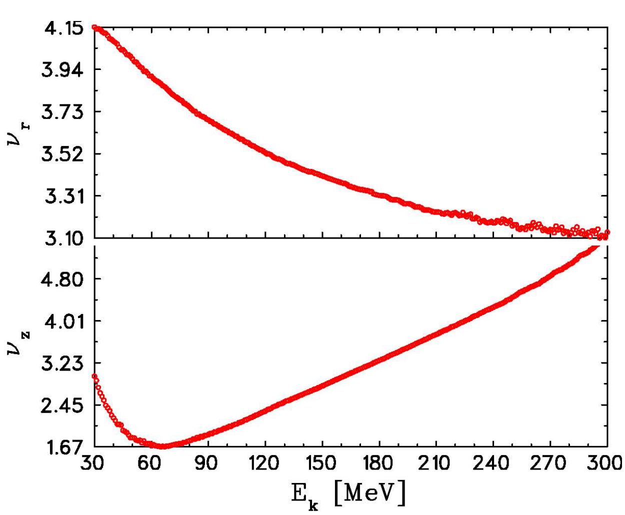

Figure 1: Tunes in the tune-stabilized medical FFAG computed by CYCLOPS.

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IFFAG

Rees [6] has proposed an isochronous radial-sector FFAG design (IFFAG) for accelerating muons from 8 to 20 GeV. This employs a novel five-magnet "pumplet" 0doFoDoFod0 lattice cell (from the Welsh word pump, pronounced pimp, for five), where the d magnets (and Fs at low energy) are reverse bending, and the d, F and D magnets each have special field profiles *B(r)*. With long drift spaces between the d magnets, and $N = 123$ cells, the circumference is 1255 m.

Méot *et al*. [7] have used the ray-tracing code ZGOUBI (originally developed for the study and tuning of mass spectrometers and beam lines.) to follow muons through a simulated field grid and confirm the orbit properties Rees predicts: good isochronism, and tunes that rise gently with energy, though ν_z exhibits some deviations (Figure 2).

Achieving isochronism and vertical focusing at such high energies is a remarkable feat. Isochronism at a kinetic energy of $(\gamma - 1)m_0c^2$ implies that the average field $B_{av} \equiv \langle B(\theta)\rangle \sim \gamma$ has a large positive radial gradient, contributing strong vertical defocusing

$$\Delta\nu_z^2 = -k = -\beta^2\gamma^2, \qquad (1)$$

where the field index $k \equiv (r/B_{av})dB_{av}/dr$. For muons at 20 GeV, $\beta^2\gamma^2 \approx 36{,}000$! Cyclotrons and FFAGs conventionally overcome this gradient defocusing by edge and spiral focusing: to first order [8],

$$\nu_z^2 \approx -k + F^2(1 + 2\tan^2\varepsilon), \qquad (2)$$

where the magnetic flutter $F^2 \equiv \langle(B(\theta)/B_{av} - 1)^2\rangle$ and ε is the spiral angle. But in Rees's lattice $\varepsilon = 0$, and F^2 is only ≈ 30 at 20 GeV. The key to this apparent inconsistency is that (2) was derived for two-magnet cells: by using more magnets, Rees has gained additional free parameters.

Because of its greater complexity, this lattice presented a greater challenge to producing an adequately detailed field map for CYCLOPS. With hard edges the tune values were sensitive to mesh size, and at some energies it was impossible to obtain orbit closure. Again the use of sinusoidal edges was effective. The tunes obtained with these (Figure 2) agree fairly well with those published by Rees and Méot, except above 15 GeV. We are currently checking our field grid to confirm that it does accurately represent Rees's lattice.

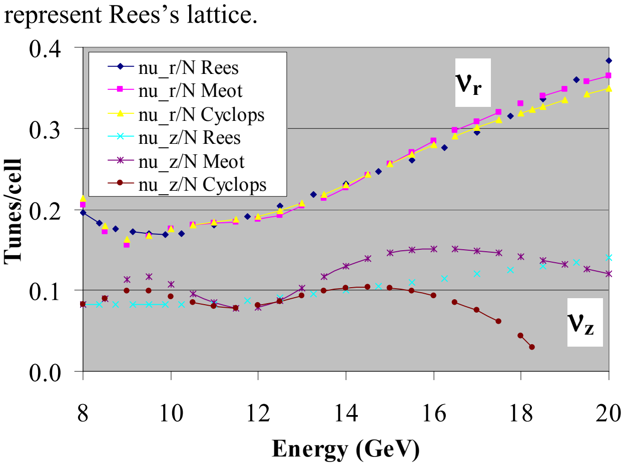

Figure 2: Betatron tunes in the isochronous IFFAG, as computed by Rees, Méot and CYCLOPS.

HIGH-ENERGY CYCLOTRONS

In the past, designs have been presented for isochronous ring cyclotrons to accelerate protons from 0.5 to 3.5 GeV and from 3.5 to 10 or 15 GeV [9, 10], to provide cw, and therefore high-intensity, beams at these energies, using the 500-MeV TRIUMF cyclotron as injector. These designs relied on spiral edge focusing to supplement the flutter factor in (2); axial stability was confirmed by tracking orbits through simulated magnetic fields using CYCLOPS. The feasibility of extracting the beam efficiently by exciting a radial resonance was also confirmed [11] using the general orbit code GOBLIN.

The high spiral angles ε required, however, lead to various practical problems: strong distorting forces on the magnet coils (particularly if these are superconducting), restricted space for rf cavities and injection and extraction equipment, and strong radial kicks during acceleration.

Cyclotrons with Reverse Bends

In view of Rees's intriguing results, and the practical difficulties presented by superconducting spiral magnets, it seemed interesting to explore how far the energies of radial-sector cyclotrons could be raised by inserting reverse-bend magnets to increase the flutter. This being an exploratory study, we made the simplest possible assumptions: N radial sectors, hard-edge magnets, no drift spaces, and equal but opposite hill and valley fields:

$$B_h = -B_v = B(r) = \gamma B_0 \,. \qquad (3)$$

The resultant cyclotron parameters are derived in [4]. Here we quote the main results. If the angular width of the hills is denoted by $2\pi h/N$, then the magnetic flutter (the same at all energies) is given by:

$$F^2 = \tfrac{1}{4}(h - \tfrac{1}{2})^{-2} - 1 \,, \qquad (4)$$

so to maintain positive axial focusing up to some maximum energy γ_m , but no further, (1) and (2) tell us that:

$$h - \tfrac{1}{2} = 1/2\gamma_m. \qquad (5)$$

Assuming that the maximum magnetic field available, B_m, is applied at maximum energy γ_m, then the "cyclotron radius" R_c (the constant factor in $R = \beta R_c$) is given by:

$$R_c = (m_0c/e)\gamma_m^2/B_m \,. \qquad (6)$$

The recipe for hill and valley field strength is:

$$B(r) = (B_m/\gamma_m)/\sqrt{\{1 - (r/R_c)^2\}}. \qquad (7)$$

Note that Symon's circumference factor [8], the ratio of the actual circumference to that obtainable with the same maximum field, but no reverse bends:

$$C = \gamma_m \,. \qquad (8)$$

At first we chose the number of sectors $N \approx 3\gamma_m$ so that $\nu_r \approx \gamma$ remains well below the $N/2$ resonance.

As examples we have studied two cases with specifications similar to those of the spiral-sector ring cyclotrons mentioned above: one accelerating protons from 1 to 4 GeV, and the other from 3 to 14 GeV. We assume a maximum magnetic field B_m = 5 T. In the first case, $\gamma_m = 5$ leads to $N = 15$, $h = 0.6$, $F^2 = 24$, and $R_c = 15.65$ m. CYCLOPS was then run on a simulated magnetic field grid with *B(r)* calculated from (7). The CYCLOPS results [4] showed that both the flutter and the axial tune were lower than predicted, ν_z becoming imaginary above 3.4 GeV, where F^2 had dropped to 10.9. This occurs because the orbit scalloping causes the field seen by a proton to vary, rather than being piecewise constant, as the derivation had assumed. To remedy this, we produced a new field map with the B contours shaped to match the orbit arcs:

$$B(r,\theta) = (B_m/\gamma_m))/\sqrt{\{1-(R_{hv}/R_c)^2\}} \qquad (9)$$

where $R_{hv}(r,\theta)$ is the radius at which the orbit through (r,θ) crosses the hill-valley boundary. With this, the full theoretical flutter was obtained – but the focusing was too strong, both vertically and radially (Figure 3)! In fact the $N/2$ resonances ($\nu = 7.5$) were reached at only $\gamma = 3.5$. The CYCLOPS tunes, though, did agree well with those obtained from a lumped-element model. The lens strengths are apparently too great for the assumptions behind (2) (and the corresponding radial equation $\nu_r \approx \gamma$) to remain valid.

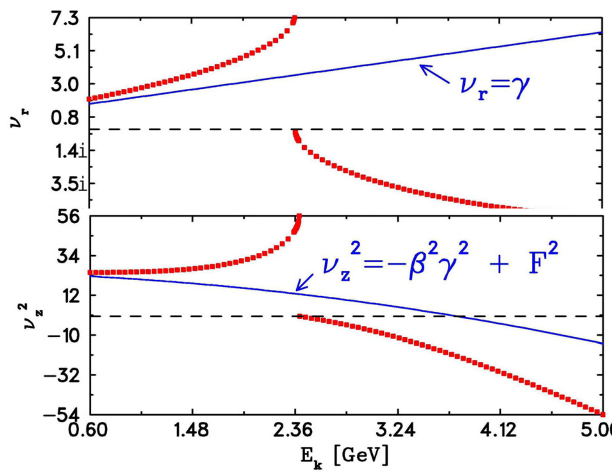

Figure 3: Tunes in a reverse-bend cyclotron with constant fields on orbit ($N = 15$, $h = 0.6$, $R_c = 15.65$ m).

Two methods of raising the top energy are possible. The first is to widen the hills, which reduces R_c, ν_r and ν_z. The lumped-element model shows stable orbits up to 3 GeV with $h = 0.65$, while R_c drops to 6.5 m (Figure 4).

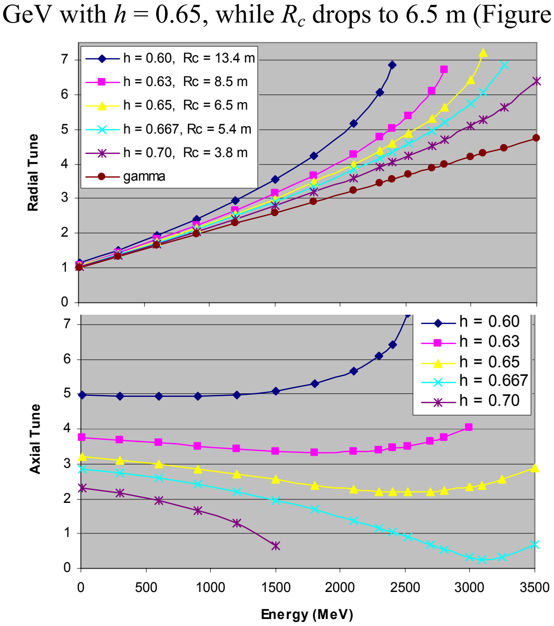

Figure 4: Tunes in a reverse-bend cyclotron with constant fields on orbit ($N = 15$, h and R_c as shown).

The second is to increase the number of sectors. With $N = 30$ and the original $h = 0.6$ the $\nu_r = 15$ resonance is raised above 5.5 GeV, while R_c remains at 14.9 m. The tunes computed by CYCLOPS are shown in Figure 5.

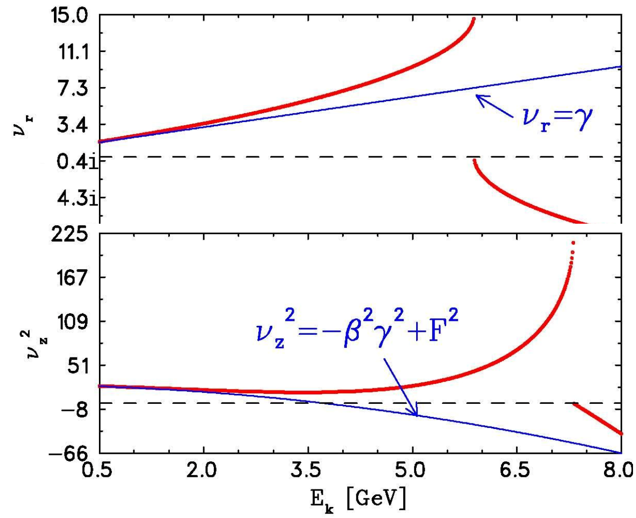

Figure 5: Tunes in a reverse-bend cyclotron with constant fields on orbit ($N = 15$, $h = 0.6$, $R_c = 15.65$ m).

For the 3-14 GeV cyclotron we took $\gamma_m = 15$, leading to $N = 45$, $h = 0.533^{\cdot}$, $F^2 = 224$, and $R_c = 140.83$ m. The tracking results for $B(r)$ independent of azimuth showed similar behaviour to those for the low-energy ring, with ν_z dropping to zero at 10.8 GeV [4]. We have yet to study the effect of tailoring the field contours to the orbits, but anticipate the same benefits as in the lower-energy case.

The radii for both rings are of course considerably larger than those for their spiral-sector counterparts (10 m and 41 m respectively), and in practice would be enlarged further by the inclusion of drift spaces for the rf cavities.

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