DESIGN OF PHOTONIC BANDGAP FIBER ACCELERATING STRUCTURE

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Abstract

Photonic crystals have been suggested for use as laser driven particle accelerator structures with higher accelerating gradients and effective damping of unwanted higher order modes. Here we selected Photonic band gap (PBG) fibers with hollow core defects to design such an accelerating structure. To achieve this design, Out-plane-wave mode in photonic crystal fiber was selected for longitudinal electric field. The out-plane-wave plane wave expansion method was deduced for confinement and the dispersive curve versus variation of k_z and speed of line for synchronization. Then super cell approximation was also introduced for calculating the defected photonic crystal structure. After the design of appropriate geometry and the dimensions of photonic crystal fiber accelerating structure, the field distribution was simulated with RSOFT Bandsolve software for this structure.

INTRODUCTION

Photonic bandgap fiber accelerator received more attention[1,2] because it has many advantages such as smaller structures, higher accelerating gradients, and the most important wake field suppression benefiting from using dielectric PBG structure.

In making this traveling wave accelerator, three requirements should be emphasized such as: longitudinal electric field, synchronization and confinement. We selected out-plane-wave in photonic crystal fiber for longitudinal electric field and deduced the out-plane-wave plane wave expansion method for confinement and calculated the dispersive curve versus variation of k_z and the speed of line for synchronization. Then super cell approximation was introduced for efficiently calculating the defected photonic crystal structure. The field distribution was simulated for this structure.

BAND GAP CALCULATION

To establish the traveling wave in photonic crystal fiber structure for particle accelerating, not only the longitudinal electric field E_z is demanded, but also the wave vector $k_z>0$ should be also considered. Here this traveling wave mode was called the out-of-plane-propagation.

Plane Wave Expansion Method for Out-Of-Plane Wave Method

The calculation of the band gap was made. Comparing with many methods, we selected a method which is one of the widely used tools and highly suited for analysis of dielectric periodic structures named with the plane wave expansion method (PWM).

For Non-magnetic materials (μ=1), the wave equation from Maxwell's equations can be deduced.

$$\nabla\times[\frac{1}{\varepsilon(\mathbf{r})}\nabla\times\mathbf{H}(\mathbf{r})]=\frac{\omega^2}{c^2}\mathbf{H}(\mathbf{r}) \quad (1)$$

Where c is the speed of light in vacuum

According to Bloch's theorem, a solution for the out-of-plane wave with $k_z>0$ to the wave equation (1) can be decomposed into:

$$\boldsymbol{H}(\boldsymbol{r})=\boldsymbol{H}(\boldsymbol{r}_\perp)e^{ik_z z}=\boldsymbol{H}_\perp(\boldsymbol{r}_\perp)e^{ik_z z}+\boldsymbol{H}_z(\boldsymbol{r}_\perp)e^{ik_z z} \quad (2)$$

Substitute (2) into Maxwell's equation (1), and after simplification, we can get the out-of-plane wave Maxwell's equation:

$$\{\nabla_\perp\times[\frac{1}{\varepsilon(\boldsymbol{r}_\perp)}\nabla_\perp\times\boldsymbol{H}_\perp(\boldsymbol{r}_\perp)]-\frac{1}{\varepsilon(\boldsymbol{r}_\perp)}\nabla_\perp(\nabla_\perp\cdot\boldsymbol{H}_\perp(\boldsymbol{r}_\perp)) \quad (3)$$
$$-(\frac{\omega^2}{c^2}-\frac{k_z^2}{\varepsilon(\boldsymbol{r}_\perp)})\boldsymbol{H}_\perp(\boldsymbol{r}_\perp)\}e^{ik_z z}=0$$

If we regard the wave vector k_z as a variable parameter, the equation can be further simplified into:

$$\nabla\times[\frac{1}{\varepsilon(\boldsymbol{r})}\nabla\times\boldsymbol{H}(\boldsymbol{r})]-\frac{1}{\varepsilon(\boldsymbol{r})}\nabla(\nabla\cdot\boldsymbol{H}(\boldsymbol{r}))-(\frac{\omega^2}{c^2}-\frac{k_z^2}{\varepsilon(\boldsymbol{r})})\boldsymbol{H}(\boldsymbol{r})=0 \quad (4)$$

To periodic structure, the distribution of dielectric constant is also periodic:

$$\varepsilon(\mathbf{r}+\mathbf{R})=\varepsilon(\mathbf{r}) \quad (5)$$

So we can get Fourier coefficients of $\varepsilon^{-1}(\mathbf{r})$ from Fourier-series expansion:

$$\varepsilon^{-1}(\mathbf{r})=\sum_{\mathbf{G}}\varepsilon^{-1}(\mathbf{G})e^{i\mathbf{G}\cdot\mathbf{r}} \quad (6)$$

According to Bloch's theorem, a solution to the wave equation(1) can be expressed as a plane wave modulated by a function with the same periodicity as the photonic crystal:

$$\mathbf{H}(\mathbf{r})=\mathbf{h}(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}} \quad (7)$$

Where $\mathbf{k}$ is the wave vector of the solution, and the periodic function $\mathbf{h}(\mathbf{r})$ can be expressed as a Fourier-series expansion in term of the reciprocal lattice vectors $\mathbf{G}$:

$$\mathbf{H}(\mathbf{r})=\sum_{\mathbf{G}}\mathbf{h}_{\mathbf{G}}e^{i(\mathbf{K}+\mathbf{G})\cdot\mathbf{r}} \quad (8)$$

Wherefore $\mathbf{h}_{\mathbf{G}}$ is the Fourier coefficients of $\mathbf{h}(\mathbf{r})$. Transfer (8) into vector form:

$$\begin{pmatrix}\boldsymbol{H}_x(\boldsymbol{r})\\ \boldsymbol{H}_y(\boldsymbol{r})\end{pmatrix}=\sum_{\boldsymbol{G}}\begin{pmatrix}\boldsymbol{h}_{\boldsymbol{G}x}\\ \boldsymbol{h}_{\boldsymbol{G}y}\end{pmatrix}e^{i(\boldsymbol{k}+\boldsymbol{G})\cdot\boldsymbol{r}} \quad (9)$$

Substitute (9) ,(5) into equation(4) and simplfy, then:

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$$\sum_{G}\begin{pmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{pmatrix}\begin{pmatrix} \boldsymbol{h}_{G'x} \\ \boldsymbol{h}_{G'y} \end{pmatrix} = \frac{\omega^2}{c^2}\begin{pmatrix} \boldsymbol{h}_{Gx} \\ \boldsymbol{h}_{Gy} \end{pmatrix} \quad (10)$$

where:

$$K_{xx} = \varepsilon^{-1}_{G-G'}[(k_y+G'_y)(k_y+G_y)+(k_x+G'_x)(k_x+G'_x)+k_z^2]$$
$$K_{xy} = \varepsilon^{-1}_{G-G'}[-(k_x+G'_x)(k_y+G_y)+(k_y+G'_y)(k_x+G'_x)]$$
$$K_{yx} = \varepsilon^{-1}_{G-G'}[-(k_y+G'_y)(k_x+G_x)+(k_x+G'_x)(k_y+G'_y)]$$
$$K_{yy} = \varepsilon^{-1}_{G-G'}[(k_x+G'_x)(k_x+G_x)+(k_y+G'_y)(k_y+G'_y)+k_z^2] \quad (11)$$

The final step required to perform analysis of a 2D triangular lattice photonic crystal is to determine the Fourier coefficients of $\varepsilon^{-1}(\mathbf{r})$:

$$\varepsilon^{-1}(\mathbf{G}) = \begin{cases} \frac{1}{\varepsilon_1} f + \frac{1}{\varepsilon_2}(1-f), \langle \mathbf{G}=0 \rangle \\ 2(\frac{1}{\varepsilon_1}-\frac{1}{\varepsilon_2}) f \frac{J_1(\mathrm{Ga})}{\mathrm{Ga}}, \langle \mathbf{G} \neq 0 \rangle \end{cases} \quad (12)$$

Where ε_1 is the dielectric constant of air hole and ε_2 is the dielectric constant of silicon. J_1 is the first order Bessel function,

$$f = \frac{\pi r^2}{\frac{\sqrt{3}}{2}a^2} \quad (13)$$

a is the lattice constant, r is the radius of air hole and f is filling fraction:

Band Gap Calculation and Result Analysis

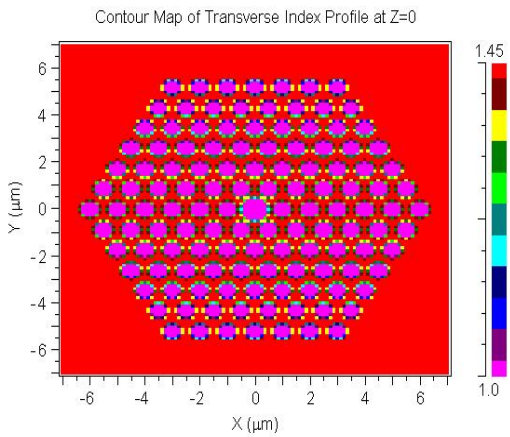

Figure 1: The transverse index profile of our PCF accelerator.

Fig.1 illustrates the transverse profile of our photonic crystal fiber accelerating structure, here the dielectric constant of air hole:

$$\varepsilon_1 = 1 \quad (14)$$

And the dielectric constant of silicon background:

$$\varepsilon_2 = 2.13\varepsilon_0 = 2.13 \quad (15)$$

The radius of air hole satisfies the relationship with lattice constant:

$$r = 0.35a \quad (16)$$

After selecting the lattice vector,the first Brilloum zone of the lattice being the same with the one of the in-plane-wave mode can be calculated and is listed in Fig.2.

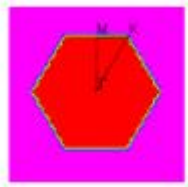

Figure 2: The first Brillouin zone of the out-of-plane wave.

Regard the wave vector k_z as a variable parameter, we can get the result of the calculation with Wolfram *Mathematica* software or R.Soft *bandsolve* software.

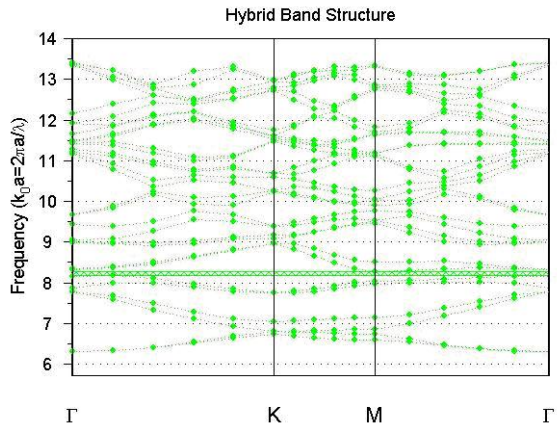

Figure 3: the Brillouin diagram calculated for k_z=8.2.

The brillouin diagram can be calculated for different k_z value. From the result, we found that there is no any band gaps existing with k_z=0, and a band gap existing with k_z=8.2 shown in Fig.3

ACCELERATING STRUCTURE DESIGN

After band gap calculating, then we can perform our design for accelerating structure. To allow uninterrupted beam passage, a center hole through the dielectric core is necessary. And a speed of light(SOL) mode confined by band gap through it.

On the other hand, after introducing a defect into the triangular crystal lattice, the periodicity will be destroyed, especially for the higher frequency near light wave. So we should find some way to calculate the effect introduced by periodicity broken.

Supercell Approximation

We use supercell approximation to estimate the effect introduced by periodicity broken. Supercell approximation is a method that we define a calculation domain much larger than a single unit cell of the crystal and regard it as a "super cell" to calculate the attribute of the defected lattice. The supercell simulation approach can establish the defect level positions. But, in order to reduce the affection of periodic boundary condition, the size of supercell must be taken as large as possible.

After introducing an air hole defect with the radius:

$$r_0 = 0.52a \qquad (17)$$

We define our 7×8 supercell of the triangular lattice with defect (Fig.4).And get the Brillouin diagram shown in Fig.5.

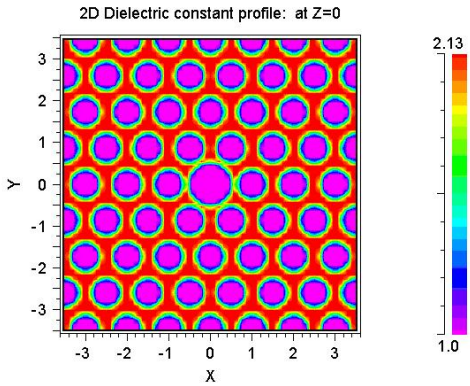

Figure 4: 7×8 super cell of the rectangular lattice with a defect.

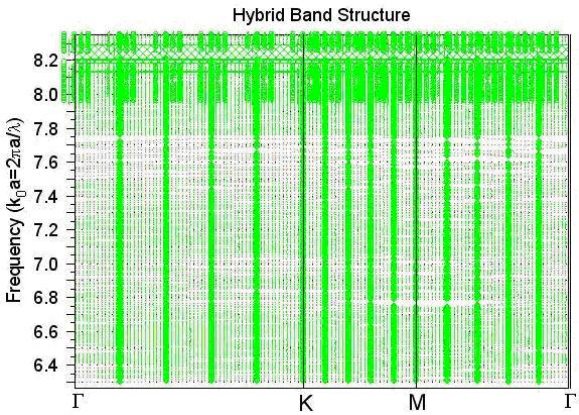

Figure 5: The Brillouin diagram by the supercell approximation.

The band gap appears between frequency band No.336 and No.337.

Accelerating Mode

To achieve accelerating mode establishment in defect hole we should search a speed of light(SOL) mode in band gap frequency.

Vary the parameter k_z from 5 to 14 and plot the band gap frequency, we can obtain the band dispersion of the defect mode. The result is shown in Fig.6.

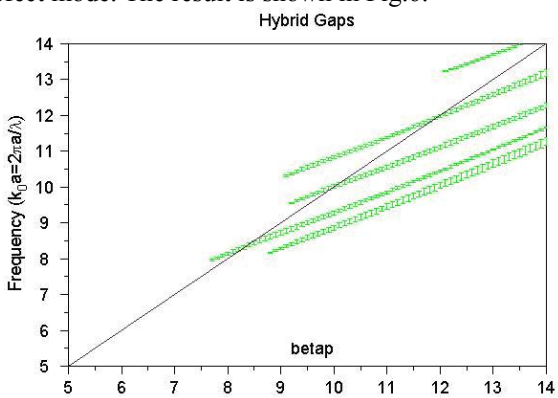

Figure 6: The band gap dispersion of the defect mode.

Mark the SOL line with purple line on Figure 6, we can obtain the synchronized accelerating normalized frequency at k_z=8.2.

Then we can get the parameters of our accelerating structure. The parameters are presented in table 1.

Table 1: Parameters of Accelerating Structure

Air hole radius *r*	599nm
Lattice constant a	1711nm
Central defect radius *r*	889nm
Circle numbers	6
Accelerating cavity radius	72.3μm
Accelerating mode frequency	2.7×10^{15} Hz

The field distribution of the structure has been calculated and given in Fig.7.

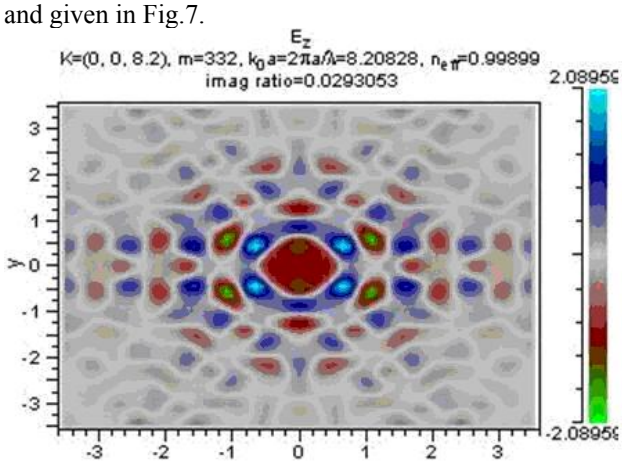

Figure 7: The field distribution of the accelerating mode.

CONCLUSION

Based on the PWM, we calculated the band gap of out-plane-wave with non-defected triangular lattice at first. After introducing the defect air hole in lattice, the supercell approximation was used to estimate the effect introduced by periodicity broken. Then we searched synchronized accelerating mode in the condition of SOL with band gap calculating result. The parameters of the structure and field distribution are presented at last.

REFERENCES

[1] X. E. Lin, Phys. Rev. Special Topics: Accelerators and Beams 4 (2001)051301.

[2] Robert J. Noble, Eric R. Colby, Benjamin Cowan, Christopher M. Sears,Robert H. Siemann, DESIGNING PHOTONIC BANDGAP FIBERSFOR PARTICLE ACCELERATION, SLAC-PUB-12571.