# TRANSVERSE IMPEDANCE LOCALIZATION USING DEPENDENT OPTICS* 

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## Abstract

Measurements of transverse impedance in the SPS to track the evolution over the last few years show discrepancies compared to the analytical estimates of the major contributors. Recent measurements to localize the major sources of the transverse impedance using intensity dependent optics are presented. Some simulations using HEADTAIL to understand the limitations of the reconstruction and related numerical aspects are also discussed.

## INTRODUCTION

The super proton synchrotron (SPS) accelerates beams from an injection energy of 26 GeV to top energy of 450 GeV which is delivered to the Large Hadron Collider (LHC). Table 1 shows some relevant parameters for the SPS. As the injector to the LHC, the SPS underwent several hardware changes (for example: removal of lepton cavities, some kickers, installation of new extractions kickers etc..) which modified its impedance. An impedance measurement campaign has been in place for several years to track the evolution due to modifications in the machine hardware [1].

Table 1: Some relevant parameters of the SPS beams

|  | Unit | Prototype |
| :--- | :---: | :---: |
| Beam momentum | $[\mathrm{GeV} / \mathrm{c}]$ | 26 |
| P/Bunch | $\left[10^{11}\right]$ | $0.05-1.3$ |
| Emittance, $\epsilon_{x, y}$ | $[\mu \mathrm{~m}]$ | 3.0 |
| Bunch Length, $\sigma_{t}$ | $[\mathrm{~ns}]$ | 0.55 |
| Betatron Tunes, $\mathrm{Q}_{x, y}$ | - | $26.12,26.19$ |
| Average $\beta$ | $[\mathrm{m}]$ | 50 |
| Main RF Frequency | $[\mathrm{MHz}]$ | 200 |

A proposal to localize the contributions of the individual sources of impedance was carried out successfully in 2004 [2]. This paper will focus on the most recent measurements performed during 2007-08 runs to improve the understanding of the individual impedance sources and potentially resolve any discrepancies between the known model and measured impedance in the SPS. The effect of the transverse impedance on the complex tune shift to first order can be approximated as a defocusing quadrupole, where the gradient can be expressed as,

$$
\begin{equation*}
\mathrm{K}_{\mathrm{eff}}=\frac{\mathrm{eN}_{\mathrm{b}}}{2 \sqrt{\pi} \sigma_{\mathrm{z}}\left(\mathrm{E}_{\mathrm{b}} / \mathrm{e}\right)} \operatorname{Im}\left\{\mathrm{Z}_{\perp}\right\}_{\mathrm{eff}} \tag{1}
\end{equation*}
$$

The corresponding linear tune shift and $\beta$-beat associ-

[^0]ated with the perturbations in $\Delta K$ can be expressed as
\[

$$
\begin{gather*}
\Delta \mathrm{Q}=\frac{1}{4 \pi} \beta_{\mathrm{k}} \Delta \mathrm{~K}  \tag{2}\\
\frac{\Delta \beta(\mathrm{~s})}{\beta(\mathrm{s})}=\frac{\beta_{\mathrm{k}} \cos \left(2\left|\phi(\mathrm{~s})-\phi_{\mathrm{k}}\right|-2 \pi \mathrm{Q}\right)}{2 \sin (2 \pi \mathrm{Q})} \Delta \mathrm{K} \tag{3}
\end{gather*}
$$
\]

The phase-beat induced is equivalent to the $\beta$-beat is more robust because it is measured independent of the hardware calibration. The SPS is equipped with approximately 100 beam position monitors (BPMs) placed around the ring at approximately $90^{\circ}$ betatron phase advance and are capable of recording turn-by-turn beam positions for 1024 consecutive turns. A Fourier transform of the BPM data immediately after a transverse kick applied to the beam can be used to deduce the linear optics. The phase advance is measured for varying intensities and fitted to a linear function similar to the tune shift

$$
\begin{equation*}
\phi_{\mathrm{I}}=\phi_{0}+(\Delta \phi / \Delta \mathrm{N}) \mathrm{N}_{\mathrm{b}} \tag{4}
\end{equation*}
$$

The slope of the linear fit $\Delta \phi / \Delta N$ is used as the effective "phase-beat" observable due to multiple defocusing impedance sources. The location of sources are reconstructed using a linear least squares method,

$$
\begin{equation*}
\Delta \tilde{\mathrm{K}}=\mathcal{A}^{-1}\left\{\Delta \tilde{\phi} / \Delta \mathrm{N}_{\mathrm{b}}, \Delta \mathrm{Q}_{\mathrm{x}}, \Delta \mathrm{Q}_{\mathrm{y}}\right\} \tag{5}
\end{equation*}
$$

where $\mathcal{A}$ is the model response matrix constructed by using the quadrupoles in the magnetic lattice. Additionally, the solution can be obtained by imposing constraints on the quadrupoles to provide only a defocusing force similar to an impedance

$$
\begin{equation*}
[\mathrm{R}, \tilde{\lambda} \mathrm{I}] \Delta \tilde{\mathrm{K}}=[\Delta \tilde{\phi}, 0]^{\mathrm{T}} \quad\left\{\Delta \mathrm{~K}_{\mathrm{i}}<0, \quad \mathrm{QDs}\right\} \tag{6}
\end{equation*}
$$

Alternate methods using orbit bumps in sextupoles are also used which is discussed in Ref. [3]. Some benchmarks of this technique and reconstruction algorithms are performed which is discussed in the following section.

## SIMULATIONS

To benchmark the reconstruction algorithms, tracking simulations using HEADTAIL [4] were performed using the effect of single and multiple impedance sources in the nominal SPS lattice ( 26 GeV ). For simplicity, the sources were placed at the existing kicker locations with an impedance of $0.5 \mathrm{M} \Omega / \mathrm{m}$ per source. Approximately $10^{5}$ particles were tracked for $10^{3}$ turns (similar to the data acquisition buffer capacity in the SPS). The centroid position of the bunch transverse distribution was recorded as a function of each turn for five different intensities. This data was used to determine the coherent tune shift due as a function of intensity. The estimated impedance from the linear slope

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of the tune shift agrees well with the input impedance in the simulation.

The turn-by-turn beam centroid positions were recorded at all the BPM locations which were used to deduce the phase advance between the consecutive BPMs as a function of the five different intensities. The phase advances were subsequently fitted with a linear equation 4 to determine the phase slope as a function of intensity. The phase advance slope in the vertical plane is shown in Figure 1 which is reconstructed in the model using a linear response matrix.


Figure 1: Bottom: Slope of the vertical phase advance between consecutive BPMs determined from a linear fit as a function of intensity. Top: Source localization from phase advance slope using linear response matrix. The top bar plot represents the location of the impedance sources used in the simulation.

The response matrix is solved for two cases, one with focusing and defocusing elements (QF, QD), and the other with only defocusing elements (QD) while constraining the sign of the gradient to produce a defocusing effect. The reconstruction from the turn-by-turn tracking data is able to identity the region of dominant impedance sources as initially input in the simulations (see Fig 1). However, the strength is distributed among the neighboring quadrupoles used in the reconstruction. Therefore, the exact location, amplitude and the sign of the impedance reconstruction is limited to the variables used and cannot be easily inferred from this technique. Additional methods using systematic orbit bumps maybe required to identity the amplitude. These benchmarks revealed that the reconstruction without constraints yields a robust solution with less parasitic sources distributed around the ring unlike the reconstruction with constraints as assumed in Ref [2].

## IMPEDANCE MEASUREMENTS

Single bunches were injected into the SPS with varying intensities and the chromaticity was set to $\sim 2-3$ units to keep the beam stable at the highest intensities while sustain coherent betatron oscillations for sufficient number of turns. Appropriate voltage matching at injection was required to suppress the strong injection oscillations and the 200 MHz RF system was ramped up subsequently to re-
duce the bunch length and improve measurement sensitivity of the beam trajectories.

In addition to intensity scan with baseline optics, two other scenarios were carried also out. A 10 mm vertical three-bump at "MKDV. 11731 " located at 550.619 m to locally increase the effective impedance and a measurement to determine the impact of a finite momentum offset $\left(\delta p / p=4 \times 10^{-3}\right)$. The bunch lengths ( $4 \sigma$ ) and the RF voltages during the tune measurements are listed in Table 2.

Table 2: Bunch lengths and RF voltages for the different beam conditions for the intensity dependent tune scan measurements.

| Date | Condition | $4 \sigma_{z}$ | $\mathrm{~V}_{R F}$ |
| :--- | :---: | :---: | :---: |
| Nov 2, 2007 | Baseline | 2.2 | 3 |
| Aug 4, 2008 | Baseline | 2.5 | 1.8 |
| Aug 5, 2008 | 10 mm V-bump | 2.5 | 1.8 |
| Aug 5, 2008 | $\delta p / p=3.7 \times 10^{-3}$ | 2.5 | 1.8 |

A dipole kick is imparted to the single bunch during the injection plateau for all intensities after approximately 500 ms and a synchronized BPM acquisition system records the beam trajectories for $10^{3}$ turns. For the tune measurement, a dedicated high resolution BPM is available. There are additionally about a 100 BPMs placed around the lattice which are capable of recording $10^{3}$ subsequent turns. Figure 2 shows the tune derived from an average of all functioning BPMs in the ring for all intensities and the three different conditions. It should be noted that BPM gain has to be substantially increased in order to measure the trajectories at low intensities $\left(<2 \times 10^{10}\right)$ which result in larger error bars. The fitted linear slopes and the computed impedance estimates from the tune shift with intensity is listed in Table 3.


Figure 2: Tune shift determined from an interpolated Fourier transform of all SPS BPM trajectories acquired over 1024 turns after a transverse kick was applied

There is a clear difference in the measured tune shifts between the baseline measurements of 2007 and 2008 probably due to the shorter bunch length as an effect of elevated voltage any residual closed orbit effects. The case with the vertical orbit bump clearly shows an increase in the tune shift slope which experimentally supports the use of local

Table 3: Coherent tune shift and the corresponding impedance estimates for different machine configurations. The tune shift is calculated from Fourier transform of approximately $10^{3}$ turn-by-turn BPM data.

| Year | Condition | Slope | $\mathrm{Q}_{0}$ | $\operatorname{Imp}[\mathrm{M} \Omega / \mathrm{m}]$ |
| :--- | :---: | :---: | :---: | :---: |
| Nov 2, 2007 | $\delta p / p=0$ | $-0.0188( \pm 1.4 \%)$ | $0.20( \pm 0.1 \%)$ | $20( \pm 0.3)$ |
| Aug 4, 2008 | $\delta p / p=0$ | $-0.0155( \pm 1.2 \%)$ | $0.18( \pm 0.07 \%)$ | $17( \pm 0.2)$ |
| Aug 5, 2008 | V-Bump $=10 \mathrm{~mm}$ | $-0.018( \pm 1.2 \%)$ | $0.18( \pm 0.2 \%)$ | $21( \pm 0.3)$ |
| Aug 5, 2008 | $\delta p / p=3.7 \times 10^{-3}$ | $-0.011( \pm 3.4 \%)$ | $0.19( \pm 0.9 \%)$ | $15( \pm 0.5)$ |

orbit bumps. The case with the momentum offset shows not only the overall tune shift (chromaticity), but also an effective reduction of the measured impedance which under study.

## INTENSITY DEPENDENT OPTICS

As a virtue of betatron oscillations sampled around the lattice, the betatron phase advance between consecutive BPMs can be measured from the Fourier transform. The tune shift observed with intensity can be expressed as the accumulated phase advance shift around the ring, $\Delta Q=\frac{1}{2 \pi} \sum_{n} \delta \phi_{(n-1) \rightarrow n}$. This approach was carried out in Ref. [2] to determine the linear slope in the betatron phase advance as a function of intensity. Figure 3 shows the lattice phase advance between consecutive BPMs calculated from the fit intercept and the corresponding slope. The three cases: baseline, 10 mm vertical three-bump at "MKDV. 11731 " located at 550.619 m and a finite momentum offset ( $\delta p / p=4 \times 10^{-3}$ ) and compared to baseline optics model of the SPS.


Figure 3: Phase advance between consecutive BPMs calculated from the linear fit of the intensity dependent optics. A comparison of phase slope two measurement performed in 2007 and 2008 is shown in the bottom plot. Note that BPMs with faulty data have been removed.

The recorded slopes at all BPM locations are used to compute the effective quadrupole perturbations in the lattice (see section ) to emulate the observed phase-beat with the aid of a linear response matrix. The quadrupole perturbation vector can be used to infer the approximate location of the largest impedance sources as shown in Figure 4 for baseline SPS lattice.

## DISCUSSION

Local impedance reconstruction using intensity dependent optics from recent measurements (2007-08) and previous measurements (Ref. [2]) point to localized sources (Figure 4). The case with no constraints yields a robust reconstruction with fewer spurious sources distributed around the ring. The amplitude and sign of the impedance


Figure 4: Slope of the horizontal (left) and vertical (right) phase advance shift with intensity and corresponding quadrupole perturbations to reproduce the observed phase-beat for baseline SPS lattice.
sources cannot be easily inferred from such a technique. However, the MKD \& MKE kickers ( $\sim 0.5 \mathrm{~km}$ ) clearly reveal themselves as one of the major contributors in the vertical plane. In addition two other locations near $1.5-2 \mathrm{~km}$ and $5-5.5 \mathrm{~km}$ regions indicate potential sources. Although, the reconstruction in the horizontal plane is distributed, the region near 2 km indicate potential sources. Dedicated measurements at these specific locations using systematic orbit bumps are planned for future.

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