STUDIES OF BUNCH DISTORTION AND BURSTING THRESHOLD IN THE GENERATION OF COHERENT THZ-RADIATION AT THE ANKA STORAGE RING *

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Abstract

In synchrotron light sources, coherent synchrotron radiation (CSR) is emitted at wavelengths comparable to and longer than the bunch length. One effect of the CSR wake field is the distortion of the bunch distribution, which increases with higher currents. In the theoretical calculations, a threshold exists beyond which the solutions begin to diverge. On the other hand, the CSR wake can also excite a microbunching instability which prevents stable emission of CSR for high currents and leads to highly intense bursts of radiation. In this paper the development of the calculated bunch shapes and the corresponding moments of the current distribution for varying bunch currents are studied. It can be shown that the bursting-stable threshold at the ANKA storage ring, in good agreement with theory, is close but not identical to the point where the bunch-shape calculations diverge.

INTRODUCTION

Coherent synchrotron radiation (CSR) provides a boost to the radiation power when compared to incoherent radiation of the same frequencies [1, 2]. The spectrum of the CSR can be obtained from the Fourier transform of the electron distribution. CSR emitted in the THz frequency range is limited from below by the cutoff of the beampipe and from above by the length of the bunches. To widen the spectum to higher frequencies, one possibility is to shorten the bunches, the other is to make use of the distortion effected by the CSR wake field that can act back on the bunches [3]. On the other hand the CSR wake field can excite microbunching [4], which prevents constant emission of CSR by causing bursts of radiation with high peak power. The full simulation of the nonlinear beam dynamics including the production of coherent THz-radiation is the long term goal of this project.

CSR WAKE FIELD AND ELECTRON DISTRIBUTION

The collective force caused by the coherent synchrotron radiation of an electron can change the energy of the particles ahead. This force on an electron at the longitudinal position z can be described by the ultrarelativistic approximation of the longitudinal CSR wake field $W(z)$ in free space, which is given by [3]:

$$W(z) = -\frac{Z_0 ce}{2\pi(3R^2)^{1/3}}\frac{\partial}{\partial z}z^{-1/3} \qquad z > 0 \qquad (1)$$

where R is the bending radius of the magnet. By using the Haissinski equation [5] the equilibrium bunch distribution $y(x)$ of the distorted bunch can be calculated:

$$y_\kappa(x) = \kappa \exp\left[-\frac{x^2}{2} \pm \int_0^\infty y_\kappa(x-z)z^{-1/3}dz\right]. \qquad (2)$$

The positive sign is taken for positive and the negative sign for negative momentum compaction factor.
x and y_κ represent the following dimensionless quantities:

$$x = \frac{z}{\sigma_0}, \qquad y_\kappa = \frac{Z_0 c}{\dot{V}_{RF}}\left(\frac{R}{3\sigma_0^4}\right)^{1/3} I(x) \qquad (3)$$

where σ_0 is the natural bunch length, I the current distribution in the bunch and $\dot{V}_{RF}$ the time derivative of the RF voltage.
The change of the dimensionless factor κ in Eq. (2) leads to different bunch shapes. The connection between bunch shapes and accelerator parameters is given by the integral $F(\kappa)$ over the bunch shape:

$$F(\kappa) = \int y_\kappa(x)dx = I\frac{Z_0 c}{\dot{V}_{RF}}\left(\frac{R}{3\sigma_0^4}\right)^{1/3} \qquad (4)$$

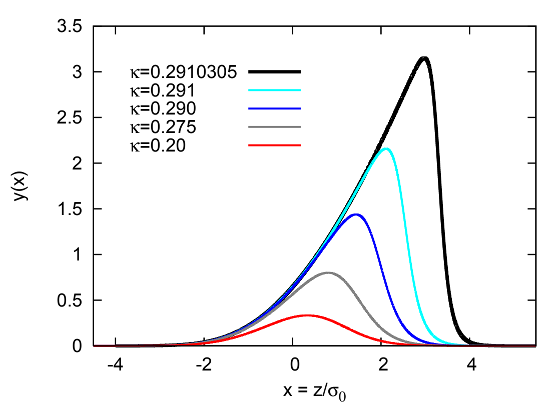

Figure 1: Calculated electron distributions y for different κ. x is the longitudinal coordinate in units of the bunch length σ_0.

* This work has partly been supported by the Initiative and Networking Fund of the Helmholtz Association under contract number VH-NG-320.

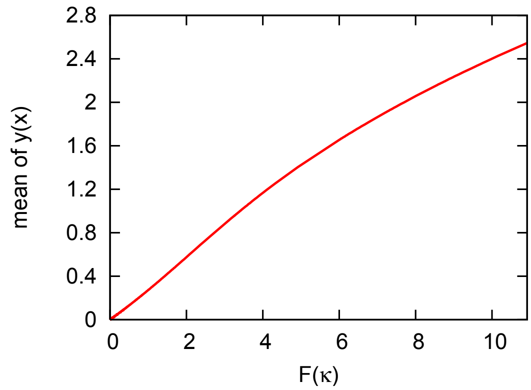

Figure 2: The dependence of the mean of the electron distributions versus $F(\kappa)$. For a constant bunch length, $F(\kappa)$ is proportional to the bunch current.

DISTORTION OF THE BUNCH SHAPE

With higher currents or shorter bunches, κ, $F(\kappa)$ and the distortion of the bunches all increase. The bunch shape develops a sharp leading edge and leans forward as Fig. 1 shows. These distributions cannot be calculated analytically, a numerical, iterative approach of Eq. (2) is needed.

Moments

Moments offer a convenient way to characterize the shape of a bunch by a set of a few characteristic numbers. Here the mean value, standard deviation and skewness are used. The mean value shows how the center of the charge distribution is affected by the CSR-wake field of the histogram, the standard deviation gives the RMS bunch length and the skewness is a measure of the asymmetry of the distribution.

The calculated moments of the bunch shape are shown as a function of $F(\kappa)$, which is proportional to the bunch current. As displayed in Fig. 2, increase in current leads to a

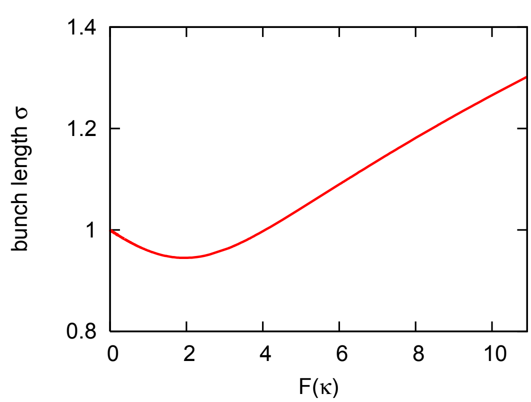

Figure 3: The bunch length σ versus $F(\kappa)$. The bunch shortens till a minimum at $F(\kappa) = 1.94$ and increases again.

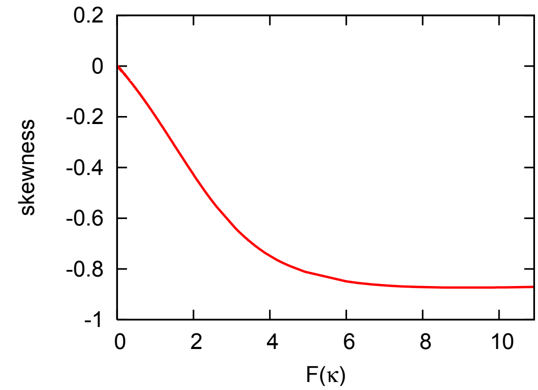

Figure 4: The skewness versus $F(\kappa)$. Negative skewness stands for a long left tail and a sharp right edge.

nearly linear shift of the mean position towards the head of the bunch for fixed bunch length. The bunch length shortens for $F(\kappa) < 1.94$ and increases for $F(\kappa) > 1.94$ again, as showed in Fig. 3.

For all calculated distributions the skewness is negative (Fig. 4) what indicates a longer left tail and a shift of the mean to higher values. The asymmetry increases rapidly with $F(\kappa)$ until $F(\kappa) \approx 4$ and stays nearly constant for $F(\kappa) > 6$.

INSTABILITY THRESHOLD

Bursting Instability

Beside the influence on the equilibrium electron distribution, the CSR wake field can cause a microbunching instability [4]. Initially small fluctuations can grow by the excitation of a wake field. A substructure develops leading to intense coherent radiation which is rapidly damped. This process is repeated again and stable emission of CSR is prevented. The radiation occurs in highly intense bursts. This instability takes place when the following condition is satisfied [6]:

$$I > const. \cdot V_{RF}\ \sigma_0^{7/3} \tag{5}$$

The observed bursting stable threshold was measured at ANKA for different bunch lengths. The THz radiation was detected at the ANKA-IR beamline with an He cooled Si bolometer at a beam energy of 1.3 GeV. The bunch length was reduced in steps by decreasing the momentum compaction factor of the ring. The stability of the signal was monitored with decaying current. The natural bunch length was obtained by linear extrapolation from the measured synchrotron frequency.

The results of two datasets are displayed in Fig. 5. The observed threshold is well described by the power law from theory, with a value $F(\kappa) = 8.75$.

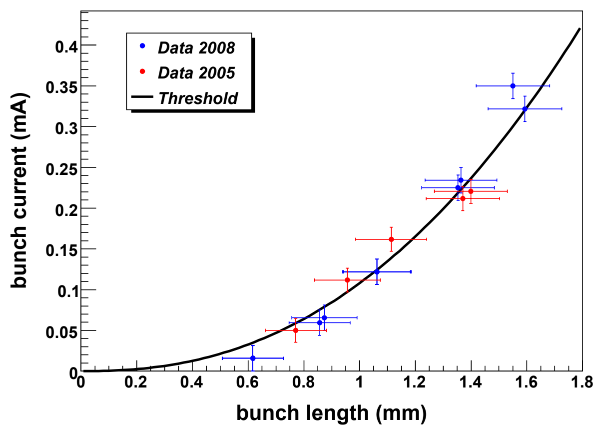

Figure 5: The observed bursting stable threshold as function of bunch current and bunch length. The black curve shows the fit to to the microbunching theory.

Numerical Threshold

As shown in [3], a threshold κ_{max} exists above which the solutions to Eq. (2) begin to diverge and no equilibrium is found. For all $\kappa < \kappa_{max}$ the factor $F(\kappa)$ rises continuously as shown in Fig. 6. From our calculations this numerical threshold lies between $0.29103051428 < \kappa_{max} < 0.29103051429$. A threshold for $F(\kappa)$ could not be found and from Fig. 6 it diverges for $F \to F(\kappa_{max})$.

To demonstrate this, $F(\kappa)$ was calculated for every iteration of Eq. (2) for different κ greater and smaller than κ_{max}, as shown in Fig. 7. The values of $F(\kappa)$ at lower iteration numbers are all the same for different $\kappa \approx \kappa_{max}$. The further κ lies from κ_{max} the earlier $F(\kappa)$ either converges, or diverges from the "shared curve", depending on whether κ is smaller or greater than κ_{max} respectively.

The "shared curve" can be described by the function

$$F(\kappa \to \kappa_{max}) = a \cdot N^b + c. \qquad (6)$$

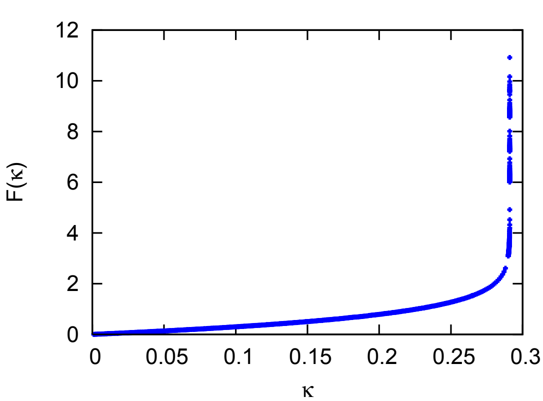

Figure 6: The development of $F(\kappa)$. Above $\kappa_{max} = 0.29103051429$ no equilibrium distribution can be found.

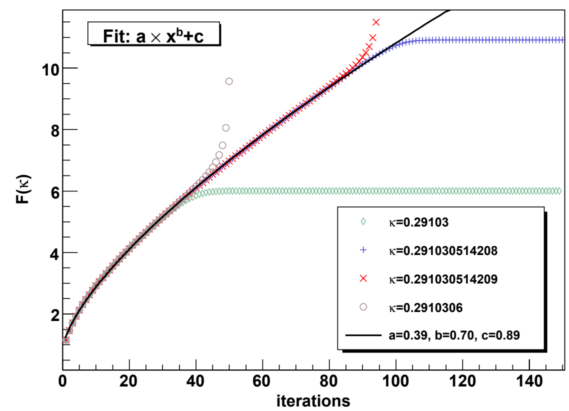

Figure 7: $F(\kappa)$ after each iteration for different κ. The fit of the collective development shows no maximum.

where N is the number of iterations. The constants are found to be $a = 0.39$, $b = 0.70$ and $c = 0.89$. Resultant from this fit no threshold for $F(\kappa)$ exists, $F(\kappa_{max}) \to \infty$.

CONCLUSION

Since only $F(\kappa)$, which is unbounded, and not κ itself is connected to accelerator paramemeters, the threshold value of κ cannot be directly associated with a machine instability. The numerical and experimental studies presented above, however, show that the stable-bursting threshold occurs in the regime where $F(\kappa)$ rapidly grows as a function of κ, i.e. as expected, in a regime where the CSR wake field leads to significant distortions of the bunch.

ACKNOWLEDGMENT

The authors would like to thank Godehard Wüstefeld for his fruitful discussions and Michael Süpfle for technical support.

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