NONLINEAR DYNAMICS STUDIES IN THE FERMILAB TEVATRON USING AN AC DIPOLE*

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Abstract

An AC dipole magnet produces a sinusoidally oscillating dipole field with frequency close to betatron frequency and excites large sustained oscillations of beam particles circulating in a synchrotron. Observation of such oscillations with beam position monitors allows direct measurements of a synchrotron's nonlinear parameters. This paper presents experimental studies to measure perturvative effects of sextupole and octupole fields, performed in the Fermilab Tevatron using an AC dipole.

INTRODUCTION

A modern synchrotron uses magnets producing nonlinear fields, such as sextupole and octupole magnets, to compensate chromaticity and instabilities of a high intensity beam. Magnet imperfections also produce nonlinear fields. In addition to compensating the corresponding effects, these nonlinear fields perturb the beam's transverse motion. For instance, the sextupole fields distort the beam's central orbit and the octupole fields shift betatron tune ν[1] depending on a beam particle's amplitude (nonlinear detuning effect). These fields also drive modes with tunes higher than the betatron tune. It is ideal if we could measure and compensate such perturbative effects to fine-tune a synchrotron, but even the measurements are not always easy and not many measurements have been done in the Fermilab Tevatron so far [1, 2].

An AC dipole magnet, a new diagnostic tool of a synchrotron [3], has been implemented to the Tevatron [4, 5, 6]. Its oscillating dipole field with frequency close to the betatron frequency can excite large sustained beam oscillations with no emittance growth. Observation of such oscillations with beam position monitors (BPMs) allows measurements of not only linear but also nonlinear properties of the beam. In this paper, we discuss experimental studies to observe the perturbative effects due to sextupole and octupole fields, performed in the Tevatron using the AC dipole.

In a linear and uncoupled synchrotron[2], turn-by-turn position of the AC dipole's excitation is given by [6, 7]

$$x_d(n;s) = A_d\sqrt{\beta_d(s)}\cos\left[2\pi\nu_d n + \psi_d(s) - \psi_d(s_{\text{ac}}) + \chi\right], \quad (1)$$

where n is number of the beam revolution, s and s_{ac} are longitudinal positions of the BPM and AC dipole, A_d and χ are constants, and ν_d is driving tune[3]. The constant A_d, determining the amplitude, is proportional to the amplitude of the AC dipole's current and to a factor $1/\sin[\pi(\nu_d-\nu)]$. We typically choose the driving tune ν_d near the betatron tune ν to make large amplitude oscillations. In the AC dipole's oscillation, the amplitude function $\beta(s)$ and phase advance $\psi(s)$ are modulated to $\beta_d(s)$ and $\psi_d(s)$, depending on the tunes ν_d and ν [6, 7].

CENTRAL ORBIT DISTORTION DUE TO SEXTUPOLE FIELDS

The central orbit distortion due to the sextupole fields is proportional to strengths of sextupole fields and A_d^2, which is proportional to the square of the amplitude of the AC dipole's current. To observe such properties of the effect, we performed an experimental study in the Tevatron using the 150 GeV proton beam. In our study, one sextupole magnet is controlled to make small changes and to test our measurement methods.

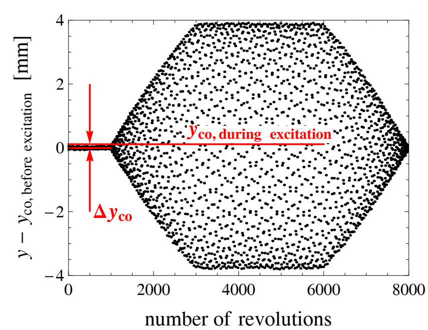

Figure 1: Turn-by-turn position of the AC dipole's excitation observed in the Tevatron, indicating a shift of the average position during the excitation. The average position before the excitation is subtracted from the data points.

Figure 1 shows turn-by-turn position of the AC dipole's excitation observed in the Tevatron. Here, the amplitude of the AC dipole's current is adiabatically ramped up (1,000th - 3,000th turns) and ramped down (6,000th - 8,000th turns), and such adiabatic processes preserve the beam emittance

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[1] In this paper, ν denotes the fractional part of the betatron tune.

[2] We only consider one dimensional motion of the beam particles.

[3] The definition of ν_d ($0 \leq \nu_d < 1$) is the following [6, 7]. We suppose that f_{rev} and f_{ac} are frequencies of the beam revolution and the AC dipole field and ν_{ac} is the fractional part of $f_{\text{ac}}/f_{\text{rev}}$. When the betatron tune ν and ν_{ac} satisfy $|\nu_{\text{ac}} - \nu| < |1 - \nu_{\text{ac}} - \nu|$, $\nu_d = \nu_{\text{ac}}$, and when $|1 - \nu_{\text{ac}} - \nu| < |\nu_{\text{ac}} - \nu|$, $\nu_d = 1 - \nu_{\text{ac}}$.

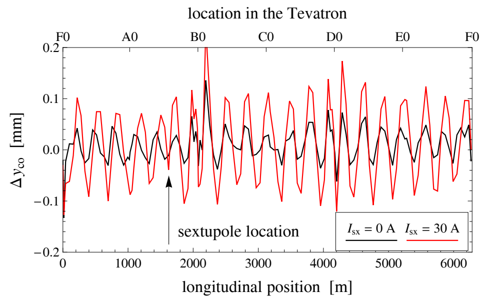

Figure 2: Central orbit distortions due to sextupole fields when the current of the controlled sextupole magnet is 0 A (nominal) and 30 A. When the current is increased to 30 A, the effect grows as expected ($\sim$80 μm) and shows a cusp structure, indicating location of the sextupole magnet.

[3, 6, 8]. We see that the average position is shifted upward while the amplitude is maximum. This shift is the central orbit distortion due to the sextupole fields. Figure 2 shows the central orbit distortion measured in this way with all the BPMs. The black and red curves represent the cases when the current of the controlled sextupole magnet is 0 A (the nominal value) and 30 A. As expected, the measurement for 30 A shows a larger central orbit distortion and also the cusp structure indicating the source location. In these measurements, the amplitude function and the oscillation amplitude at the location of the controlled sextupole magnet, $s_{\rm sx}$, are about $\beta_d(s_{\rm sx}) \simeq 94$ m and $A_d\sqrt{\beta_d(s_{\rm sx})} \simeq 4$ mm, and the effective strength of the sextupole magnet is about $B''\ell/(B\rho) \simeq 420$ nrad/mm^2 when its current is 30 A [9]. In such a condition, this sextupole magnet is equivalent to a dipole magnet with a deflection angle $\frac{1}{4}A_d^2\beta_d(s_{\rm sx})B''\ell/(B\rho) \simeq 1.7$ μrad [6] and the expected central orbit distortion is about 80 μm at the BPM locations, where the amplitude function is $\beta(s) \simeq 95$ m. We can see that this estimate is not far from the measurement in the figure. We note that some sextupole magnets are also used in the condition of our study and the black curve represents the effects of such residual sextupole fields.

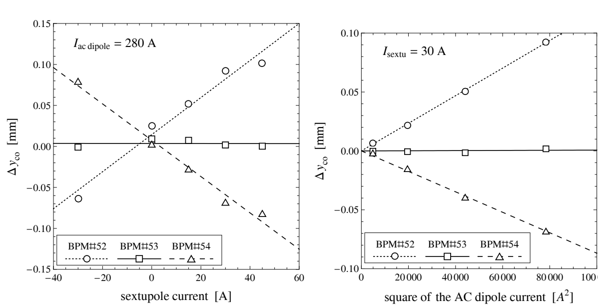

Figure 3: Central orbit distortions observed by three BPMs. The effect is proportional to the current of the controlled sextupole magnet (left) and the square of the amplitude of the AC dipole's current (and hence A_d^2) (right). The difference of the slopes is due to phase dependence of the effect.

Figure 3 shows the measured central orbit distortion, when the current of the controlled sextupole magnet is set to various values while holding other quantities constant (left) and when the amplitude of the AC dipole's current is set to various various values, changing the oscillation amplitude, while holding other quantities constant. We can observe that the effect is proportional to the current of the given sextupole magnet and also the square of the amplitude of the AC dipole's current, as expected. In this way, the AC dipole allows us to observe changes of the central orbit distortion due to the sextupole fields in the Tevatron.

NONLINEAR DETUNING EFFECT DUE TO OCTUPOLE FIELDS

The nonlinear detuning effect due to the octupole fields is proportional to strengths of octupole fields and also A_d^2. To observe such properties of the effect, we performed another experimental study with the 150 GeV proton beam in the Tevatron. Here, strength of one group of octupole magnets[4] is controlled to make small changes and to test our measurement methods.

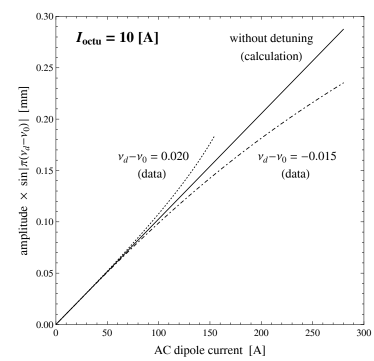

Figure 4: Amplitude of the driven beam (multiplied by a factor $\sin|\pi(\nu_d-\nu_0)|$) vs. amplitude of the AC dipole's current. The detuning effect either suppress or enhance the beam's amplitude. The deviation of the beam's amplitude from the linear growth allows to determine the detuning effect.

As we discussed previously, the amplitude of the AC dipole's excitation is proportional to the amplitude of the AC dipole's current and the factor $1/\sin[\pi(\nu_d-\nu)]$. Figure 4 shows relations, observed by one BPM, between the beam's oscillation amplitudes and the amplitude of the AC dipole's current. Here, the beam's amplitude is multiplied by a factor $\sin|\pi(\nu_d-\nu_0)|$, where ν_0 is the betatron tune of a beam particle in the limit of a zero oscillation amplitude. In our study's condition, the detuning effect raises the betatron tune along with the growth of the beam's amplitude. In such a condition, if the driving tune is smaller than the betatron tune, the betatron tune moves away along with the beam's amplitude growth and the beam's amplitude is suppressed through the factor $1/\sin[\pi(\nu_d-\nu)]$, compared to

[4]The group referred to as **OD**, consisting of 18 octupole magnets [10].

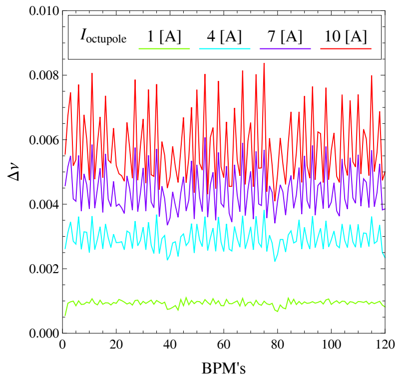

Figure 5: Tune shifts measured with all the BPMs for different currents of the controlled octupole magnets. The driving tune and the maximum amplitude of the AC dipole are fixed to $\nu_d - \nu_0 = -0.015$ and 280 A, producing 5 mm oscillation amplitude in the arc.

the linear growth in the case of no detuning effect. On the contrary, if the driving tune is larger than the betatron tune, the betatron tune approaches to the driving tune along with the beam's amplitude growth and the beam's amplitude is enhanced. By observing such modulations of the beam's amplitude, we can measure the detuning effect [11].

Figure 5 shows the tune shift measured in this way with all the BPMs, when the current of the controlled octupole magnets are changed from 1 A to 10 A (the nominal is 7 A) and when the maximum amplitude and driving tune of the AC dipole are kept the same to produce ∼5 mm oscillations in the arc. For our octupole magnets, the effective focal length is 260 km when its current is 1 A and the beam's amplitude is 5 mm [9]. The amplitude functions at the locations of these octupole magnets are about $\beta(s) \simeq 90$ m. Under these conditions, when the current of these controlled octupole magnets is changed by 3 A, the estimated tune shift is about 0.0015, which is roughly consistent with the gaps between curves. These measurements are based on the beam's amplitude modulations but the amplitude function of the AC dipole's excitation $\beta_d(s)$ also depends on the betatron tune. The growth of the deviations over BPMs is expected due to the change of $\beta_d(s)$ induced by the detuning effect. In this way, the AC dipole allows us to observe changes of the nonlinear detuning effect due to the octupole fields in the Tevatron.

MODES WITH HIGHER TUNES

In addition to the effects discussed in previous sections, when the beam is driven by the AC dipole, the sextupole and octupole fields drive modes with tunes $2\nu_d$ and $3\nu_d$, for each, and magnitudes of these modes nonlinearly depend on the constant A_d. Figure 6 shows the Fourier spectra of the turn-by-turn positions of the AC dipole's excitation. We can see that the peaks at $2\nu_d$, $1-2\nu_d$, $3\nu_d$, and $1-3\nu_d$, corresponding to the modes of the sextupole and octupole fields, appear only for the larger excitation, indicating the

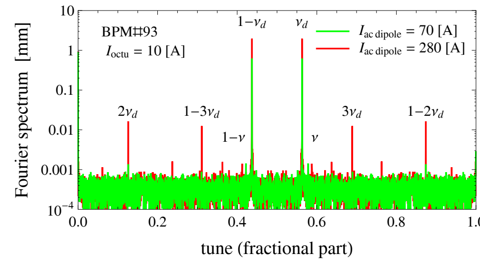

Figure 6: Fourier spectra of turn-by-turn positions of the AC dipole's excitations. The peaks at $2\nu_d$, $1-2\nu_d$, $3\nu_d$, and $1-3\nu_d$, corresponding to the modes driven by the sextupole and octupole fields, appear only for the larger excitation, indicating these modes depend on the oscillation amplitude of the AC dipole's excitation.

dependence of these modes on A_d. By measuring heights of these peaks with BPMs around a synchrotron, we can measure corresponding resonance driving terms [12, 13].

CONCLUSION

Our studies in the Tevatron demonstrated that the AC dipole allows us to measure changes of the central orbit distortion due to the sextupole fields and the nonlinear detuning effect due to the octupole fields. We also demonstrated that the AC dipole allows us to observe the higher tune modes driven by these fields. The measurements presented in this paper are non-destructive to the beam quality and can be performed on a routine basis, and hence may be useful for a quick diagnostic of a synchrotron's nonlinear fields.

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