NUMERICAL STUDY OF COLLECTIVE EFFECTS FOR MUON BEAMS

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Abstract

The study of Muon beam optics is crucial for future Neutrino Factory and Muon Collider facilities. At present, the GEANT4-based simulation tools for Muon beam tracking such as G4beamline and G4MICE only do single particle tracking without collective effects taken into account. However, it is known that collective interaction such as space charge and wakefields for muons (in matter or vacuum) are not ignorable. As the first step, space charge computation is implemented into muon tracking. The basic algorithm is particle-to-particle interaction through retarded electro-magnetic field. The momentum impulse due to collective effects is applied to every particle at each collective step, and the G4beamline main code is used for tracking. Comparisons to LANL Parmela are illustrated and analyzed. Optimizations of the algorithm are also underway to gain less computing time and more accuracy. Moreover, the idea of enhancing ionization cooling efficiency by utilizing the collective effect due to the polarized charges in matter appears to be possible, and preliminary estimates have been made.

INTRODUCTION

The proposed Neutrino Factory and Muon Collider are both muon beam facilities. Compared to the current accelerator facilities that are made for proton, electron or heavy ion acceleration, the difference is that the muon beam will interact with matters such as absorbers, scintillators, etc. during transportation. Therefore the interaction between muons and those materials must be taken into account. Geant4 [1], which is able to do simulations of the passage of particles through matter, is the best candidate to do muon beam tracking. G4Beamline [2] is one of the tracking programs employing the Geant4 packages.

At present, Geant4 is only able to do single particle tracking. In other words, it does not take care of the interaction between two beam particles, and between a beam particle and beamline components. These interactions, including the well-known space charge and wakefields, are not negligible in the low-energy end of the current muon beamline design; and can also introduce the same collective effects as those in regular accelerator facilities such as collective instabilities. As the first step to estimate the collective effects for a muon beam, the numerical computation of space charge is implemented into the G4beamline program.

In order to accelerate a muon beam efficiently and obtain good luminosity, it must be cooled (reduce the emittance) before it reaches the acceleration section. Stopping power, which describes the energy loss of incident particle per unit length, is the key factor to determine the cooling efficiency in ionization cooling [3]. Previous studies [4] in the 1970s suggested that the stopping power computed by the Bethe-Bloch equation [5] would be enhanced by the coherent interference between the electric fields of the incident particles passing through matter. As an application, it could be used to improve the ionization cooling efficiency. However, the previous analysis is not sufficient for muon cooling, and more investigation is required.

SPACE CHARGE ALGORITHM

In this section, the numerical algorithm of space charge computation in G4beamline is introduced, and some simple results in comparison with LANL Parmela [5] are provided.

Geant4 is an Object-Oriented program written in C++, including many classes and functions responsible for program control, tracking, implementing physics processes, etc. G4Beamline employs these features to do single particle tracking. In order to realize the collective computation, a new run manager has been developed and collects all of the tracks into a single track vector; it is coupled with a new (collective) process that is assigned to every particle. The detailed algorithm for collective computation is thus implemented and encapsulated inside the new collective computation class.

Because Geant4 and G4beamline are based on single particle tracking, to avoid dramatic change of the fundamental program structure, the method of macro-particle to macro-particle tracking through electro-magnetic interaction is utilized. Unlike Particle-In-Cell (PIC) codes, the electro-magnetic field is solved by Lienard-Wiechert potentials [6]. In order to do that, the intersection of the trajectory of the source particle and the light cone is determined by interpolation (Fig. 1), then the retarded field emitted from the intersection (source) is computed and the momentum kick is applied to the tracking particle. After this collective computation is applied to all particles, the particles are stepped forward by Geant4. The whole process can be illustrated with the flow chart in Fig. 2:

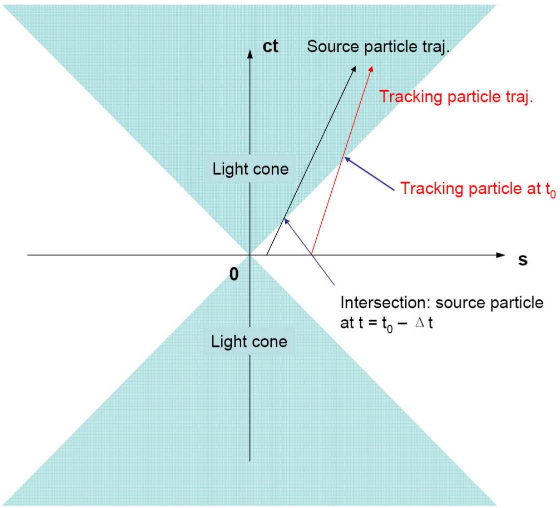

Figure 1: Schematic of finding intersection point on light cone to compute the retard fields.

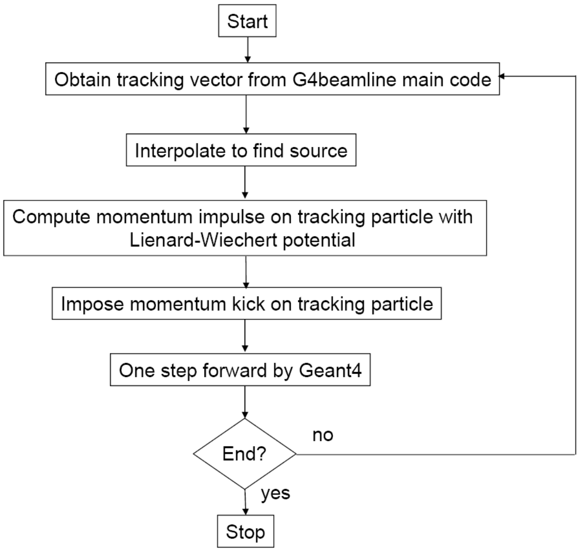

Figure 2: Flow chart of space charge algorithm.

The electro-magnetic field generated by a moving charged particle derived from Lienard-Wiechert potential can be expressed in G-units as (Eq. 1) [6]:

$$\vec{B} = \left[\vec{n} \times \vec{E}\right]_{ret}$$

$$\vec{E}(x,t) = e\left[\frac{\vec{n}-\vec{\beta}}{\gamma^2\left(1-\vec{\beta}\cdot\vec{n}\right)^3 R^3}\right]_{ret} + \frac{e}{c}\left[\frac{\vec{n}\times\left\{\vec{n}-\vec{\beta}\right\}\times\dot{\vec{\beta}}}{\left(1-\vec{\beta}\cdot\vec{n}\right)^3 R}\right]_{ret} \tag{1}$$

where relativistic $\vec{\beta}$ is in the direction of particle speed, $\vec{n}$ points to the direction of radiation. $\vec{E}$ and $\vec{B}$ are the electric and magnetic field calculated at previous time t_{ret} at the target location, respectively.

In the algorithm, the electro-magnetic field is computed at each collective step. It is the sum of the contributions from all the particles within the bunch at some previous time except the target particle itself.

The idea of macro-particle is used in the algorithm. Each macro-particle consists of multiple real beam particles with a certain distribution and non-zero radius. In our case, to simplify the problem, we assume the distribution is Gaussian. Many discussions have been carried out [7, 8] about the optimization of the internal distribution of a macro-particle. However, Gaussian distribution is the most popular one and is able to obtain the reasonable results close to reality. The x, y, z extensions of a macro-particle are determined by the ratio of the standard derivation of the bunch in all three directions and the total number of macro-particles in the bunch; also it can be adjusted by multiplying a “radius factor”.

In the preliminary tests of the space charge program, a 1.6 nC, 200 MeV/c muon bunch with 1000 macro-particles is tracked down in a drift space of 2 m long with a 0.05 nanosecond step size. This is the regular case of the muons in MICE [9] generated from pion decay. To simplify the tracking, the muon bunch is uniformly distributed within a sphere of 1 mm radius, and with zero transverse velocity before tracking starts. The comparisons are with LANL Parmela [10]. The results of muon beam transport in a drift space with 1000 macro-particles and a set of values for the “radius factor” are shown below (Fig. 3). Note that the transverse emittance here is the square root of the product of the horizontal and vertical emittances of the beam.

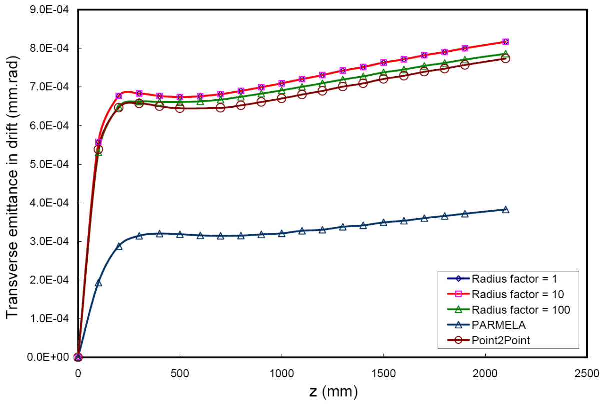

Figure 3: The normalized RMS transverse emittance with “radius factor” of 1, 10 and 100, and point-to-point simulation considering particle as an ideal infinitesimal point, in comparison with the LANL Parmela result.

The comparison with Parmela shows a factor of 2 difference. However, this amount of variation is common among current space charge codes [11] (Fig. 4); we conclude that in some sense the algorithm reasonably describes the space charge mechanism.

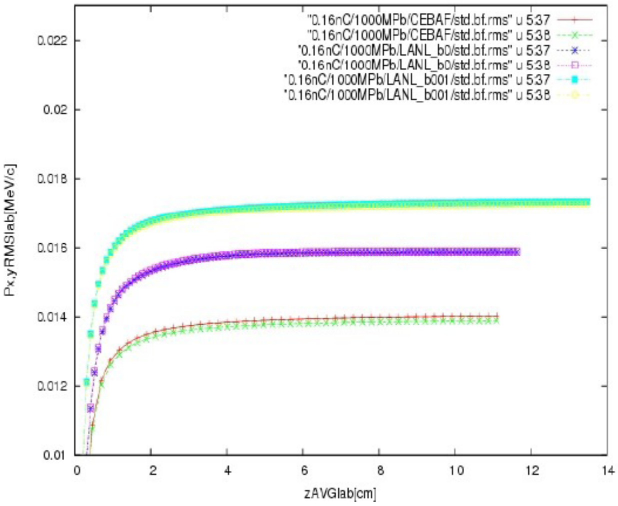

Figure 4 [11]: The comparison of RMS momentum by CEBAF Parmela (red), LANL Parmela with impact factor 0 (green) and LANL Parmela with impact factor 0.001 (blue). The beam is of 0.16 nC charge and $\beta\gamma = 0.1$

OPTIMIZATION OF ALGORITHM

Because the algorithm is based on a particle-to-particle computation, it is accurate but slow. In order to increase the computing speed, two well-established algorithms can be introduced. One is the field-in-cell method based on Particle-In-Cell, another is the leapfrog method.

The idea of field-in-cell is to establish a mesh covering the entire region of the bunch, then calculate the electro-magnetic field on mesh points, then compute the field on particles with the "area-weighting method" [12]. The computing time can thus be radically reduced because the number of grid points is much smaller than the number of macro-particles in a bunch. However, the simulation shows that at some steps some non-uniform wings are developed that introduce huge emittance growth; hence this method seems not very appropriate in this case.

The leapfrog method is to do one extrapolation by utilizing the results of two previous steps. If the step size is small enough, this method is rather accurate and frugal with time. In our case, the relativistic β of the macro-particle is approximated by the leapfrog method every "real" tracking step and then the new three-dimensional momentum of the tracking particle is computed. With this method, the computing time can be reduced by more than 50%, while the outputs are still close to those without leapfrog.

STOPPING POWER

In ionization cooling, the stopping power (energy loss per particle per unit length in absorber) plays a critical role. The growth of the transverse emittance of the beam in absorber can be described as [3]:

$$\frac{d\varepsilon_n}{ds} = -\frac{1}{\beta^2}\frac{1}{E}\frac{dE}{ds}\varepsilon_n + \frac{\beta\gamma}{2\beta^4}\frac{E_s^2}{E^2}\frac{\beta_\perp}{L_R} \tag{2}$$

where ε_n is the normalized transverse emittance, dE/ds is the stopping power, $\beta_\perp (= \beta_x = \beta_y)$ is the beta function, E_s is the characteristic scattering energy, L_R is the radiation length of the absorber medium, E is the beam particle energy, and β is v/c. The first term is the cooling term, proportional to the stopping power; the second term is the heating term due to multiple scattering. The stopping power is beam energy related and can be described by Bethe-Bloch equation [5], in which the analysis is for a single particle and the impact from other particles is neither studied nor taken into account. However, in 1977, R. A. McCorkle, et al [4, 13] suggested that there should be an extra enhancement factor of the stopping power due to the coherent interference of the electric fields of incident particles. Considering the angular dispersion of incident beam, the enhancement factor can be estimated by [4]:

$$N_{ck} \approx \frac{2n_b}{n\gamma\Delta\theta^2} \tag{3}$$

where n_b is the density of incident beam, $\Delta\theta$ is the transverse angular spread of the beam, n is the density of plasma electrons induced by incident particles. Estimation shows this factor could be bigger than 1 in the last stage of muon cooling [14], which is significant and could be possibly utilized to improve the cooling efficiency. Note that their analysis is insufficient for ionization cooling because it only includes the kinetic energy loss, omitting consideration of transverse effects. Thus more work needs to be done to complete the analysis and confirm its validity.

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