ABOUT NON RESONANT PERTURBATION FIELD MEASUREMENT IN STANDING WAVE CAVITIES

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Abstract

We discuss the use of non resonant bead pull technique for measuring fields in standing wave accelerating structures. From the Steele perturbation theory, one can derive the relation between the magnitude and phase of the field in the cavity and the complex reflection coefficient. The effect of the bead size, the calibration of the bead and the comparison with the more common resonant techniques are addressed. As an example, we discuss the measurement on a X-band bi-periodic cavity proposed for linearizing emittance at the Frascati photo-injector SPARC.

INTRODUCTION

The standard perturbation ("bead-pull") technique used to measure field inside resonant cavities is based on the Slater formula

$$\frac{\Delta\omega}{\omega_0} = \frac{\omega_p - \omega_0}{\omega_0} = -k_{SL}|E|^2/U, \qquad (1)$$

where ω_0 (ω_p) is the unperturbed (perturbed) resonance frequency of the cavity; $|E|$ is the absolute value of the field at the bead position, U the average energy and k_{SL} a constant. Equation 1 is relevant for accelerating cavities with only electric field on the cavity axis while more general results can be found, for example, in Ref. [1]. Practical hints on how to perform precise Slater measurements are discussed in Ref. [2].

Non resonant perturbation theory was first proposed by Steele [3] and allows the measurement of electromagnetic field in the perturbing bead position by measuring the complex variation of the reflection coefficient at a given frequency ω, i.e.

$$\Delta S_{11} = S_{11,p}(\omega) - S_{11,u}(\omega) = -j\omega k_{ST} E^2/P_{inc}, \qquad (2)$$

where $S_{11,p}$ ($S_{11,u}$) is the perturbed (unperturbed) reflection coefficient, P_{inc} the average power entering the e.m. structure, k_{ST} a constant and E the electric field at the bead position and at frequency ω. Non resonant perturbation method (in the following referred to as Steele method) is the only way to measure e.m. field in non resonant RF structures through a bead pull technique; it is used in resonant structures when one is interested also in the phase behavior of the field. Equation 2 is written only for electric field, but a more general relation can be found in Ref. [3].

In this paper we discuss the characterization of resonant cavities using Steele method and we compare the results with the Slater formula approach. In particular we focus on how to calibrate the beads to get field measurement in physical units (in order to be able to measure for example R/Q factor) and on the field phase measurement. We apply our results to the brazed prototype of the X-band bi-periodic cavity proposed for the SPARC high brilliance photo-injector [4].

The Slater and Steele formula for resonant cavities are obviously tightly connected, e.g. the unloaded quality factor is

$$Q_0 = \frac{\omega_0 U}{\left(1 - |S_{11,u}|^2\right) P_{inc}}, \qquad (3)$$

and they must give the same results for the field at the bead position. Moreover, theory states that $k_{SL} = k_{ST}$ for slightly perturbing beads (i.e. infinitely small objects).

The calibration constants k_{SL}, k_{ST} depend only on the bead and are normally computed performing a measurement on a cavity with known field in the bead position and

$$k_{SL} = \frac{|\Delta\omega|/\omega_0}{|E|^2/U} \quad \text{or} \qquad (4)$$

$$k_{ST} = \frac{|\Delta S_{11}|}{\left(1 - |S_{11,u}|^2\right) Q_{0,u}|E|^2/U}, \qquad (5)$$

ΔS_{11} being measured at the frequency $\omega = \omega_0$.

SINGLE CELL CAVITY

Considering a single mode in a cavity, one can derive from circuital model the expression of the reflection parameter S_{11} [1, 5] at a frequency ω

$$S_{11}(\omega) = \frac{\beta - 1 - jQ_0\delta(\omega, \omega_{res})}{\beta + 1 + jQ_0\delta(\omega, \omega_{res})}, \qquad (6)$$

where ω_{res} is the resonant frequency and for $\omega \approx \omega_{res}$

$$\delta(\omega, \omega_{res}) = \frac{\omega}{\omega_{res}} - \frac{\omega_{res}}{\omega} \simeq \frac{2(\omega - \omega_{res})}{\omega_{res}}. \qquad (7)$$

Therefore one can write Eq. 2 as

$$\Delta S_{11} = -j\frac{Q_0\delta}{1 + jQ_L\delta}\frac{1 - |S_{11,u}|^2}{2} \text{ with } \delta = \delta(\omega_0, \omega_p) \qquad (8)$$

$$\text{and} \quad \angle\Delta S_{11} = \frac{\pi}{2} - \arctan(Q_L\delta) \quad \text{for } \delta < 0. \qquad (9)$$

Moreover comparing with the Slater formula of Eq. 1,

$$k_{ST} = \frac{k_{SL}}{\sqrt{1 + (Q_L\delta)^2}}, \qquad (10)$$

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which implies that $k_{ST} \to k_{SL}$ only if $Q_L\delta \to 0$.

Figure 1 shows the ratio between the bead calibration constant with respect to the induced normalized resonant frequency shift δ for two S-band pillbox cavity modes. k_{ST}/k_{SL} is computed from Eq.s 4, 5 using reflection measurements and analytical formulas for $|E|^2/U$. The measurements are performed for three cylindrical beads of different sizes: a 1 mm long dielectric cylinder with 1.8 mm diameter (circles in Fig. 1), a 3 mm long Teflon cylinder with 3 mm diameter (diamonds) and 4 mm long dielectric cylinder with 3 mm diameter (triangles). Dashed curve are the theoretical expectation from Eq. 10.

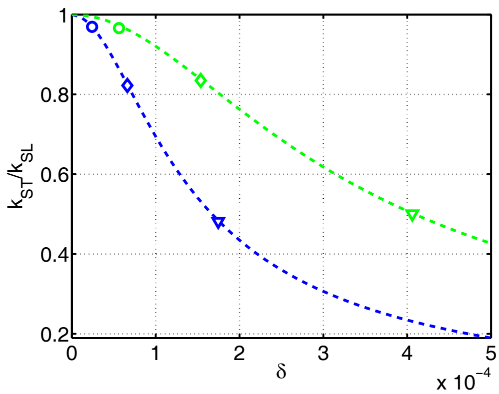

Figure 1: Ratio of the measured calibration constant k_{ST}/k_{SL} with respect to δ for TM_{010} (blue curve, f_0=1.913 GHz) and TM_{020} (green curve, f_0=4.392 GHz) modes. Points are measurements with three different beads, while the dashed lines are the theoretical expectations of Eq. 10. Uncertainty of the measurements are within the size of the experimental points.

The phase of the accelerating field can be measured with Steele method with Eq. 2, but if the single resonance model is applicable, it can also be inferred following Eq. 9 for $\angle\Delta S_{11}$, i.e. measuring $Q_L\delta$ with Slater method. To clearly see this behavior in our 4 cm long pillbox with 12 cm diameter, we can chose a mode not constant along the cavity axis, that is the TM_{011} mode and the results are shown in Fig. 2 showing a good agreement.

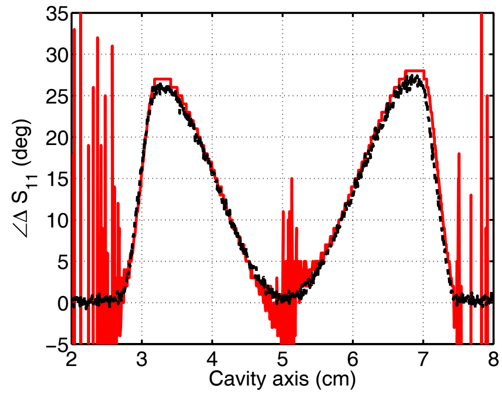

Figure 2: Phase of the ΔS_{11} (red curve) compared with the estimation of Eq. 9 (black curve). Measurement on the S-band calibration cavity resonating on the TM_{011} mode at 3.997 GHz, with Teflon cylindrical bead (3 mm of diameter and 4 mm of length).

Table 1: Cavity $\pi/2$ Mode Parameters After Tuning

	After brazing	Before brazing	Simulation
Q_0	6801	5815	7412
β	0.85	0.72	0.97
R/Q (Ω/m)	9520 ± 200	9150 ± 200	8300

SPARC BI-PERIODIC CAVITY

A bi-periodic X-band accelerating section for linearizing the longitudinal phase space in Frascati high-brilliance photo-injector (SPARC) has been proposed and the copper prototype has been realized [4]. In this paper we present measurements on the recently brazed prototype to focus some relevant topics of the measurement technique. A comparison between the brazed prototype and the previous one is shown in Table 1.

As shown for the single resonance case, if the bead is so small that $Q_L\delta \ll 1$, then k_{ST} do not depend on δ, i.e. on the field under measurement and $k_{ST} \simeq k_{SL}$. Figure 3 shows the measurements done with a 0.5 mm diameter dielectric spherical bead traveling on the bi-periodic cavity axis at the $\pi/2$ mode frequency. Both the Slater (red curve) and the non resonant Steele (blue curve) methods show the same behavior and they are in good agreement with numerical HFSS [6] simulations and also with the previous measurements on the not brazed cavity as shown in table 1. Further improvements to the measurements would be the estimation of the effect of the wire carrying the bead which may be not negligible due to the small size of the bead used. The calibration constant k_{SL} have been estimated via Slater field measurement of the fundamental mode of a X-band pillbox (with 1 cm diameter) after comparing the results with theoretical field values.

When it is not possible to use a bead such that $Q_L\delta \ll 1$, one may apply the k_{ST} corrected according to Eq. 10. An example is shown in Fig. 4 where the $\pi/2$ (tuned) mode is measured with a 2 mm long copper cylindrical bead with 1 mm diameter. One can compare the frequency shift measured according to Steele (blue curve) from Eq. 4-5 (and considering $k_{ST} = k_{SL}$), i.e.

$$\frac{\Delta\omega}{\omega_0} = \frac{|\Delta S_{11}|}{\left(1 - |S_{11,u}|^2\right) Q_{0,u}}, \tag{11}$$

to the one directly measured with Slater method (red curve). They are different if the correction on the Steele data is not applied according to Eq. 10 (black curve). The variation of $\Delta\omega/\omega_0$ (and thus δ) along the cavity (even if the structure is tuned) is a measurement artifact due to the

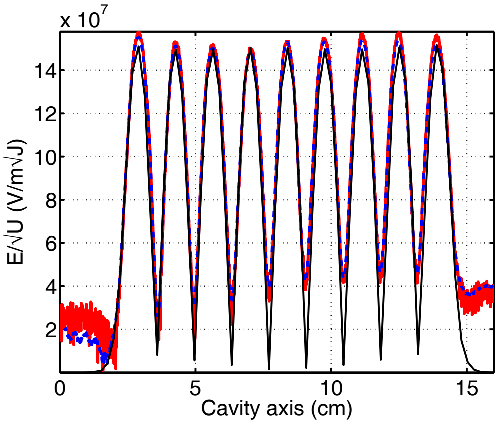

Figure 3: Accelerating electric field on the bi-periodic cavity axis after tuning. Slater results (red curve) and Steele results (blue curve) are compared against HFSS numerical simulation (black curve). ($f_0 = 11.41$ GHz)

fact that the frequency shift induced by the bead in the $\pi/2$ mode being measured is comparable with the distance of adjacent modes frequencies.

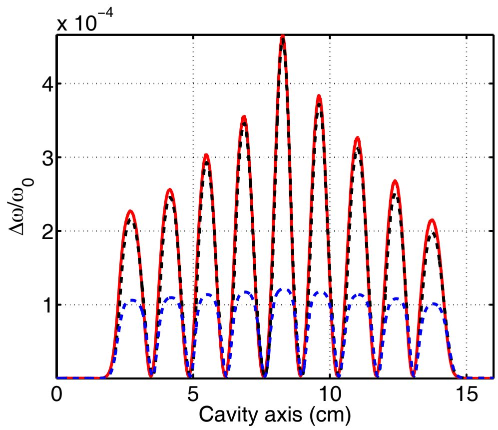

Figure 4: Comparison between Slater measurement (red curve), non resonant (Steele) measurement corrected to account the bead size (black curve) and not corrected (blue curve). ($f_0 = 11.41$ GHz).

Measuring the phase of the field along the cavity axis is possible using $\angle\Delta S_{11}$ and Fig. 5 shows the results for three different beads. The phase shift between the cell is not constant even for the smaller bead (red curve) because it is a multi-cell cavity and the single resonance model of Eq. 6-9 do not strictly apply. Nevertheless the effect of an additive depending on $Q_L\delta$ is clear in the measurement done with a bigger bead (blue line). The phase measured with an even bigger bead (green curve) is affected by the measurement artifact discussed before for Fig. 4.

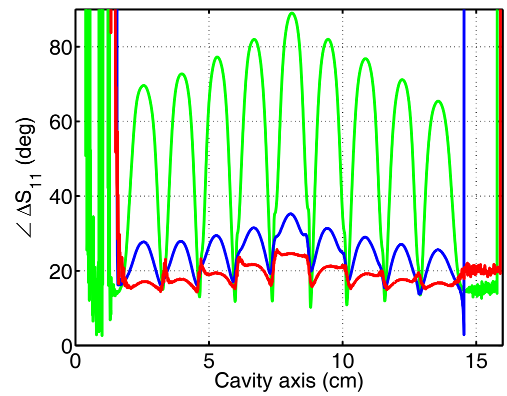

Figure 5: Measured phase of ΔS_{11} in a Steele measurement for different beads: 0.5 mm diameter dielectric sphere (red curve), 1 mm long dielectric with 0.5 mm diameter (blue curve) and 2 mm long copper cylinder with 1 mm diameter (green curve) ($f_0 = 11.41$ GHz).

CONCLUSIONS

We discussed the non resonant perturbation method applied to standing wave cavity comparing the results with the more common Slater technique, both on a single cell S-band cavity and on the brazed bi-periodic X-band cavity designed for advanced photo-injector. We focused on the calculation of the bead calibration constant and the effect of bead dimensions on it. We compared the results also against the single resonance model which is well suited for single cell cavities showing that can be applied also to field amplitude measurement of multi-cell cavities.

The calibrated non resonant perturbation method can be used in measurements of field in resonant cavities provided that the bead is small enough, that is $Q_L\delta \ll 1$; in that case the calibration constant can be measured in a known structure (e.g. a pillbox) and it is identical to the Slater calibration constant. The non resonant perturbation method is also attractive in characterizing RF structures where a standing wave part is tightly connected to a traveling wave one.

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