LATTICE STUDIES FOR A HIGH LUMINOSITY MUON COLLIDER*

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Abstract

Recent advances in the High Temperature Supercoucting magnet technology and ionization cooling theory have re-launched the interest of the physics community in the realization of a high energy, high luminosity Muon Collider (MC). Here we briefly outline the challenges set by a MC optics design, give an overview of previous attempts and propose a new approach.

INTRODUCTION

The large muon energy spread requires large momentum acceptance whereas the required luminosity calls for β^* in the mm range. To avoid luminosity degradation due to the hour-glass effect, the bunch length must be comparatively small. To keep the needed RF voltage within feasible limits the momentum compaction factor must be as small as possible.

A low β^* means high sensitivity to alignment and field errors of the Interaction Region (IR) quadrupoles and large chromatic effects which limit the momentum acceptance and require strong correction sextupoles, which eventually limit the Dynamic Aperture (DA).

Finally, the ring circumference should be as small as possible, luminosity being inversely proportional to it.

Several, more or less mature, designs may be found in the literature, covering a wide range of beam parameters. They differ on the chromaticity correction strategy and on the arc design for achieving small α_p.

CHROMATIC CORRECTION

It is known that the low-beta quadrupoles excite a large chromatic beta-wave. Usually this perturbation is corrected by using sextupole families in the arcs. The first sextupole must be in a knot of the IR chromatic wave ie at $m\pi/2$ from the low-beta quadrupoles, while the number of sextupoles and phase advance between sextupole families is chosen so as to avoid exciting the lowest order third-order resonances.

This classical approach has been applied to a 3 mm β^* MC design [1], but did not have the required momentum acceptance. The momentum compaction issue was not addressed there.

As an alternative, one can use “special sections” with large beta functions and dispersion next to the low-beta region: after the first sextupole located at a knot of the IR chromatic wave, a pseudo $-I$[1], section is inserted between it and a ”*twin*” sextupole compensating the non-linear kick. In practice such kind of schemes may be prone to unavoidable focusing errors.

Eventually, if sextupoles correcting horizontal and vertical chromaticity are not orthogonal, a *non-interleaved* correction scheme may be considered.

Some 3 mm β^* MC optics using this concept are described for instance in [2-5].

Extremely small momentum compaction values are obtained by using flexible momentum compaction modules for the arcs[6]. The rings are about 8 km long. In the best case, the resulting stability momentum range is about 0.15%.

A similar approach for the IR chromatic correction of a 3 mm β^* MC optics has been applied by K. Oide[7]. It uses non-interleaved sextupoles for both IR and arcs.

The arcs are made with 2.5π cells with negative momentum compaction factor. The ring α_p is 5×10^{-5}. The optics studies included quadrupole fringe field effects. By optimising 22 families of sextupoles and higher multipoles, a momentum stability range of $\pm0.6\%$ and a DA aperture of 3σ, for ϵ_N=25μm and a beam energy of 750 GeV, were attained. The ring is 5.7 km long. The extremely large peak value of β_y (900 km) and the very strong sextupoles make this design instructive, but likely unfeasible [8].

More recently a different approach has been proposed. The chromatic beta-wave excited by the low-beta quadrupoles may be described by [9]

$$\frac{dB}{ds} = -2A\frac{d\mu}{ds} \quad \text{and} \quad \frac{dA}{ds} = 2B\frac{d\mu}{ds} + \sqrt{\beta(0)\beta(\delta)}\Delta K$$

with

$$B \equiv \frac{\Delta\beta}{\beta} \quad \text{and} \quad A \equiv \beta\Delta\left(\frac{\alpha}{\beta}\right)$$

Going through the low-beta quadrupoles, A becomes nonzero, but as long as the phase does not change, β (and the phase advance) for off-energy particles stay unperturbed . A *local* correction with sextupoles is possible if the dispersion in such a region is made non-vanishing. If we allow $D'_x \neq0$ at the Interaction Point (IP), then we can use dispersion generated far from the IR [10].

Otherwise one needs to insert relatively strong bending magnets in the IR region. The first attempt in this direction had a $\pm$6.5 m long free drift between the low-beta quadrupoles, but it had small DA. In a second design [11] a 4 m long, 7.5 T bending magnet was introduced at 2.5 m from the IP, enhancing the dispersion at the IR sextupoles. The resulting DA is greatly improved.

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[1] $\mu = \mu_0 + \pi$ and $\beta = \beta_0$, while $\alpha \neq \alpha_0$

IR OPTICS FOR NON-INTERLEAVED SEXTUPOLES

In a more recent design we try to combine the local chromaticity correction with the non-interleaved scheme.

Required average luminosity for a 2× 750 GeV collider is $\geq 2\times10^{34}$ cm^{-2} s^{-1}. Machine parameters vary depending on the available number of muons and their emittance. Expectations are summarised in Table 1 for two possible cooling scenarios [12]. A value of β^*=1 cm in both planes

Table 1: Muon Parameters

	high transv. emitt.	*low transv. emitt.*
$N_b \times N_\mu$	$1 \times 20 \cdot 10^{11}$	$10 \times 1 \cdot 10^{11}$
$\Delta p/p$	0.1%	1%
ϵ_N	25 μm	2 μm

has been chosen has a compromise between luminosity and feasibility.

In the present design we have kept the peak beta asymmetry of the [7] design. In particular $\hat{\beta}_x$ and the the IR horizontal chromaticity are much smaller than in the vertical plane. The larger IR vertical chromatic wave is corrected "in loco" ie with a sextupole at $\Delta\mu_z \simeq 0$ from the low beta quadrupoles. Unlike [7], as the transformation across the IP between low-beta quadrupoles is a pseudo $-I$ one, we rely on the sextupole on the other side of the IP[2] for compensating the non-linear kick. A second sextupole in the first knot of the horizontal chromatic wave correct the IR horizontal chromaticity. A pseudo $-I$ section is inserted between this sextupole and a "twin" sextupole which compensates the non-linear kick[3]. Unlike[11], no bending magnet has been inserted between IP and first quadrupole.

The arcs have been realized with 90 degree 35 m long FODO cells. The maximum reachable dipole field is assumed to be about 10 T. The cell dipoles are quite long (14 m) in order to minimize the total number of cells needed to close the ring, and thus the total length. The resulting small cell chromaticity should not require a non-interleaved chromaticity correction in the arcs. To get the required small α_p a dispersion perturbation is introduced by altering the dipole symmetry: one over each 3 FODO cells has weaker reverse polarity dipoles. The resulting dispersion is large, while the β functions and phase advances are almost unchanged. The ring is 3588 m long (one IP) and the resulting momentum compaction is 7.7×10^{-5}.

Fig. 1 and 2 show the IR Twiss and MAD-X chromatic functions respectively, while in Fig. 3 and 4 IR and arc dispersion respectively are shown. Fig. 5 shows tunes vs. momentum error. The energy range in which the optics is stable is $\pm0.96\%$, almost sufficient even for the low

[2] this requires D'_x=0 at the IP and therefore the insertion of dipoles as close as possible to the IP

[3] Although the horizontal chromaticity is small the use of a twin sextupole in the horizontal plane turned out to be necessary for achieving a large DA.

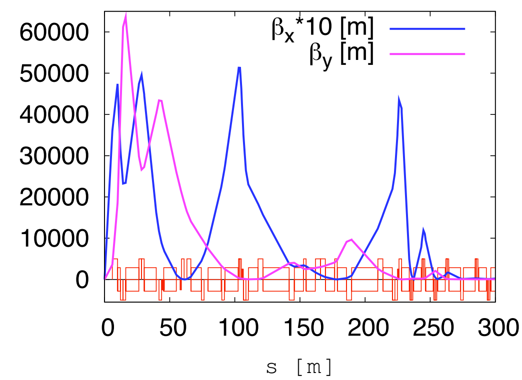

Figure 1: IR Twiss functions.

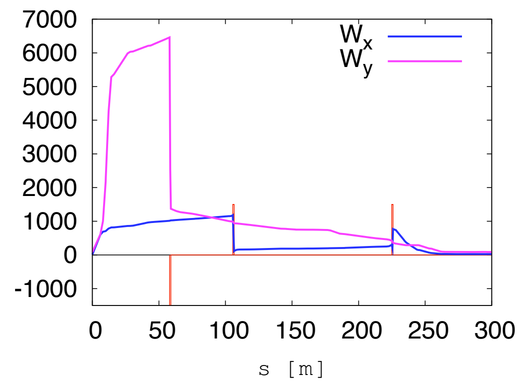

Figure 2: IR chromatic functions and sextupole locations.

transverse emittance scenario. Problematic is the large dependence of α_p on energy. We have not tried to use the up to 6 families of sextupoles available in the arcs for correcting such an effect but it looks unlikely that a solution may be found without compromising the energy stability range and with reasonable sextupole strengths. Very likely a re-design of the arcs will be needed.

DYNAMIC APERTURE

Due to the short muon lifetime, long time tracking is not needed. We tracked particles over 1000 turns correspond-

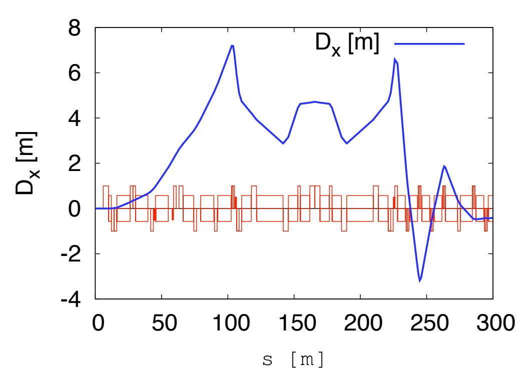

Figure 3: IR dispersion.

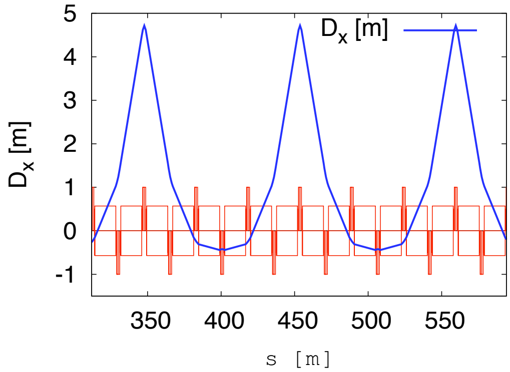

Figure 4: Arc dispersion (9 cells).

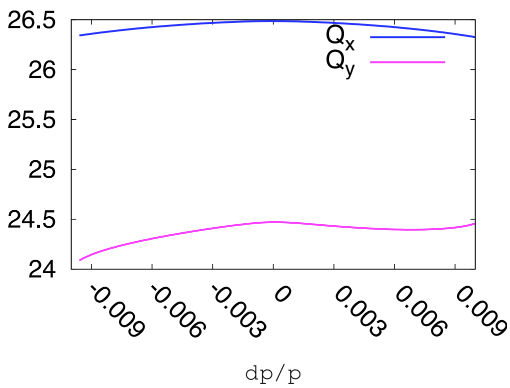

Figure 5: Tunes vs. dp/p.

ing to the time needed for the beam current to decay by about a factor 2. The detuning with amplitude has been corrected with 2 octupoles by using MAD8 STATIC module. The detuning coefficients after correction are shown in Table 2. The blue and magenta points in Fig. 6 denote the

Table 2: MAD-8 STATIC Detuning Coefficients

dQ_1/dE_1	0.60×10^4 m^{-1}
dQ_1/dE_2	0.20×10^2 m^{-1}
dQ_2/dE_2	-0.50×10^4 m^{-1}

square of the oscillation amplitudes, A_x and A_y, times γ, of stable and unstable particles respectively. The DA is 6.7 σ for ϵ_N=25 μm. Having interleaved sextupoles in the arcs and having dropped the vertical IR twin sextupole did not affect negatively the DA. It is worth noting that beam-beam issues have been not yet addressed. Beam-beam interaction may change the phase advance across the IP thus affecting the the $-I$ transformation. The use of a plasma[13] could be envisaged.

SUMMARY AND OUTLOOK

After shortly reviewing some of the previous designs, we have presented an optics for a high luminosity Muon Collider based on non-interleaved correction of the IR chromatic wave. The larger chromatic wave, vertical in this design, is corrected "in loco". The energy range, within the machine is stable, is about $\pm 0.96\%$.

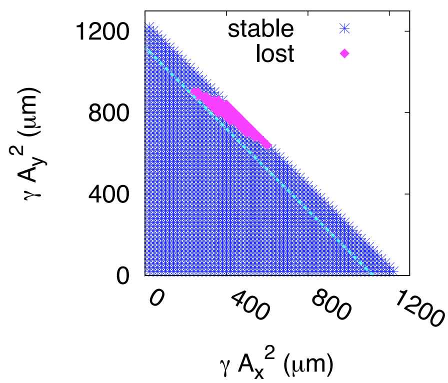

Figure 6: On-energy DA.

The Dynamic Aperture for on-energy particles is large without having to resort to an error prone non-interleaved scheme in the arcs too.

The large 2th order momentum compaction is unlikely to be corrected by using different sextupole families and will require a re-design of the arcs.

REFERENCES

[1] N. Gelfand, Private Communication. .

[2] C. M. Ankenbrandt et al., "Status of muon collider research and development and future plans", PRST-AB, vol.2, 081001 (1999).

[3] C. Johnstone, A. Garren, "A ring lattice for a 2 TeV Muon Collider", PAC97, Vancouver, May 1997, p.411.

[4] C. Johnstone, A. Garren, "An IR and Chromatic correction design for a 2 TeV Muon Collider", Proceedings of 1996 DPF/DPB Summer Study on New Directions for HEP, Snowmass 1996,p.218.

[5] P. Snopok, http://www.cap.bnl.gov/mumu/conf/MC-080317/agenda.html

[6] S. Y. Lee, K. .Y. Ng and D. Trbojevic, "Minimizing Dispersion in Flexible-momentum Compaction Lattices", Phys.reV.E **48**, 3040 (1993).

[7] https://mctf.fnal.gov/databases/lattice-repository/collider-ring/oide

[8] MCTF 2007 Report (FERMILAB-TM-2399-APC)

[9] B. W. Montague, "Linear Optics for Improved Chromaticity Correction", LEP Note 165 (1979).

[10] P. Raimondi, F. Zimmermann,"Performance of a Compact Final-Focus System for a 30 TeV MC", CERN SL Note 2001-020 (2001).

[11] Y. Alexahin, E. Gianfelice-Wendt, these Proceedings.

[12] Y. Alexahin, "Status of Medium Emittance MC Scenario", http://conferences.jlab.org/muon/program.html

[13] G. V. Stupakov and P. Chen, "Plasma Suppression of Beam-Beam Interaction in Circular Colliders", Phys. Rev. Lett. 76. 3715 (1996).