LINEAR AND NONLINEAR BEAM OPTICS STUDIES IN THE SIS18

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Abstract

The GSI heavy ion synchrotron SIS18 will be used as a booster for the SIS100 synchrotron of the new FAIR facility. The linear corrections and measurements are a necessary step before the nonlinear field errors can be applied. A tune response to a change in a sextupole magnet strength for a certain Closed Orbit (CO) deformation is used to verify beta-functions of the SIS18 model at the location of the ring's sextupoles for chromaticity correction. The progress in development of Nonlinear Tune Response Matrix (NTRM) technique [1, 2] to diagnose nonlinear field components is presented.

INTRODUCTION

In the Orbit Response Matrix (ORM) method the CO response to the steering angle change provides information on the linear field errors. The NTRM technique extends the ORM method with the difference that the tune response to the steering angle change is measured. At the moment in the SIS18 the tune can be measured with much higher precision than the CO. We are taking an advantage of measuring the tune instead of the CO to retrieve experimentally β-functions, dispersion and nonlinear field components of the machine. A nonlinear element can be a lattice sextupole as well as a dipole magnet nonlinear error. A sextupole produces additional focusing $\tilde{k}$ depending on the closed orbit position. The CO distortion in the sextupole produces the tune shift, which contains information on linear and nonlinear optics.

LINEAR OPTICS STUDIES

The first order contribution to the machine tunes with respect to the distorted CO [3] in the presence of nonlinearities is

$$\Delta Q_{x,y} = \pm \frac{1}{4\pi} \int_0^C \beta_{x,y}(s)\tilde{k}(s)ds\,, \tag{1}$$

where $\tilde{k}$ is an extra linear focusing component of the ring's nonlinear elements, C is the circumference of the ring. The tunes with respect to the CO are given by $Q_x = Q_{x0}+\Delta Q_x$ and $Q_y = Q_{y0}+\Delta Q_y$. Here Q_{x0} and Q_{y0} are the tunes of the linear lattice with the closed orbit corrected. If N_l thin normal sextupolar elements each of integrated strength K_{2l} located at s_l are included in the ring, then Eq. (1) becomes a sum

$$\Delta Q_{x,y} = \pm \frac{1}{4\pi} \sum_{l=1}^{N_l} \beta_{x,y\,l} K_{2l} x_{CO\,l}\,, \tag{2}$$

where the horizontal and vertical $\beta_{x,y\,l}$ are taken at the location s_l; $x_{CO\,l}$ is the horizontal CO at the location s_l deformed by a setting of N_t steering angles. The value of $x_{CO\,l}$ calculated via the ORM is

$$x_{CO\,l} = \sum_{t=1}^{N_t} M^x_{lt}\theta_{xt}\,, \tag{3}$$

so that Eq. (2) reads

$$\Delta Q_{x,y} = \pm \frac{1}{4\pi} \sum_{l=1}^{N_l}\sum_{t=1}^{N_t} \beta_{x,y\,l} K_{2l} M^x_{lt}\theta_{xt}\,. \tag{4}$$

Choosing one sextupole at $l = \bar{l}$ and changing its strength of $\Delta K_{2\bar{l}}$, and taking one steerer $t = \bar{t}$, then from Eq. (4) we obtain β-function at the location $s_{\bar{l}}$

$$\beta_{x,y\,\bar{l}} = \pm \frac{4\pi}{\Delta K_{2\bar{l}}} \frac{\Delta Q_{x,y}}{\Delta x_{CO\,\bar{l}}} = \pm \frac{4\pi}{\Delta K_{2\bar{l}}} \frac{{}_{x,y}Q^x_{\bar{t}}}{M^x_{\bar{l}\bar{t}}}\,, \tag{5}$$

where ${}_{x,y}Q^x_{\bar{t}} = \Delta Q_{x,y}/\theta_{x\bar{t}}$. The SIS18 has 12 normal

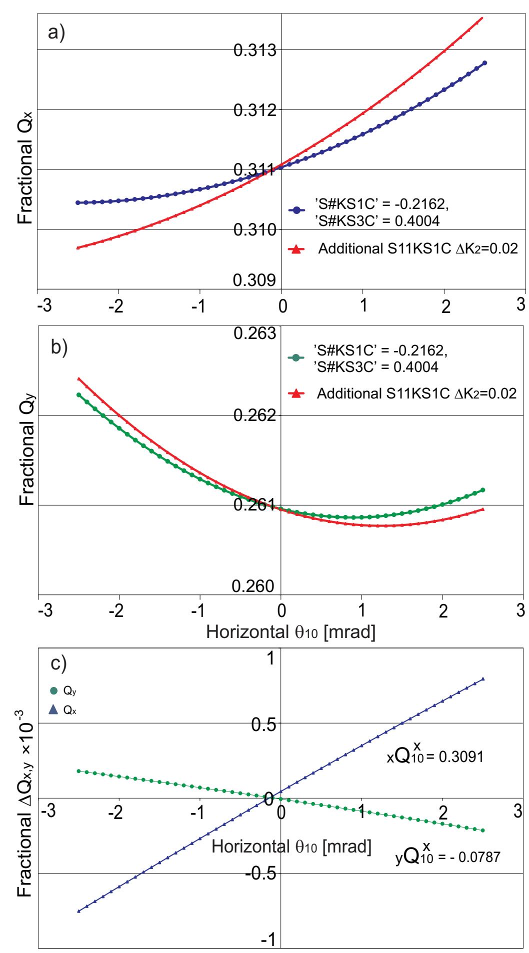

Figure 1: Simulation to the β-function measurement.

independently powered sextupoles, united in two groups for the horizontal (S#KS1C) and vertical (S#KS3C) chromaticity correction (here # = 1,3,5,7,9,11). Perturbing each of them consecutively with a certain ΔK_{2l} allows to measure 12 horizontal and vertical β-functions. Coherent beam oscillations were excited by a fast Q-kick of about 0.15 mrad given on 45° in both X- and Y- planes. The beam particle tune was computed using the FFT of the coordinates over 2048 turns with data filtering [4]. The sextupoles were activated with their chromaticity settings: (S#KS1C) with $K_2 = -0.2162$ m^{-2} and (S#KS3C) with $K_2 = 0.4004$ m^{-2}. The CO was varied using the steerer #10 (S10MU1A). The coherent oscillations are measured for each steering setting of the perturbed CO. After the sextupole S11KS1C was increased by $\Delta K_2 = 0.02$ m^{-2} the procedure was repeated. The simulation for the horizontal and vertical tune responses is presented in Fig. 1a and b. Subtracting two parabolas differential linear slopes are obtained (Fig. 1c). Using Eq. (5) the $\beta_{x,y\,11}$ are found.

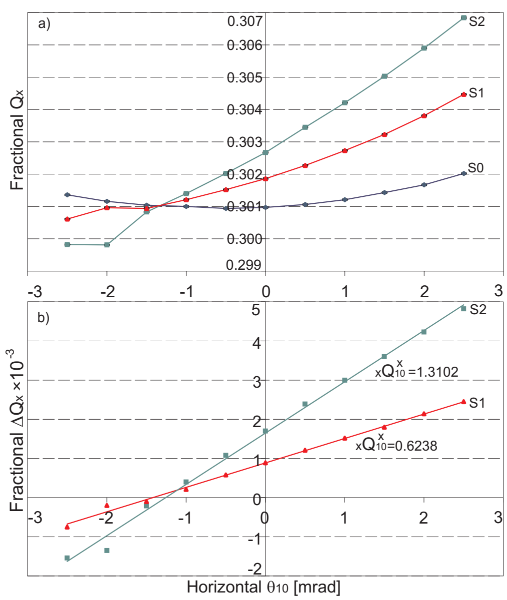

Figure 2: Test of the proposed β-function measurement.

According to the simulation the accuracy of the retrieved β-functions is better then 0.5 %. For the distorted vertical/horizontal CO with a kick of 0.5-2 mrad the relative error increases to 2-10 %. A measurement for conditions close to the simulations is presented in Fig. 2. The $\beta_{x\,11}$ obtained for two different excitations of the sextupole S11KS1C are given in Table 1, the corresponding model values are $\beta_{x\,11} = 29.06$ m and $\beta_{y\,11} = 7.43$ m.

Table 1: $\beta_{x\,11}$ Retrieved from Measurement Data

ΔK_2 , m^{-2}	$\beta_{x\,11}$, m	Rel. error, %
S1: 0.04	29.49	1.5
S2: 0.08	30.97	6.5

Eq. (1) allows a verification of dispersion at the location of sextupoles s_l after the β-functions are found. By altering the mean beam momentum, the CO is changed $\Delta x_{CO\,l} = D_l \Delta p/p$, which inserted in Eq. (5) yields

$$\frac{\Delta Q_{x,y}}{\Delta p/p} = \pm\frac{1}{4\pi}\Delta K_{2l}(D\beta_{x,y})_l\,, \tag{6}$$

then we find the product $(D\beta_{x,y})_l$. To test Eq. (6) the sextupole S11KS1C was varied with $\Delta K_2 = -0.374$ m^{-2}. The chromaticity change produced in both X- and Y- planes was measured. Using the model values for $\beta_{x\,11}$ and $\beta_{y\,11}$ the dispersion at the location of the sextupole S11KS1C was experimentally estimated, see Table 2.

Table 2: D_{11} Retrieved from Measurement Data

Model	Exp. using $\beta_{x\,11}$	Exp. using $\beta_{y\,11}$
D_{11}: 3.16 m	(3.48 ± 0.35) m	(3.54 ± 0.36) m

NONLINEAR OPTICS STUDIES

The other important application of Eq. (1) is the reconstruction of sextupolar field errors of the ring's main dipoles. The NTRM technique to diagnose nonlinear field components in circular accelerators was tested in [1, 2] with two sextupolar errors. We extend here its validation by using six probing normal sextupolar errors. The six chromatic sextupoles (S#KS1C) and six horizontal steerers were chosen arbitrary in the ring (Fig.3). The measurement conditions were adjusted close to those of the previous tests for two sextupolar errors [1, 2]. A one turn injection was optimized to create a 'pencil' like beam to exclude finite beam size effects on the tune. The tune response with chro-

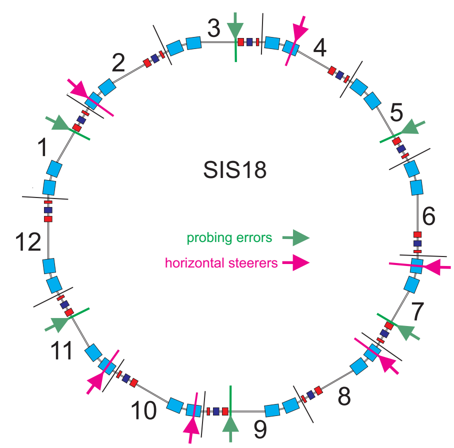

Figure 3: SIS18 sextupolar magnets and steerers used.

matic sextupoles powered on (referred to the setting $S0$) is measured. Then six sextupoles for chromatic correction get a small extra probing strength error, and the tune response is re-measured for the same CO deformation (setting $S1$), see Fig. 4. By subtracting the two tune response curves, the resulting differential tune response (Fig. 4c) depends solely from the extra probing error added to the sextupoles. When the normal probing errors are excited, only the horizontal deformation of the CO can reveal them

$$_xQ_t^x = \frac{1}{4\pi}\sum_{l=1}^{N_l}\sum_{t=1}^{N_t}\beta_{x\,l}K_{2l}M_{lt}^x\,, \tag{7}$$

where $\Delta Q_x =_x Q_t^x \theta_{xt}$. As the probing errors are folded linearly into the terms ${}_xQ_t^x$, the experimental task is to measure the differential tune response and obtaining ${}_xQ_t^x$. The procedure for measuring ${}_xQ_t^x$ was repeated for the other five horizontal steerers. Solving the linear system of Eqs. (7) with the simulated $\beta_{x\,l}$ and M_{lt}^x the set of probing errors K_{2l} was found. The results obtained from the simulations and experiments are summarized in Table 3.

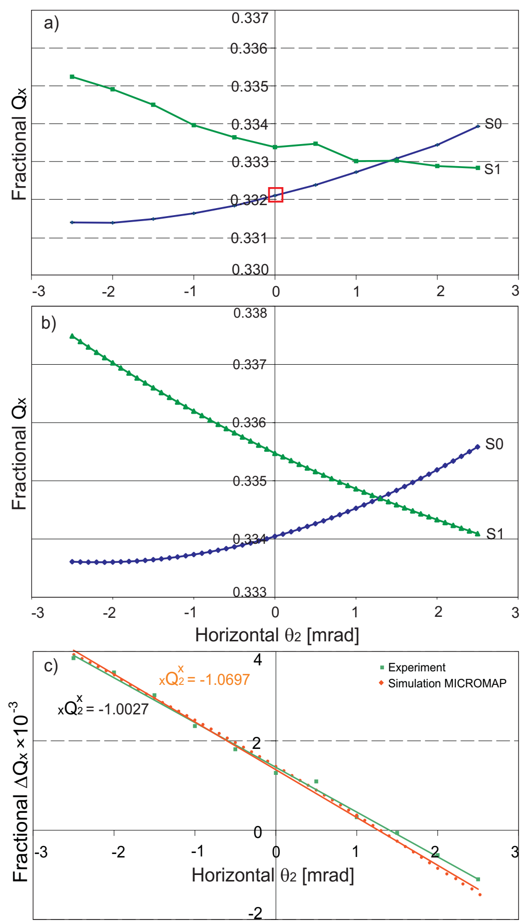

Figure 4: Measured a) and calculated with MICROMAP b) fractional part of the horizontal tune vs. horizontal steering angle θ_2 (S02MU1A). The corresponding differential tune responses d).

Table 3: Additional Strengths Applied in the Sextupoles

Normal sextupolar error, l	ΔK_2	Cal.	Exp.	Rel. Err., %
	$\times 10^{-2}$, m^{-2}			
1	3	3.011	2.634	12.2
2	-4	-4.001	-4.440	10.0
3	1	1.000	0.773	22.7
4	-1	-1.001	-1.758	75.8
5	-2	-2.001	-1.786	10.7
6	2	2.020	1.480	26.0

Note that the six probing normal sextupolar errors are of the order of natural errors ($K_2 \approx 0.01$ m^{-2}). The accuracy in reconstruction of the six sextupolar field errors is about 10-25%, except for the error $l = 4$. The drawback of the performed tune measurements is that the working point (tune value for not distorted CO marked with a red square in Fig. 4a) was set close to the third order resonance. Although a relatively large beam loss was observed, it was possible to obtain the tune value from the measured spectra with sufficient precision. For this reason, the reproducibility of the solution has been checked numerically for horizontal tunes in the range `4.31:4.35`. It is important to note that Fig. 4c shows indirect test of sextupolar- and steerer-magnets' calibration.

APPLICATION AND OUTLOOK

A possible lattice model with nonlinear components is made with one sextupolar error located in each period of the SIS18. In this case the total change in the horizontal tune produced by the 12 natural sextupolar errors $K_{2\lambda}$ and 12 chromatic sextupoles K_{2l} is

$$4\pi\, {}_xQ_t^x = \sum_{t=1}^{N_t} [\sum_{\lambda=1}^{N_\lambda} \beta_{x\,\lambda} K_{2\lambda} M_{\lambda t}^x + \sum_{l=1}^{N_l} \beta_{x\,l} K_{2l} M_{lt}^x]\,. \quad (8)$$

The effect of chromatic sextupoles [the second term in Eq. (8)] is included as the precision of the tune evaluation is often limited by the signal decoherence. However, a tune measurement over 2048 turns was performed without chromatic sextupoles, see Fig. 5. The measured tune response is produced only by the systematic and natural errors of the machine. The parabolic behavior can be a result of the presence of octupolar and coupled sextupolar field components [1]. The linear contribution of the normal sextupolar field errors ${}_xQ_2^x$ is linear in θ (red marker in Fig. 5). Obtaining ${}_xQ_t^x$ for the other 11 horizontal steerers will complete the left-hand side of the linear system of Eqs. (8) for finding the 12 natural sextupolar components $K_{2\lambda}$ in the each period of the SIS18.

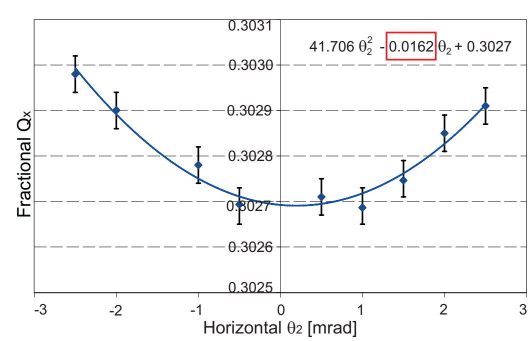

Figure 5: Measured fractional part of the horizontal tune vs. horizontal steering angle θ_2 (S02MU1A) without chromaticity corrected.

REFERENCES

[1] G. Franchetti *et. al.*, Phys. Rev. ST **11**, 094001 (2008).

[2] A. Parfenova *et. al.*, Proceedings EPAC 2008 (THPC066).

[3] S.Y. Lee, "Accelerator Physics", 2004.

[4] R. Bartolini *et. al.*, Particle Accelerators **5**, p. 147 (1996).