# SINGLE PARTICLE DYNAMICS IN THE UNIVERSITY OF MARYLAND ELECTRON RING 

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## Abstract

We undertake a study of the single particle dynamics in a model of the University of Maryland Electron Ring. This accelerator uses a low energy electron beam to study the effects of space charge on beam dynamics. However, due to this low energy, other effects that are seldom taken into account in high energy accelerators become important to the single particle dynamics of the beam. The simulation is performed using COSY Infinity, which has the effects of the earth's magnetic field added to it. When the simulated trajectories are compared to measured beam positions there is good agreement through the ninth section of the ring, at which point the difference between predicted and observed diverges. A method of calculating map elements corresponding to the measured data will be used to determine where issues with the ring that could cause these problems might be found.

## INTRODUCTION

The University of Maryland Electron Ring is a low energy (10Kev) Electron re-circulating ring. It is approximately 3.8 meters in Diameter and is designed to study high space charge beams under controlled conditions. The acceleration for the ring currently occurs in the electron gun, which uses thermal emission, and whose current is controlled via a series of apertures.


Figure 1: COSY Infinity produced Diagram of the University of Maryland Electron Ring. RC stands for Ring Chamber each one contains a bpm which can be switched for a phosphor screen, 4, 10, and 16 have current monitors instead.

The ring is composed of an injection line that enters into a Y-shaped injection section. Seventeen sections, each containing four quadrupoles and two dipoles, comprise the rest of the ring. Each section has an interchangeable BPM and phosphor screen. The fourth, tenth, and sixteenth sections do not have those diagnostics and instead are used for resistive wall current monitors [1], The layout is analogous to that in Figure 1.

Recently a series of experiments were performed on the beam with the purpose of more fully understanding the single particle dynamics of the UMER lattice. While performing this analysis an unknown issue caused difficulty with respect to predicting data past the ninth beam position monitor.

First we will present a brief overview of the initial simulations and the difficulties with calculating past the ninth position monitor, then a new method for map benchmarking will be elucidated. Finally, experimental results will be used to help confirm the procedure.

## SIMULATION AND THEORY

The single particle dynamics code that is being used for this research is COSY Infinity 9.0. This is a map based code that uses differential algebras to allow for map elements to be calculated to arbitrary order. Furthermore it uses normal form algorithms to calculate the betatron tunes using only the map, and thus no costly particle tracking. [2]


Figure 2:. Comparison of horizontal position data for the UMER 7 mA beam using the Beam position Monitors Provided. Blue represents simulated predictions while black represents measured.

Since UMER is so small and has such low energy, the earth's magnetic field is a non-trivial issue. This is implemented in the COSY simulation as a kick within the map. As can be seen in Figure 2 there is good agreement between the measured values and the computed values through the section labelled RC 9. Afterwards there is strong disagreement with theory.


Figure 3: Plot of measured trajectories at the various beam position monitors through the pinhole scan.

One way of benchmarking code within a beam that may have un-anticipated issues would be to calculate the actual map elements themselves. Theoretically if both the position and momentum could be measured then the map elements could be calculated. Using equations 1,2 , where the tilde indicates that it is the measurement after one iteration of the lattice, multiple tildes mean multiple iterations.

$$
\begin{align*}
\widetilde{x_{n}} & =(x \mid x) x_{n}+\left(x \mid p_{x}\right) p_{x n}  \tag{1}\\
\widetilde{p_{x}} & =\left(p_{x} \mid x\right) x_{n}+\left(p_{x} \mid p_{x}\right) p_{x n} \tag{2}
\end{align*}
$$

Unfortunately finding the momentum, or in our coordinate system the angle, is impossible directly since current methods require at least two beam position monitors separated by a large drift [3]. Since UMER does not have this, a new method is needed.

One method would be to choose our coordinate system in a way that would make calculating the map elements more simple. The simplification is made by choosing our initial assumptions carefully. As a first guess we assume that the ring can be described linearly, and that there is negligible coupling. In practice the ring shows little coupling after injection.

Furthermore we make some assumptions about the statistical properties of the angles used, so we use the ( $x$, a) system instead of ( $x, p_{x}$ ). We assume that the coordinate, a, has zero average.

$$
\begin{equation*}
<a>=0 \tag{3}
\end{equation*}
$$

We also assume that a has a standard deviation which is a known constant, which for this derivation we normalize to 1 .

$$
\begin{equation*}
\sqrt{\frac{\sum_{n}\left(<a>-a_{n}\right)^{2}}{n}}=\sigma_{a} \tag{4}
\end{equation*}
$$

These two statements require that we have a multitude of measurements of particles going through three identical lattices.

Equation 3 lets us calculate the $(\mathrm{x} \mid \mathrm{x})$ value directly:

$$
\begin{equation*}
(x \mid x)=\frac{<\tilde{x}>}{<x>} \tag{5}
\end{equation*}
$$

In order to calculate ( $\mathrm{x} \mid \mathrm{a}$ ) we subtract a value for x from the mean value of $x$. Since the standard Deviation of a is 1 this drops out, leaving a value for $(\mathrm{x} \mid \mathrm{a})$.
$\sqrt{\frac{\sum_{n}\left(<\widetilde{x}>-\widetilde{x_{n}}-(x \mid x)\left(<x>-x_{n}\right)\right)^{2}}{n}}=(x \mid a)$
By using another two iterations of the map (a|x) and (a|a) are calculated:

$$
\begin{array}{r}
(a \mid x)=\frac{\widetilde{\widetilde{x}}>-(x \mid x)<\widetilde{x}>}{<x>(x \mid a)} \\
(a \mid a)=\frac{\approx}{<\bar{x}>-(x \mid x)<\widetilde{x}>}-\frac{<\widetilde{x}>}{<x>(x \mid a)(a \mid x)}-\frac{<x>}{<x} \tag{8}
\end{array}
$$

Thus for any lattice with a multitude of particles for which there is position data at four consecutive applications of the map, the elements can be calculated for x and a .

This can be easily tested on a simplified model of the ring. First we neglect the earth's field, and use eighteen identical sections. In order to show the usefulness of this technique for finding problems, there will be a 2 mm offset placed directly before RC 7 .

| COSY Trace | -0.7879119899645564 |
| ---: | :--- |
| RC 1 Trace | -0.7879119899645504 |
| RC 3 Trace | -8.932929167121786 |
| RC 5 Trace | -0.2130762504790107 |
| RC 7 Trace | -0.2804785104716530 |
| RC 9 Trace | -0.7879119899645519 |

Here it is clearly visible that the trace changes when the map element calculation first begins using the offset point for its calculations the trace changes.

## EXPERIMENTS

The current of the UMER beam is controlled by using rotating a set of apertures in front of the cathode. As part of an unrelated experiment it was decided to move the smallest aperture across the cathode vertically at halfmillimetre intervals. This created seven separate beams that were then measured at the various beam position monitors. The horizontal results of this scan are shown in Figure 1. Each of these beams was 0.6 milliamps and can therefore be considered as though their centroids are single particles.

These particles are going to form the basis for the calculation of map elements. Since each iteration needs to be over the same map, and three even numbered beam position monitors have been replaced by current monitors, a double period has been chosen using the odd numbered beam position monitors. The results of this scan are shown in Figure 3.

The map elements are now calculated using equations 5 through 8, with an example here:

$$
\left(\begin{array}{ll}
(x \mid x) & (x \mid a)  \tag{9}\\
(a \mid x) & (a \mid a)
\end{array}\right)=\left(\begin{array}{cc}
-2.01045 & 14.2143 \\
-0.332264 & 2.78216
\end{array}\right)
$$

The question now becomes how this method is useful in comparing measurements to code. While the maps created in the simulation and the maps created from the above method are based on different coordinate systems, there are methods for comparing them. Since the method involves a linear transform between $\mathrm{p}_{\mathrm{x}}$ and a and that the matrix is symplectic, then the betatron tune should remain the same, which means that the trace of the two maps should be the same, no matter what the coordinate transform.
Table 1. Trace data taken using the pinhole scan method.

| Map starting at <br> BPM \# | Trace <br> (calculated) | Trace <br> (measured) |
| :---: | :---: | :--- |
| 1 | 1.105 | $0.771716 \pm .136$ |
| 3 | 0.88587 | $0.445086 \pm .019$ |
| 5 | 8.47313 | $0.823861 \pm .17$ |
| 7 | -0.01043 | $3.28036 \pm .0033$ |
| 9 | -0.17871 | $-0.865915 \pm .036$ |
| 11 | 0.29599 | $2.41813 \pm .401$ |

As Table 1 shows there may be disagreement both between simulated and measured. There is also disagreement between traces based on where the map element calculation begins. This can have multiple causes. Either the lattice can change dramatically, this would be the case if one of the quadrupoles within the three lattices used was different from the others, or it can signal that the map has a constant term which usually indicates that there is an offset, or misalignment.

We begin to see the first major change in the trace with the map which was calculated from BPM 5. This is
because that is the first map that includes BPM 11, which is where the difficulties began.

It should be noted that with regard to the first trace in each instance, these are large most likely due to the offset errors from injection. The later traces which don't include those do not show such a large amplitude. This is most likely because the assumption that the average a is zero may not be valid enough if the offset imposes a large kick. Furthermore data is not shown for maps starting with RC 13,15 , and 17 because they pass through the Yinjection section and there is no evidence that the lattice should be the same as the rest of the ring.

If it is safe to assume that the first three sections under consideration are offset free, then it is possible to calculate the offset based on the difference in trace between the two sets of sections. However once the map moves beyond the first instance of an offset, then the difference in traces will contain evidence of both offsets and angular displacements which we have not yet determined how to differentiate.

## CONCLUSIONS

The University of Maryland Electron Ring is capable of performing many interesting experiments. While it was designed for high space charge operation its unique size, energy, and ambient conditions all make its low space charge operation an interesting and non-trivial problem. Furthermore the flexibility that its design possesses as well as its ease of use make it useful as a test bed for not only research in space charge, but also new beam diagnostics and steering algorithms.
The ability to move the aperture and thus create additional simultaneous data-points allows a user to compute the elements of an idealized transfer map which can be used both for computer benchmarking, and for misalignment diagnosis.

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