HIGH PRECISION BEAM ENERGY MEASUREMENT WITH CHERENKOV RADIATION IN AN ANISOTROPIC DISPERSIVE METAMATERIAL LOADED WAVEGUIDE*

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Abstract

A new method of measurement of charged particle energy is considered. This is based on measurement of frequencies of modes generated in a waveguide loaded with a cylindrical layer of an engineered or natural medium. This method can be realized with the help of different materials. Anisotropic materials with "plasma-like" dispersive properties can be used for high precision energy measurements at relatively small values of the Lorentz factor γ. A thin layer of simple non-dispersive isotropic dielectric is convenient for measurement of larger values of γ. The range of values of γ to which these techniques are sensitive can be extended through the use of resonant dispersive metamaterials.

GENERAL RESULTS

Cherenkov radiation is extensively used for the detection of charged particles and in beam diagnostics [1]. We have developed a new method of determination of the energy of charged particles based on measurement of frequencies of waveguide modes [2–6]. Metamaterials demonstrate advantages for this purpose compared with conventional media. Problems of development of metamaterials for Cherenkov detectors are discussed in ref. [7]. In the present paper, we consider some prospects for the use of both metamaterials and traditional dielectrics on basis of certain "macro-models". This analysis assumes that the "metamedium" can be treated as possessing an effective permittivity or permeability.

Unlike the work of refs. [2–6], we will now take into account the presence of a vacuum channel and analyze the influence of the channel radius on the waveguide modes. Different models for the material of the Cherenkov radiator will be considered. The main focus is the case when the material of the cylindrical layer is a non-magnetic medium characterized by a diagonal permittivity tensor

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}, \tag{1}$$

The basic model of dispersion is of the form

$$\varepsilon_\perp = \varepsilon_c\,, \quad \varepsilon_\parallel = \varepsilon_c - \omega_p^2\left(\omega^2 + 2i\omega_d\omega - \omega_r^2\right)^{-1}, \tag{2}$$

where ω_p is a plasma frequency, ω_r is a resonant frequency, ω_d is an attenuation parameters, and ε_c is some constant exceeding 1. Some peculiarities of the particular case of a non-dispersive isotropic dielectric will be noted as well.

It is assumed that the medium forms a cylindrical layer in the waveguide with some radius a. The main axis of the medium (i.e. the z-axis) coincides with the waveguide axis. A vacuum channel has radius b, and the thickness of the medium layer is $d = a - b$. The charged particle bunch moves along the z-axis with a velocity $\vec{V} = c\beta\vec{e}_z$, and Lorentz factor $\gamma = \left(1-\beta^2\right)^{-1/2}$. The transverse dimension of the bunch is negligible, and longitudinal distribution of the charge is determined by the Gaussian function $\exp\left(-\zeta^2/\left(2\sigma^2\right)\right)$, where $\zeta = z - Vt$ and σ is much less than the typical wavelength.

Our analysis is based on the mode expansion of the wave field behind the bunch:

$$\begin{Bmatrix} E_\rho \\ E_z \\ H_\phi \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} E_{0\rho}^{(m)}(\rho) \\ E_{0z}^{(m)}(\rho) \\ H_{0\phi}^{(m)}(\rho) \end{Bmatrix} \exp\left\{-\frac{\omega_m^2\sigma^2}{2V^2} + \frac{i\omega_m\zeta}{V}\right\}, \tag{3}$$

where ρ, ϕ, z are cylindrical coordinates. The mode frequencies $\omega_m = 2\pi\nu_m$ are determined by the equation

$$\begin{aligned} &s\,I_1(k_0b)\left[J_0(sb)N_0(sa) - J_0(sa)N_0(sb)\right] - \\ &-\varepsilon_\parallel k_0\,I_0(k_0b)\left[J_1(sb)N_0(sa) - J_0(sa)N_1(sb)\right] = 0, \end{aligned} \tag{4}$$

where

$$k_0 = \omega(c\beta\gamma)^{-1}, \quad s^2 = \omega^2V^{-2}\varepsilon_\parallel\varepsilon_\perp^{-1}\left(\beta^2\varepsilon_\parallel\varepsilon_\perp - 1\right), \tag{5}$$

$J_n(\xi)$ are Bessel functions, and $I_n(\xi)$ are modified Bessel functions.

For high precision measurement of beam energy, the dependence $\omega_m(\gamma)$ should be sensitive, and the amplitudes of the modes should be not very small. Therefore it is better to use low frequencies (<30 GHz, as a rule) because of the exponential factor in (3).

"PLASMA-LIKE" MATERIAL

Previous publications [2–6] were devoted to the case of waveguide filled fully with anisotropic "plasma-like" ($\omega_r = 0$) medium. Now we consider the influence of vacuum channel for this model of the medium.

It should be noted that obtaining "plasma-like" response of metamaterials at frequencies 1 – 30 GHz is not a simple problem. This response takes place in a

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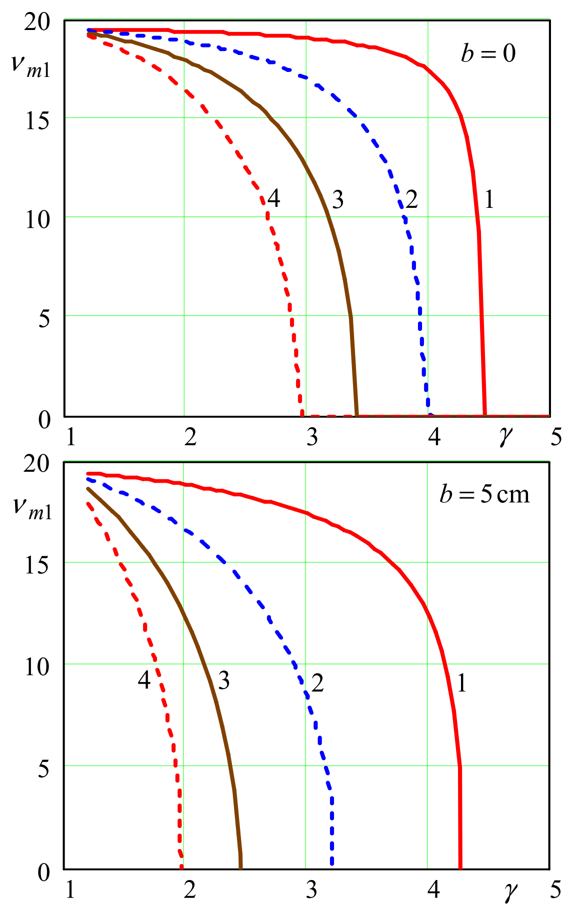

Figure 1a: Dependence of the frequencies ν_{m1} (GHz) on γ in the case of anisotropic "plasma-like" medium filling the waveguide fully (above) or partially (below); $a = 10\,\text{cm}$, $\nu_p = 20\,\text{GHz}$, $\varepsilon_c = 1.05$; mode numbers m are indicated near the curves.

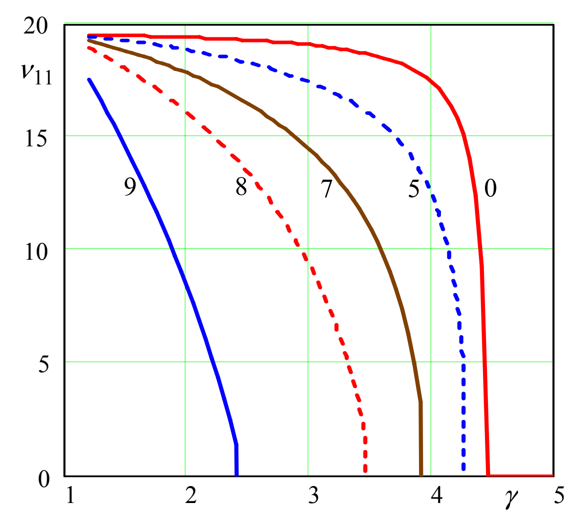

Figure 1b: Dependence of the frequency ν_{11} (GHz) on γ in the same case for the channel radius values indicated near the curves (cm).

system of infinitely thin periodic conductors if the electric field of the wave is parallel to the wires. However for other wave polarizations the formula for $\varepsilon_{\|}$ is more complex because of the effects of spatial dispersion [8]. Some structures described by the formula (2) are found in ref. [9]; one of them consists of wires loaded with plates.

Analysis based on the model (1) and (2) shows that there are two series of mode frequencies. The low-frequency series (ν_{m1}) can be generated under the condition $\beta^2\varepsilon_c < 1$, and the high-frequency series (ν_{m2}) can be generated for $\beta^2\varepsilon_c > 1$. The frequency ν_{11} is most convenient for measuring relatively small values of the Lorentz factor. Figures 1a and 1b show that choice of the radius of the vacuum channel can be used to obtain a desirable dependence $\nu_{11}(\gamma)$. For measuring γ with high precision in some narrow range close to the limiting value $\gamma_{\max}$, it is better to use a channel with a small radius. For measurement of γ over a wider range, it is convenient to have a smoother dependence. This can be achieved by use of a larger of channel radius (5 – 9 cm for Fig.1b). It is interesting that the dependence $\nu_{11}(\gamma)$ is close to linear for a thin enough "plasma-like" layer (Fig.1b, $b = 9\,\text{cm}$).

NON-DISPERSIVE MEDIA AND RESONANT MATERIALS

A "plasma-like" medium is convenient for measuring relatively small γ but not very comfortable for bigger γ because this requires very small values of ε_c. It is interesting that good results can be achieved through the use of non-dispersive isotropic dielectrics ($\varepsilon_{\perp} = \varepsilon_{\|} = \varepsilon = const$). The key factor for this technique consists in optimization of the thickness of the layer d.

Dependence of $\nu_m(\gamma)$ on γ increases with decreasing d. Therefore, the use of thin layers of simple dielectric provides essential advantages. Of course, there is some limitation on decreasing d because the amplitude of the mode that is considered must be large enough for measurement. However, as a rule, this limitation is not very restrictive.

Figures 2a-b show mode frequencies in the case of a simple dielectric layer. One can see that using layers with $d = 2 \div 10\,mm$ allow determination of values of $\gamma \sim 10 \div 30$ at least (for other parameters as indicated). For $d \le 1\,mm$ the frequencies exceed 30 GHz, and the amplitudes driven by a realistic bunch can be too small for measurement.

A shortcoming of this technique consists in its limitation on γ from below. For example, if $b = 9.5\,\text{mm}$ then the range $5 < \gamma < 10$ corresponds to very high frequencies (Fig.2) and, accordingly, small amplitudes

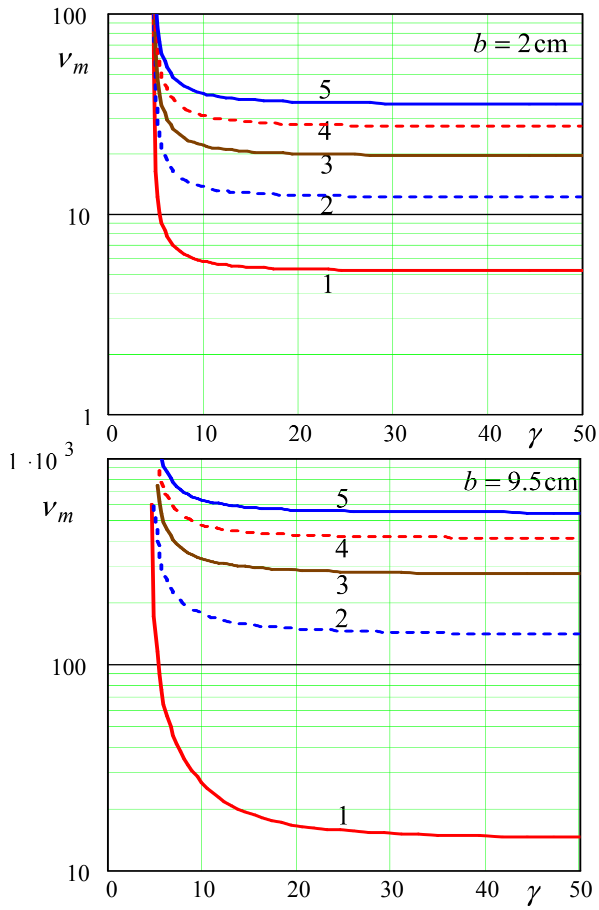

Figure 2a: Dependence of the frequencies ν_{m1} (GHz) on γ in the case of non-dispersive isotropic dielectric for thick (above) and thin (below) layers. Numbers of modes m are indicated near the curves; $a = 10\,\text{cm}$, $\varepsilon = 1.05$.

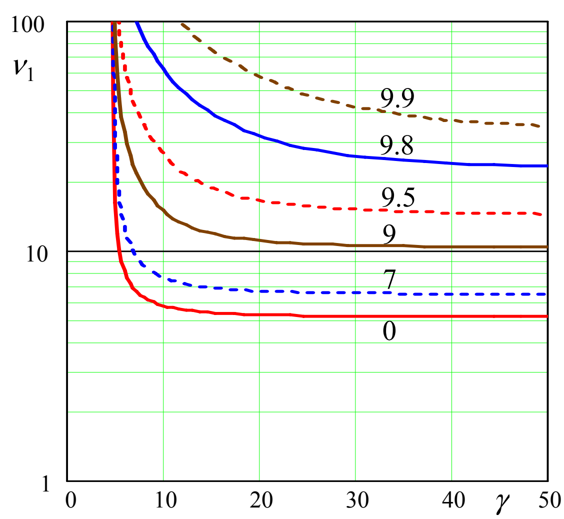

Figure 2b: Dependence of frequency ν_1 (GHz) on γ in the same case for the channel radius values indicated near the curves (cm).

for bunches with $\sigma \geq 3\text{mm}$. This defect can be partially eliminated with the use of a resonant material. Such a metamaterial can be made in the form of a periodic system of short thin parallel wires.

Figure 3 demonstrates the effect of resonant dispersion in the case of a relatively thin layer of material. The medium parameters indicated correspond to wire length ~5mm, wire radius ~0.5mm, and the transverse period ~5mm. One can see that the essential dependence $\nu_{11}(\gamma)$ takes place at least for $5 < \gamma < 30$, and the frequency does not exceed 30 GHz. Other frequencies $\nu_{m1}(\gamma)$ are practically constant in this situation.

In conclusion, it should be underlined that application of different materials and optimisation of the layer thickness give good prospects for creation of a technology for noninvasive bunch diagnostics for different energy ranges.

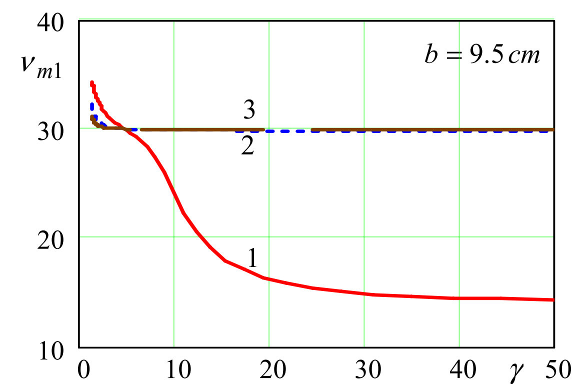

Figure 3: Dependence of the frequencies ν_{m1} (GHz) on γ for thin layer of resonant anisotropic material. Numbers of modes are indicated near the curves; $a = 10\,\text{cm}$, $\varepsilon_c = 1.05$, $\nu_p = 20\,\text{GHz}$, $\nu_r = 30\,\text{GHz}$.

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