THEORY AND APPLICATIONS OF LATTICE WITH NEGATIVE MOMENTUM COMPACTION FACTOR

Yu. Senichev*, Institute of Nuclear Physics, Forschungszentrum Juelich, D-52425 Juelich, Germany

Abstract

A possible solution for avoiding transition energy crossing is a lattice with a negative momentum compaction factor. The lattice developed for this purpose is based on the resonantly correlated curvature and gradient modulations in arcs with integer tunes in horizontal or both planes, and is known as the "resonant" lattice. This method was first adopted for the TRIUMF and Moscow Kaon Factories project [1,2]. It was then applied in the SSC low energy booster [3], the CERN Neutrino Factory [4], and in the Main Ring of the Japanese proton accelerator research complex facility [5,6]. For the superconducting option of the High-Energy Storage Ring lattice in the FAIR project, the same principle was also applied [7]. It is also one of the candidates for PS2 in CERN [8]. Due to its special features, the "resonant" lattice can also be used as a lattice with stochastic cooling where different arcs have different mixing factors, while the dynamic aperture is conserved for the whole machine [9]. On the basis of the theory of "resonant" lattices for synchrotrons with complex transition energy developed in [5,10], the application of such lattices in various accelerators is outlined. The "resonant" lattice also appears to be useful for electron machines. In synchrotron-light sources, in particular, the minimum momentum-compaction factor and the minimum modulation of the dispersion function are both simultaneously required to have a small horizontal emittance [11,12].

MAIN ASPECTS OF «RESONANT» LATTICE THEORY

With a specially correlated modulation of the quadrupole gradient $K(s)$ and orbit curvature $\rho(s)$, and a particular choice of betatron oscillation frequencies ν, the theory of "resonant" lattices makes it possible to get interrelated dispersion variations D(s) and 1/ρ(s) along the equilibrium orbit and a negative momentum compaction factor (MCF)

$$\alpha = \frac{1}{2\pi}\int_0^{2\pi} \frac{D(\vartheta)}{\rho(\vartheta)} d\vartheta \le 0 \qquad (1)$$

The general principles of construction of "resonant" lattices detailed in [5,10] are based on the solution of the equation for the dispersion $D(\vartheta)$ in the biperiodic structure with the cell periodicity and superperiodicity

$$D'' + K(\vartheta)D = 1/\rho(\vartheta) \qquad (2)$$

In the optics with the horizontal tune ν and the superperiodical curvature modulation $1/\rho(\vartheta) \sim Be^{i\omega\vartheta} + 1/\overline{R}$, the dispersion solution is $D(\vartheta) \sim Ae^{i\nu\vartheta} + \frac{B}{\nu^2-\omega^2}e^{i\omega\vartheta} + \overline{D}$. Since MCF is the average value of the function $D(\vartheta)/\rho(\vartheta)$, it can be written as the sum

$$\alpha = \frac{\overline{D}}{\overline{R}} + \frac{\overline{\widetilde{D}(\vartheta)\cdot\widetilde{r}(\vartheta)}}{\overline{R}} \qquad (3),$$

where $\widetilde{D}(\vartheta) = Be^{i\omega\vartheta}/(\nu^2-\omega^2)$ and $\widetilde{r} = B\overline{R}e^{i\omega\vartheta}$ are the functions oscillating about the averages.

In an ordinary regular FODO lattice without the superperiodic modulation, the oscillating component gives $\overline{\widetilde{D}(\vartheta)\cdot\widetilde{r}(\vartheta)} = 0$, and the minimum value of MCF $\alpha = \frac{\overline{D}}{\overline{R}} = \frac{1}{\nu^2}$ is limited by the horizontal tune ν.

In the "resonant" lattice, the oscillating components contribute the term $\widetilde{D}(\vartheta)\cdot\widetilde{r}(\vartheta) = B^2/2(\nu^2-\omega^2)$, which in the case $\nu \le \omega$ can result in a negative MCF. The same can be shown for the gradient modulation $K(\vartheta) + \varepsilon k(\vartheta)$. Here, we omit the long intermediate computations and quote the final formula for the case when the functions of gradients $\varepsilon\cdot k(\vartheta) = \sum_{k=0}^{\infty} g_k \cos k\vartheta$ and/or orbit curvature $\frac{1}{\rho(\phi)} = \frac{1}{\overline{R}}\left(1 + \sum_{n=1}^{\infty} r_n \cos n\phi\right)$ are jointly modulated[5,10]:

$$\alpha = \frac{1}{\nu^2}\left\{1 + \frac{1}{4(1-\omega/\nu)}\cdot\left[\left(\frac{\overline{R}}{\nu}\right)^2 \frac{g_k}{[1-(1-\omega/\nu)^2]} - r_k\right]^2\right\}, \qquad (4)$$

where g_k and r_k are fundamental harmonics producing the maximum effect on MCF.

Indeed, if both the gradient function and the orbit curvature function are modulated with identical frequency, the second term in (4) may make an appreciable negative contribution to MCF provided that the value $|1-\omega/\nu|$ is small and $\nu \le \omega$.

In addition, from (4), there follows an obvious condition of antiphase modulation of the gradient and curvature function $g_k r_k < 0$, which allows correlated variation of the momentum compaction factor with the aid of these functions.

«RESONANT» LATTICE WITHOUT γ_{tr} CROSSING

The most important application of the "resonant" lattice is the magneto-optic structure without γ_{tr} crossing. Based

* On leave from Institute for Nuclear Research, RAS, Moscow, e-mail: y.senichev@fz-juelich.de

on the above reasoning, the "resonant" lattice method with simultaneous orbit curvature and quadrupole gradient modulation with an approximately identical contribution of both modulations to the final value of the momentum compaction factor is most effective. From (4) it is easy to derive the following equation for arbitrary fundamental harmonics g_k and r_k giving $\alpha \approx -1/\nu^2$ and $\gamma_{tr} \approx i\nu$

$$|r_k| \le \left(\frac{\overline{R}}{\nu}\right)^2 \left|\frac{g_k}{1-(1-\omega/\nu\ )^2}\right| \qquad (5)$$

Usually the machine lattice consists of arcs and straight sections. For the dispersion in straight sections to be zero, the arc consisting of S_{arc} superperiods should have a phase advance of radial oscillations that is a multiple of 2π, i.e. ν_{arc} should be an integer. This means that the phase advance in one superperiod should be $2\pi\nu_{arc}/S_{arc}$. On the other hand, for MCF to be controlled, it is reasonable to take the minimum possible difference $\nu_{arc} - S_{arc} = -1$. Thus, many ratios exist between S_{arc} and ν_{arc}: (4:3), (6:5), (8:7), (10:9), … It is obvious that in all ratios, the number of superperiods S_{arc} is taken to be even, while the betatron oscillation frequency takes on odd values. In this case, the phase advance of the radial oscillations between the cells located in different superperiods and separated by $S_{arc}/2$ superperiods is

$$2\pi \cdot \frac{\nu_{arc}}{S_{arc}} \cdot \frac{S_{arc}}{2} = 2\pi \cdot \frac{\nu_{arc}}{2} = \pi + 2\pi n\,, \qquad (6)$$

which corresponds to the condition of first-approximation compensation for the nonlinear effects of sextupoles located in these cells. This remarkable property also applies to higher multipoles in any element (see Fig. 1).

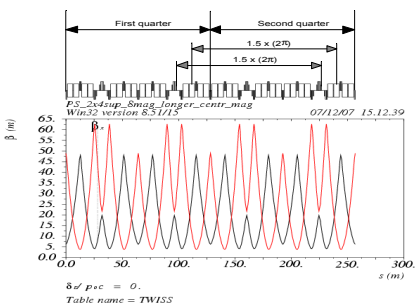

Figure 1: Half of arc with (S_{arc}:ν_{arc})=8:6.

«RESONANT» LATTICE FOR STOCHASTIC COOLING

Another application of the "resonant" lattice is as an advanced lattice for stochastic cooling. It is known that to intensify the stochastic cooling process, the mixing factor between the pick-up and kicker should ideally be as large as possible. On the other hand, in the case of mixing between the kicker and pick-up, the mixing factor should be smaller. It can be seen that the "resonant" lattice has a remarkable feature: the gradient and the curvature modulation amplify each other when they have opposite signs, while they can compensate for each other when they have the same signs (see formula 4):

$$\frac{1}{4(\omega/\nu-1)} \cdot \left(\frac{g_k}{[1-(1-\omega/\nu)^2]} \mp |r_k|\right)^2 \approx 2 \quad \text{or} \quad 0 \qquad (7)$$

Thus, in such a lattice, we can make two arcs with different slip factors: $\eta_{pk} = 1/\nu^2 - 1/\gamma^2$; $\eta_{pk} = -1/\nu^2 - 1/\gamma^2$. In case $\gamma \approx \nu$, one of the arcs is isochronous when the slip factor is $\eta_{pk} \approx 0$ and the other slip factor is $\eta_{kp} \approx -2/\nu^2$. As an example, we will consider the SC option for the HESR lattice [9] with different slip factors. Both arcs have four-fold symmetry with superperiodicity S=4. The phase advance per arc is $\nu_{x,y} = 3.0$ in both planes. Each superperiod consists of three FODO cells with 4 or 6 bending magnets and 3 families of quadrupoles (fig. 2).

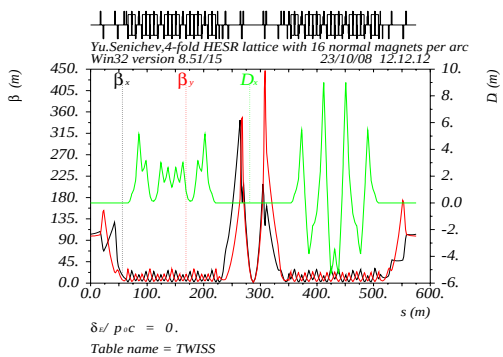

Figure 2: The advanced HESR lattice with two different arcs.

The different momentum compaction factors are reached mainly due to the dispersion function change, and the β-function changes insignificantly. Two families of sextupoles are used for the chromaticity correction, and their nonlinear influence is self-compensated inside each arc. Due to this fundamental advantage despite two different arcs, the dynamic aperture does not suffer in comparison with the option of two identical arcs.

«RESONANT» LATTICE FOR SYNCHROTRON LIGHT SOURCES

The third application of the "resonant" lattice is the synchrotron light source. Since the horizontal emittance depends upon the horizontal dispersion function η_x, as $\varepsilon_x \propto \langle H \rangle_{dipole}$, where $H = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2$, the lattice with small electron emittances therefore requires the smaller dispersion function and, as a consequence, the stronger sextupoles are needed in order

to correct the chromaticity. At the same time, it is well known that the sextupoles dramatically decrease the dynamic aperture due to their nonlinear action. In this case, we can use the "resonant" lattice where the sextupolar terms are aimed to be smaller, and each pair of sextupoles have the same effect as one octupole. The nonlinear tune then plays the role of a stabilizing factor. Such a lattice has to be classified as a special lattice [11], since the sextupolar term is effectively suppressed, but the nonlinearity remains under control and strong. This method together with the smallest emittance results in a large dynamic aperture. For comparison, the dynamic aperture of a modified circular Chasman-Green lattice with the same number of sextupole families is smaller by a factor of four.

Furthermore, there is currently much interest in magnetic lattices, which can be operated over a range of momentum compaction factors. This provides several advantages and the possibility of working without sextupoles.

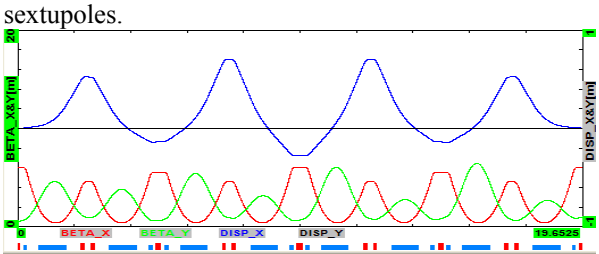

Figure 3: The arc lattice with tuneable momentum compaction factor.

Figure 3 shows the arc with a tuneable momentum compaction factor. In this lattice, one cell is formed by two adjoining cells, and each second focusing quadrupole is replaced by a short combined function bend magnet with positive gradient BG(G>0). As a result, the arc consists of four cells and each half cell has a structure: trim QF+short BG(G>0)+long BG(G<0)+QF. The trim quadrupole is used for fine tuning. The dispersion function is equal to zero at the ends of the arc. This is automatically caused by an integer tune number for the arc. The matching sections are not needed. Due to the low beta function and zero dispersion function in the middle of each long bending magnet, the electron beam emittance has a rather low value [12].

CONCLUSION

We have developed a multi-application "resonant" lattice with the following distinguishing features:

- ability to achieve the negative momentum compaction factor using resonantly correlated curvature and gradient modulations;
- gamma transition variation in a wide region from $\gamma_t \approx \nu_x$ to $\gamma_t \approx i\nu_x$ with quadrupole strength variation only;
- dispersion-free straight section;
- independent optical parameters of arcs and straight sections;
- two families of focusing and one of defocusing quadrupoles;
- separated adjustment of gamma transition, horizontal and vertical tunes;
- convenient chromaticity correction method using sextupoles;
- first-order self-compensating scheme of multipoles and a large dynamic aperture.

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REFERENCES

[1] N. Golubeva, A. Iliev, Yu. Senichev, The new lattices for the booster of Moscow Kaon Factory, Proceedings of International Seminar on Intermediate Energy Physics, v. 2, p. 290, Moscow, 1989

[2] U. Wienands, et al., A racetrack lattice for the TRIUMF Kaon Factory Booster, HEAC XV, Hamburg, 1992

[3] E. Courant et al., Low Momentum Compaction Lattice Study for the SSC Low Energy Booster, PAC, p. 2829, San-Francisco, 1991

[4] H. Schönauer, et al., A slow-cycling proton driver for a Neutrino Factory, Proceedings of EPAC, p. 966, Vienna, 2000

[5] Yu.Senichev, A «resonant» lattice for a synchrotron with a low or negative momentum compaction factor, submitted in Particle Accelerator, KEK Preprint 97 40, 1997

[6] Y. Ischi, et al., Lattice design of JHF synchrotrons, APAC, Tsukuba, 1998

[7] Yu. Senichev, et al., Lattice Design Study for HESR, EPAC, Lucerne, 2004, p.653

[8] Yu. Senichev, The lattice with imaginary γ-transition for the CERN proton synchrotron PS2, CERN-2007-005, BEAM-07, p. 171

[9] Yu. Senichev, The advanced HESR lattice for improved stochastic cooling, COOL-07, Bad Kreuznach, Germany, 2007, p.102

[10] Yu. Senichev and A. Chechenin, Journal of Experimental and Theoretical Physics, 2007, vol. 105, No. 5, pp. 988–997

[11] Yu. Senichev, The proposed racetrack lattice for the synchrotron light source "Astrid II", EPAC, Stockholm, 1998, pp. 642

[12] Yu. Senichev, Low periodicity lattice for Third generation synchrotron light sources: avoidance of dynamic aperture reduction by sextupole compensation, PAC, New York, 1999, p. 2442