# STATISTICAL MODELING OF DC SPARKS

Y. I. Levinsen, Univ. of Oslo, Norway and CERN, Geneva, Switzerland A. Descoeudres, S. Calatroni, M. Taborelli, and W. Wünsch, CERN, Geneva, Switzerland

### Abstract

The understanding of electrical breakdowns has a critical role in the design of the RF accelerating cavities for the CLIC linear collider. In this context a new statistical model of the conditioning process and breakdown rate evolution is presented. The model is applied to a DC spark system with tip-plane electrode geometry charged from a capacitance. The approach requires few assumptions, but makes several predictions. The saturated breakdown field dependence on electrode gap distance, the breakdown rate dependence on field and the "spitfest" (grouped breakdowns) are among the phenomena described by this simple model.

# **INTRODUCTION**

In order to better understand the relevance of the results of a DC spark test system compared to RF structures, a simple statistical data analysis model has been constructed. The model has several goals including:

- Recreate a field dependent breakdown rate based on a few simple principles.
- Better understand the statistical nature of sparks in the DC test system, for example during the conditioning process.
- Study effects of electrode geometry on breakdown field.

Macroscopic consequences of the breakdown phenomenon have been simulated/modeled in the past. One example is the simulation of the success rate when constructing superconducting accelerating cavities made of niobium [1]. Another example are the microscopic particle in cell (PIC) simulations of breakdowns [2].

## THE MODEL

The present model is built around an idea of progressive surface modification of the cathode, due to either breakdown or field emission. Although clearly identified cathode and anode are used here, the model can be extended to the RF case. Field emitting sites are distributed randomly over the cathode within a given boundary radius. This radius is set around  $200 - 300 \,\mu$ m, consistently with SEM pictures of breakdown craters.

The field emission current is defined from Fowler– Nordheim field emission equation,

$$I = \frac{A_e 1.54 \cdot 10^6 (\beta E)^2 e^{10.41 \phi^{-0.5}} e^{\frac{-6530 \phi^{1.5}}{\beta E}}}{\phi}.$$
 (1)

In this equation,  $A_e$  is the emitting area, and E is the macroscopic field so that  $\beta E$  is the local field at the tip of the protrusion.  $\phi$  is the work function for the given material. For metals  $\phi$  is in the range 3.5 – 5 eV, and is set to 4.5 eV in the model in accordance with values for copper. The value of the emitting area is not needed in order to find the  $\beta$  in an experiment, since

$$\frac{d[ln(I/E^2)]}{d[1/E]} = \frac{-6530\phi^{1.5}}{\beta}.$$
 (2)

Each emitter is assigned a given field enhancement factor,  $\beta$ . These values are taken from a Gaussian distribution with a fitting mean and standard deviation. In literature an exponentially decaying function is often used [1]. The difference is not as big as one might think, since one can consider the Gaussian distribution to be the high- $\beta$  tail of the exponential distribution. The choice of a Gaussian allows a significant reduction in computing time, although an exponential function might explain conditioning as discussed below.

The most important assumption of the present model is the strict rule for breakdown. Field emitters are breakdown precursors and we impose that there is a limiting local field  $\beta E$ . If any emitter exceeds this limit a breakdown occurs. This is consistent with literature [3].

As mentioned, it is assumed that breakdowns and field emission can modify the cathode surface, while the anode is unchanged. A breakdown is assumed to completely redistribute the emitters in a defined area around the breakdown site (i.e. new  $\beta$  values are randomly chosen from the distribution). One can then mimic the conditioning process by taking the new  $\beta$  values from a distribution with a lower mean and a smaller standard deviation. This gives the same effect as using an exponentially decaying distribution, where conditioning would be the process of removing some of the emitters in the high- $\beta$  part of the distribution, so that the distribution is cutoff after some breakdowns.

When the surface is subject to field emission only, in absence of breakdowns (like in the breakdown rate experiments described below), the field emission is assumed to add a small  $\Delta\beta$  to the previously defined  $\beta$ 's of the emitters. This perturbation is randomly chosen from a distribution with mean equal to zero and low variance, and there is no sign preference (so it is equally probable that field emission improves or worsens the surface).

The model is compared with the results obtained with the two DC spark test systems available at CERN, described in detail in [4]. These systems have an anode tip of 1.15 mm radius, a plane cathode, and typical electrode gap d in the range of  $10 - 100 \,\mu$ m. There are two main experimental

**T06 - Room Temperature RF** 

**Radio Frequency Systems** 

modes – spark cycle mode and breakdown rate mode. In the former, a given voltage V is applied. If no breakdown is detected, the voltage is increased and the new voltage is applied to the electrodes. In case of a breakdown, the voltage is registered and the cycle is reset to a starting voltage, usually sufficiently low to avoid any breakdown. The mean breakdown field after conditioning from such an experiment is reported as the "saturated breakdown field".

In a breakdown rate measurement a fixed voltage is applied repeatedly to the electrodes. Whether or not a breakdown took place is registered after each attempt. The ratio between number of breakdowns and total number of attempts is reported as the breakdown rate, which is a function of the voltage. Breakdown rate experiments in the DC spark test system are time consuming because of the low duty cycle. Breakdown rates down to about  $10^{-3}$  can be measured with a reasonable accuracy, whereas the breakdown rate information needs to be extended at least three orders of magnitude lower to compare to the relevant range measured for RF accelerating structures. Moreover, it is not perfectly clear if the two environments produce the same conclusions with respect to material rankings etc.

## RESULTS

#### Breakdown Rate

In general, there is no well established theory for the voltage dependence of the breakdown rate, and several empirical formulas (exponential dependence, power law) have been used to fit the data. Breakdown rate data simulated with the present model and displayed in Fig. 1 show a trend similar to what is observed in experiments [5]. After many attempts without a breakdown, the overall  $\beta$  will be lowered because the field emission is always "attacking" the emitters with the highest local field  $\beta E$ . This is the reason for the hysteresis observed in Fig. 1, that is the difference between starting the simulation at a high voltage and reducing it, and starting at a lower voltage and increasing it. Similar observations have been made experimentally with the DC spark test system. The steepness of the curves in Fig. 1 is dependent on how much the field emission modifies the emitting sites, which is in turn related to the value of  $\Delta\beta$  mentioned before. The lower  $\Delta\beta$  is the less effect field emission has, and the steeper the curve. On the boundary (no modification at all), the line will be vertical.

The trend as it is shown in Fig. 1 is highly dependent on the parameters, e.g. the density of emitters, the amount of effect of field emission, etc. In addition, it is observed that when increasing the field up to values resulting in a high breakdown rate, the breakdowns rather abruptly start to come in groups. This happens when the average  $\beta$  of the new emitters distributed after a breakdown times the field value is close to the limiting  $\beta E$  field. This is the so-called "spitfest" and has also been observed in the RF testing facilities [6]. This gives a very rapid and unstable change in the breakdown rate.



Figure 1: The breakdown rate as a function of field V/d. The electrode gap is set to 20  $\mu$ m and 100 000 breakdown attempts are simulated for each field value. The red curve is for a downwards field scan, the green curve for an upwards scan

# Field Emission

The model has been also used to calculate the field emission current from a given distribution of field emitters, as would be measured in the DC spark test system. A calculation of  $\beta$  from the field emission current as a function of electrode gap can be seen in Fig. 2. 50 emission sites are placed on the cathode, with a mean  $\beta$  value of 40 and a standard deviation of 5. The sharpest emitter has a  $\beta$  of 54.6 in this example. From the measured I(V) curve a value of  $\beta$ of 37 for a gap of 20 µm would instead be reported, if the field is calculated as V/d as customary (measurement uncertainties are not taken into account). The cathode protrusion contributing most to the field emission current might be off-axis from the tip of the anode, and the macroscopic field at its position would then be lower than the field defined as V/d, because of the spherical shape of the anode tip. This purely geometrical effect explains why the measured  $\beta$  can be lower than the intrinsic value, whereas perfect plane-plane electrode geometry would not show such a feature. The simulation result is similar to what was discovered by Alpert et. al [3]. The field distribution on the cathode flattens as the electrode gap distance is increased, hence the "actual"  $\beta$  plotted in Fig. 2 moves towards the maximum  $\beta$  value of the distribution.

## Saturated Breakdown Field

The fluctuation of the measured breakdown field during a typical spark cycle experiment [7, 8] is simply described in the present model by the redistribution of the field emitters after each breakdown. The range of variation of the field is then reflected in the standard deviation of the Gaussian distribution of the  $\beta$  of the emitters.

Due to the geometry effect seen in Fig. 2 for the used electrodes shape, a gap dependence on the saturated breakdown field is also predicted by the model, given as the red line in Fig. 3. The green line in the same figure shows experimental data from copper electrodes in the DC spark



Figure 2:  $\beta$  dependence on electrode gap spacing for a field emission experiment, simulated as a measurement of current as a function of voltage (red curve), where  $\beta$  is calculated assuming a field V/d. The "actual"  $\beta$  value for the given gap is also plotted (green curve). This is calculated assuming the correct value of the local field found at any emitting site, generally lower than the macroscopic V/dfield.

test system, with a behaviour similar to what the model predicts.



Figure 3: The model predicts that the average breakdown field will drop when the electrode gap is increased, an effect of the electrode geometry (red line). Similar behaviour is observed in experiments (green dots).

### **SUMMARY**

This simple model can already give explanations for several interesting aspects of electrical breakdown. One of the most important things is that considering a local field limit for breakdown ( $\beta E$ ) instead of the macroscopic applied field seems to be well in line with the experimental results [9]. Furthermore, the model predicts that if one could make explicit the "softness" of a material with respect to field emission, one would possibly have more information about the field dependence of the breakdown rate of the ma-

### **Radio Frequency Systems**

**T06 - Room Temperature RF** 

terial. First experimental evidence of this correlation has been measured with the DC spark test system.

As a last point, the model predicts that there would be an advantage in minimizing the area of regions of high electric field (this is a consequence of what is shown in Fig. 2).

This is a work in progress and more details could probably be added to give a more physically correct picture.

### REFERENCES

- J. Wiener and H. Padamsee. Improvements in field emission: An updated statistical model for electropolished baked cavities. *DESY-TESLA*, 02 2008.
- [2] G. R. Werner. Probing and Modeling Voltage Breakdown in Vacuum. PhD thesis, Cornell University, August 2004.
- [3] D. Alpert, D. A. Lee, E. M. Lyman, and H. E. Tomaschke. Initiation of electrical breakdown in ultrahigh vacuum. J. Vac. Sc. & Tech., 1964.
- [4] M. Kildemo. New spark-test device for material characterization. Nuclear Instruments and Methods in Physics Research, 2004.
- [5] S. Döbert. High-Power RF Tests Results: 30 GHz. Presentation at CLIC Workshop, 2008.
- [6] C. Adolphsen, et al. Processing Studies of X-Band Accelerator Structures at the NLCTA. In *Proceedings of PAC*, 2001, p. 478.
- [7] M. Taborelli, S. Calatroni, and M. Kildemo. Breakdown resistance of refractory metals compared to copper. In *Proceedings of EPAC*, 2004. Prepared for 9th European Particle Accelerator Conference (EPAC 2004), Lucerne, Switzerland, 5-9 Jul 2004.
- [8] A. Descoeudres, T. Ramsvik, S. Calatroni, M. Taborelli, and W. Wünsch. dc breakdown conditioning and breakdown rate of metals and metallic alloys under ultrahigh vacuum. *Phys. Rev. ST Accel. Beams*, 12(3):032001, Mar 2009.
- [9] S. Calatroni, A. Descoeudres, Y. Levinsen, M. Taborelli, W. Wünsch. DC Breakdown Experiments. In *Proceedings* of the Thirteenth Advanced Accelerator Concepts Workshop, AIP Conf. Proc., 2009, Volume 1086, pp. 359-364.