TEMPERATURE AND STRESS RISE INDUCED BY CRACKS IN ACCELERATING STRUCTURES*

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Abstract

The achievable gradient of accelerating structures is limited by dark current capture, RF breakdown and cyclic fatigue [1, 2]. We consider one effect related to cyclic fatigue, the temperature and stress rise caused by the increase in ohmic heating near cracks. We made detailed analysis and simulations of the temperature and stress distribution in the vicinity of cracks of different shapes in the presence of a surface heat flux, and compared the results to the case of a smooth metallic plane. This analysis gives some insight of the cyclic fatigue leading to the formation of microcracks and crack growth.

INTRODUCTION

Microcracks in accelerating structures can cause two effects: first, magnification of the RF electric field due to the sharp ends (protrusions on coarse surface) of the structure results in field emission and even dark current [1]; second, penetration of the RF magnetic field into a crack enhances the ohmic heating if the crack is aligned with the RF magnetic field. The enhanced heating can lead to cyclic fatigue and therefore damage to the structure [3, 4]. Below, we focus on the second effect and consider a crack aligned with the RF magnetic field.

Examples of cracks in a structure are shown in Fig. 1. The length of the crack on the left is 15 μm and the one on the right is 25 μm.

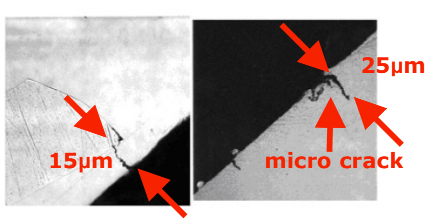

Figure 1: Cracks in accelerating structure (reproduced from Ref. [5]).

THEORETICAL ANALYSIS OF ELEVATED TEMPERATURES

Infinite Metallic Surface without Cracks

The time dependent elevated temperature in the case of a smooth planar surface is the solution of a 1-D heat transfer equation [6],

$$T(y,t)-T_0=\frac{q}{k}2\sqrt{\alpha t}\left\{\sqrt{\frac{1}{\pi}}\exp\left(-\frac{y^2}{4\alpha t}\right)+\frac{y}{2\sqrt{\alpha t}}Erfc\left(-\frac{y}{2\sqrt{\alpha t}}\right)\right\} \quad (1)$$

Here q is the incident heat flux, k is the thermal conductivity, and α is the thermal diffusivity. Profiles of the elevated temperature (1) for several instants of time are shown in Fig. 2 for a heat flux equal to q=400MW/m^2 on a large copper surface (This will be our reference case). This flux corresponds to magnetic field H~170 kA/m for an 11.424 GHz source. The temperature rises with time reaching $\Delta T = 17^\circ K$ at the surface at time $t = 1.9\mu s$.

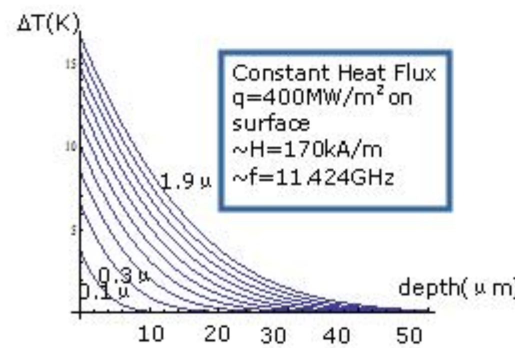

Figure 2: Elevated temperature profiles at ten time intervals, for a smooth, uncracked metallic surface.

Metallic Plane with a Crack

Let us consider now the same surface in the presence of a crack as shown in Fig. 3a.

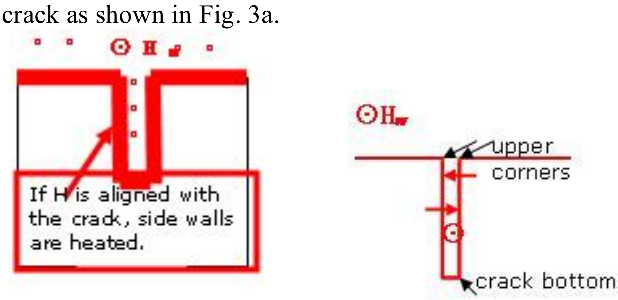

Figure 3 (a): Sketch of a structure with a crack. (b): Maximum temperature rise takes place at the upper corners of the crack.

Assuming the magnetic field **H** is aligned with the crack, the same heat flux is applied on the side walls of the crack as on the surface of the metal plane. When the crack width is small enough, the additional elevated temperature at the top of the crack is found to be

$$\Delta T = \frac{\alpha q}{\sqrt{\pi}k}\int_0^t d\tau[4\alpha(t-\tau)]^{1/2} Erf(\frac{d}{[4\alpha(t-\tau)]^{1/2}}) \quad (2)$$

where d is the depth of the crack. For a deep crack and sufficiently early times, $d > (4\alpha t)^{1/2}$, the additional temperature rise is equal to the 1D result on the surface. Thus, the presence of the crack effectively doubles the temperature rise.

Figure 4 shows the elevated temperature at the top of the crack obtained by adding the contributions from the planar surface (1) and crack (2) for the case of copper material and our reference heat flux. Again we see that the elevated temperature is close to twice the planar surface value.

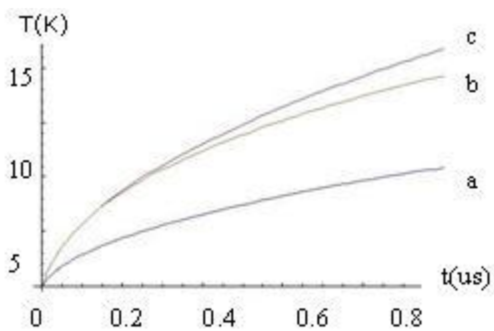

Figure 4: Line (a) is the temperature rise vs time for the infinite plane surface; Line (b) is the temperature rise at the crack upper corners; Line (c) is the doubled Line (a).

Figure 5 shows the elevated temperature from an ANSYS [7] simulation of a conductor with a V-shaped crack of opening width w=1um and depth d=8um, at t=1us. The temperature rise at the upper corner of the crack is $\Delta T = 19.723^{\circ} K$, which is consistent with the theoretical estimates (1) and (2).

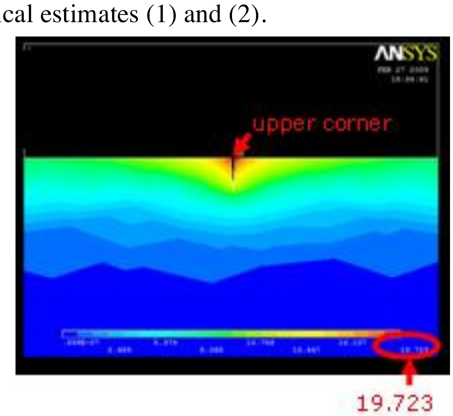

Figure 5: Temperature contour plot at the snapshot of 1µs.

STRESS STRAIN ANALYSIS

There are three sets of equations necessary to uniquely solve for the stress, strain and displacement in the body of the material in the presence of an elevated temperature profile.

These are the force balance relation,

$$\nabla \cdot \overleftrightarrow{\sigma} = 0 \quad (3)$$

where $\overleftrightarrow{\sigma}$ is the stress tensor. The relation between the strain tensor $\overleftrightarrow{\varepsilon}$ and material displacement **u,**

$$\varepsilon_{ij} = \frac{1}{2}[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}] \quad (4)$$

And the stress-strain relation (Hook's Law) including the coefficient of thermal expansin,

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \delta_{ij}(\lambda Tr(\overleftrightarrow{\varepsilon}) - (3\lambda + 2\mu)\alpha_T \Delta T) \quad (5)$$

where λ and µ are Lame elastic constants, δ_{ij} is the Kroneker delta function and α_T is the coefficient of linear thermal expansion.

These equations are solved by ANSYS with the appropriate boundary condition that no stress is transferred across a free surface, and the displacement far from the crack or surface vanishes. The stress in the material is then characterized by the spatially dependent (real) eigenvalues of the local (symmetric) stress tensor. Von Mises stress is one measure of the severity of the stress. It is the RMS value of the differences in the eigenvalues of the stress tensor. Expressed in terms of the elements of the stress tensor, the von Mises stress in the two dimensioned limit is

$$\sigma_e = \sqrt{\frac{1}{2}[(\sigma_x - \sigma_y)^2 + 2\sigma_{xy}^2]} \cdot \quad (6)$$

For the case of a planar conductor with a temperature profile T(y) given by (1), one finds ($\sigma_y = \sigma_{xy} = 0$)

$$\sigma_e = \frac{\sigma_x}{\sqrt{2}} = 22.98 MPa \quad (7)$$

at the surface of the conductor for the reference parameters.

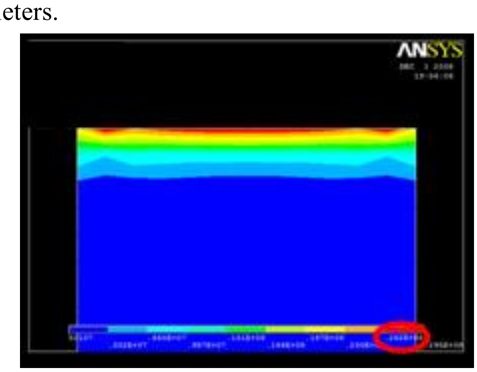

Figure 6: Contour plot of von Mises stress on the smooth metallic plane.

This estimate can be compared with the results of ANSYS simulations, which for the case of a planar surface is 26.2 MPa (Fig.6). In the case of an 8µm deep crack with a rectangular shaped bottom, the computed stress is 125 MPa (Fig.7), and if the crack has a V-shaped bottom the stress is 66.2 MPa. So, due to the existence of cracks, stresses are several times larger than for planar surfaces.

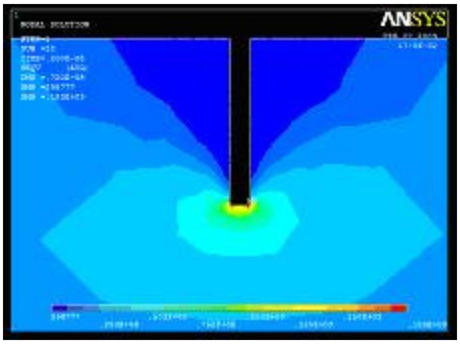

Figure 7: Contour plot of von Mises stress in the vicinity of a crack on the plane.

EFFECTS OF CRACK GROWTH

The stress at the bottom of V-shaped cracks with opening angle arctan(1/20) is shown in Fig.8 as a function of crack depth. When the crack depth is small there is a factor of two enhancement in the stress at the bottom of the crack. As crack depth increases the enhancement decreases. The large enhancement for crack depth can drive the growth of cracks.

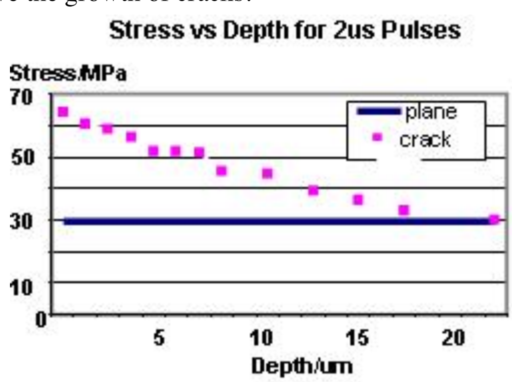

Figure 8: Stress vs crack depth at 2μs pulses.

CONCLUSION

We made detailed theoretical analysis of the heating and induced stress in a metallic planar surface with and without cracks. Both 1-D and 2-D heat transfer equations and stress, strain-displacement relations were considered. Theoretical estimates were compared with results from ANSYS.

Due to the existence of the crack, the temperature doubles on the upper corner. Computation and simulation also provide necessary temperature profiles for further stress and strain analysis.

Analysis shows that stress at the crack bottom is several times higher than that of the smooth plane surface. This can drive crack growth. As the crack grows deeper, stress decreases and approaches the value of the smooth plane surface. At this point the RF enhancement to crack growth stops.

ACKNOWLEDGEMENT

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