RESONANT THIRD-INTEGER EXTRACTION FROM THE PS2

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Abstract

For the proposed PS2 accelerator several extraction systems are needed, including a slow third-integer resonant extraction. The requirements are presented together with the conceptual considerations for the sextupole locations and strengths, the separatrices at the extraction elements and the aperture implications for the overall machine. Calculations of the phase space separatrices have been computed with a new code for the physics of slow resonant extraction, which is briefly reviewed. Implications for the extraction equipment design and for the injection-extraction straight section optics are discussed.

INTRODUCTION

The existing CERN PS is a combined function synchrotron with an injection energy of 1.4 GeV and a top energy of 25 GeV. It forms the core of the CERN complex, presently providing many proton beams for the different users, and in 2009 celebrates its 50th birthday. The replacement of the existing PS with a modern, reliable, flexible and robust synchrotron is an important part of the future CERN programme. A separated-function PS2 is assumed, with a circumference of 1346 m and parameters given in Table 1. The injection energy is 4.0 GeV for p+, and 135 MeV/u (1.3 GeV p+ equivalent) to accept heavy ions from LEIR. The extraction energy range is 20 to 50 GeV and the conceptual design of slow extraction based on the 3rd integer resonance for a possible fixed-target physics program is described.

Table 1: Main parameters of the proposed PS2 accelerator

Parameter	Unit	Value
p+ injection energy (kinetic)	GeV	4.0
Extraction energy (kinetic)	GeV	20-50
Circumference	m	1346.4
Beam intensity	p+	1.0×10^{14}
Maximum beam power	kW	~300

The basic design of the PS2 depends on the approach to transition crossing. The present preference is for a racetrack geometry with an imaginary γ_{tr} lattice [1] based on a negative momentum compaction cell, which would avoid transition crossing. The long straight section (LSS) for injection and extraction [2], Fig. 1, will be a 90° FODO insertion and a cell length of about 22 m, giving ~10 m free drift per half-cell. A symmetric doublet is placed at the centre of the straight to accommodate the H^- injection system.

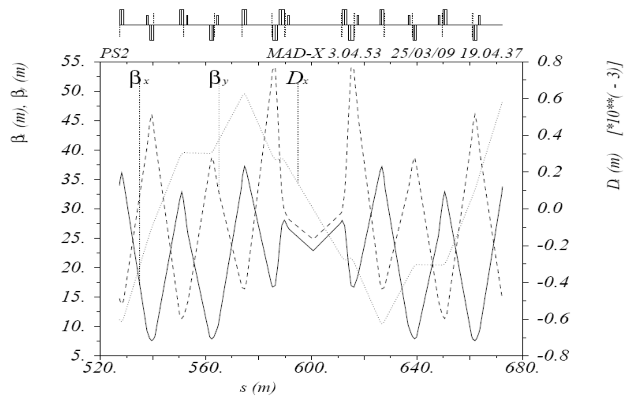

Figure 1: Beta functions in PS2 injection-extraction straight for Q_x =13.320 (horizontal dispersion is less than 1 mm).

BASIC CONSIDERATIONS

3rd Integer Resonance

In a normalised coordinate system the equation for radial betatron motion in the presence of a sextupole perturbation is

$$\frac{d^2x}{ds^2} + Q^2x = kx^2 \cos n\Phi$$

where x is the displacement from the equilibrium orbit, Q is the horizontal tune, $\Phi = \int ds/Q\beta$, k is the sextupole strength and n = 1, 2, … The resonance condition is met if Q = n/3, and in this case the motion of the particles is unstable everywhere in the phase plane. If Q is slightly different from resonance then small amplitude oscillations are stable. The tune Q for a single particle is expressed as a contribution of its difference ΔQ from the unperturbed machine tune Q_o, and the contribution depending on the chromaticity (δQ/Q)/(δp/p) and its momentum offset Δp/p

$$Q = Q_o + \Delta Q + \left(\frac{\partial Q}{\partial p}\right)\cdot \Delta p = Q_o\left\{1 + \frac{\Delta Q}{Q} + \left(\frac{\partial Q/Q}{\partial p/p}\right)\cdot\frac{\Delta p}{p}\right\}$$

Separatrix orientations and particle rotation in phase space depend on the signs of the sextupole perturbation and the sign of ΔQ. In practice this means arranging Δp and ΔQ in order to maintain $(Q_{res}-Q_o)\times(Q_{res}-Q) > 0$, such that approaching the resonance from above or below the n/3 tune does not change the orientation of the separatrix.

The area of the stable triangle can be expressed as the distance DQ_{res} from the resonant tune and the normalised sextupole strength $\overline{K}_s$

$$A_0 = 48\pi\sqrt{3}\cdot\left(\frac{\Delta Q}{\overline{K}_S}\right)\cdot\pi$$

The distance from the origin $\bar{x}_F$ of the fixed point on the $\bar{X}$ axis is derived from simple geometry:

$$\bar{x}_F = \frac{2\sqrt{A_0}}{\sqrt[4]{27}}$$

The spiral step (amplitude increase in 3 turns) can be shown to be equal to

$$\Delta x = \frac{3}{4}\left|\bar{K}_s - \Delta x'^2\right|$$

Extraction Sextupole Locations

The electrostatic (ES) to magnetic septum (MS) phase advance is 78°, which means the extracted separatrix (branch 3) angle at the ES should be about 40 degrees assuming first quadrant extraction. This provides optimum opening at the MS and clearance for the other separatrix branches: for angles <40° the first branch approaches the ES, and for >40° branch 2 approaches the MS, Fig.2. The phase advance from sextupoles to the ES should thus be 110, 230 or 350 degrees for +ve sextupole polarity, or 50, 190 or 310 degrees for -ve polarity.

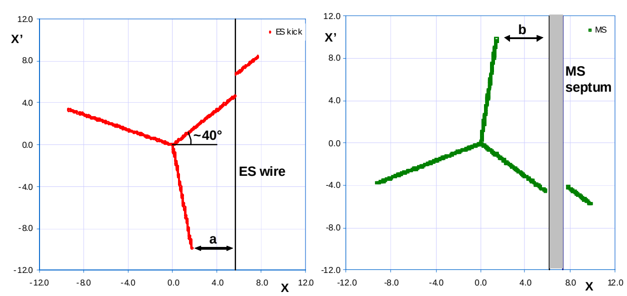

Figure 2: With a phase advance of 78° from ES to MS, the extracted separatrix should have an angle of about 40° with respect to the X axis, to optimise clearances **a** and **b**.

Ideally the normalised dispersion at the extraction sextupoles should be small compared to the rest of the machine, to avoid a modification of the chromaticity while the tune changes. This is not, however, a major concern, since extraction is anyway likely to be carried out with natural chromaticity, to make use of the extra tune spread which reduces the required sextupole strength and allows better control of the spill. Locations with $\left|D_x/\sqrt{\beta_x}\right|$ below about 0.1 $m^{½}$ were sought, to be compared to the peak and RMS values of 1.5 and 0.4 $m^{½}$ respectively. Large β_x values were preferred and a free drift of ≥70 cm between lattice elements required. About 20 potential locations were found in the dispersion suppressor, in the arcs and in the two LSS.

SEPARATRIX TRACKING

A numerical code *fppexe* to calculate the resonant extraction separatrices has been developed in Matlab™, based on the algorithms used in the existing CERN Fortran routine *prtags* [3]. The new version allows for much faster optimisation and provides a graphical user output – it was verified against the existing code using the SPS resonant extractions as a test cases. The program numerically computes the fixed points in normalised phase space using the *fminsearch* routine function, computes the stable area and tracks the outgoing separatrices. The spiral step of the extraction separatrix is derived, and the program also matches the stable area to the required emittance, either by modifying the particle momentum Δp or the tune offset ΔQ. The program treats a single particle rather than an ensemble, and is used to define the extraction parameters and element strengths.

RESULTS

Four different combinations of sextupole positions were evaluated numerically, using in each case four magnets:

- V1: in arcs with D_x ~0.4m, β_x ~20m, space 0.7 m;
- V2: in DS with D_x ~0.2m, β_x ~24m, space 0.8 m;
- V3: in RF LSS with D_x 0m, β_x ~20-28m, space >4m;
- V4: mixed locations (2 in DS, 1 per LSS) with D_x of 0 – 0.2m, β_x of 9 - 25m.

With the natural chromaticity (δQ/Q)/(δp/p) of -1.654, the separatrices were tracked with *fppexe* for normalised sextupole strengths of 3.0 m^{-2}. An example output is shown in Fig. 3 for version 4, which uses sextupoles in two DS and one in each LSS. The sextupole branches are very straight, with the downward pointing non-extracted branch exhibiting the right sense of curvature, and the spiral step at the septum amplitude about 8.8 mm (real), to be compared to an estimated physical width of the septum wire of 50-100 µm. The losses are therefore expected to be reasonable, of the order of 0.6-1.1%.

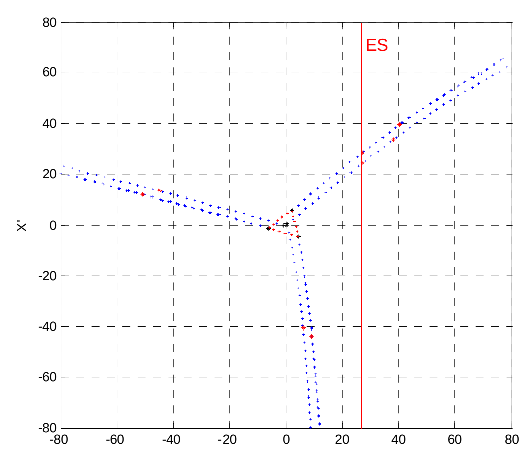

Figure 3: Separatrices obtained for extraction sextupole configuration version 4 (normalised mm / mrad).

For versions 1 and 3 the separatrices were fairly similar, however for configuration 2 using the DS locations the situation was different. In this case one sextupole location has large phase errors of 27° with respect to the ideal multiple of 60° to the ES, which means that the kicks are partially cancelling and the spiral step at the ES was only 4.4 mm. All else being equal much stronger sextupoles would be required for these positions, and the separatrix curvature would be greater.

Sextupole Magnet Parameters

With a normalised sextupole strength $\mathbf{K}_s$ of 3.7 m^{-2} per magnet, the real maximum strength k_s is found for the lowest β_x to be

$$k_s = \mathbf{K}_s \sqrt{\frac{\beta_x}{\beta_N^3}} = 3.0 \cdot \sqrt{\frac{60}{20.6^3}} = 0.25 m^{-2}$$

From the relation $k_n = \frac{e}{p} \cdot \int a_n dl = \frac{0.3}{p\left[\frac{GeV}{c}\right]} \int a_n dl$ we find that $\int a_3 dl = 41.7 Tm^{-1}$. This should be easily achievable with a magnetic length of 0.5 m and a pole tip radius of the order of 70 mm (for comparison the 0.7 m long SPS extraction sextupoles have a pole tip radius of 60 mm and provide 166 Tm^{-1} at nominal current).

PS2 Aperture Requirements

For the version 4 solution the maximum amplitude $\mathbf{X}$ in normalised phase space is 56.6 mm (defining $\mathbf{X} = x\sqrt{\beta_x / \beta_N}$ and assuming β_N = 60 m). The real (x, x') coordinates were tracked around the ring for the final 3 turns before extraction. This gives a required good field region / aperture of 65.6 mm in the lattice QFs, when tolerances for orbit (5 mm) and the dispersion contribution for δp of 0.001 are included, Fig. 4. Note that the actual excursions will be less than this except in the region between the final sextupole and the ES. The aperture requirements for the overall machine could, if critical, be reduced somewhat by reducing the separatrix angle at the ES location from the value of almost 45° seen in Fig. 3 – for example the same spiral step for a separatrix an angle of 25° gives an overall aperture requirement of 53.5 mm.

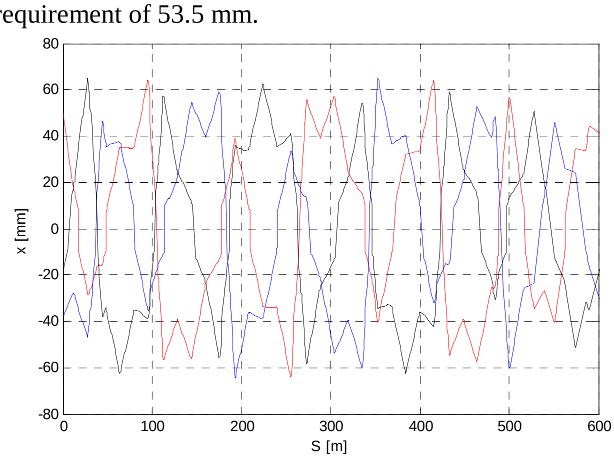

Figure 4: Separatrices tracked around PS2 (one arc and one LSS shown), including orbit, tolerances and dispersion.

Extraction Channel Elements

The extraction channel for the PS2 is presently assumed to comprise horizontal bumpers HB (not shown), the thin ES, a thin MS1 and a thick MS2. An initial optimisation of the element locations has been made for some of the tracked separatrices, illustrated in Fig. 5. The extracted beam is assumed to pass through a coil window gap in the downstream quadrupole doublet with an angle of ~20 mrad. The element strengths and main assumed parameters are given in Tab. 2. It can be seen that these are all fairly comfortable. The good field region of these quadrupoles should extend to about 85 mm.

Table 2: Main parameters of extraction elements

Element	k [mrad]	E [kV/cm] or B [T]	L [m]	Thickness [mm]
ES	0.82	68	6.0	0.1
MS1	3.6	0.22	2.8	5
MS2	21.6	0.66	5.6	16
HB	2.1	0.72	0.5	-

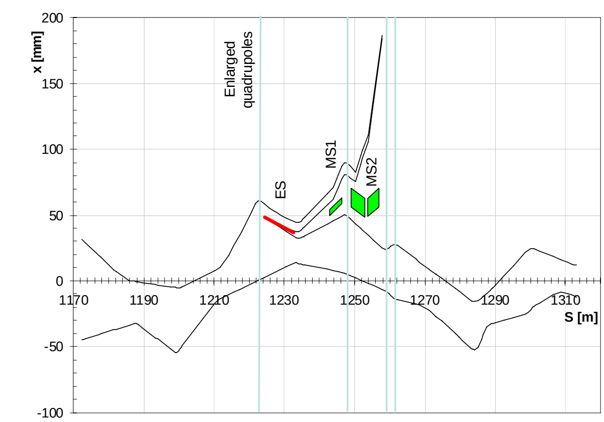

Figure 5: Layout of extraction channel elements with tracked separatrices. Enlarged quadrupoles are indicated.

CONCLUSION

The proposed slow extraction from the PS2 is feasible with the layout examined. The required element strengths seem comfortable. The aperture / good field constraints on the overall machine may be further optimised if required – the separatrix angle at the ES may be adjusted by choosing other sextupole locations, or dynamically by using orthogonal sextupole pairs at 15° intervals around the ideal phases. The parameters for the extraction elements including special extraction quadrupoles, septa and bumpers have been derived.

REFERENCES

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[3] P.Strolin, "Note on the Use of the FORTRAN Program "PRTAGS"", CERN Program Library, Long Write-Up T301, 967.