# DESIGN AND DIFFRACTIVE MODELING ON A SINGLE LENS SHAPER 

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## Abstract

This paper introduces a single lens laser beam shaper which is capable of redistributing a beam with a Gaussian profile to a super-Gaussian profile. Both geometrical and diffractive optical modelings are performed on a typical single lens shaper that shows significant reduction of destructive effects on the beam uniformity over those with sharp-edges.

## INTRODUCTION

The basic principle of laser beam reshaping by a pair of aspheric lenses was proposed by Frieden [1] and Kreuzer [2], and later by Shealy and co-workers [3][4]. Since then, a lot of work has been done to optimize design and get admirable experimental results. A recent study shows it is possible to shape an arbitrary beam profile into a desired one with only a single aspheric lens instead of two-lens system on which the popular refractive beam shaper was based and designed for many years [5-6]. Here, the design of a new single lens shaper with two convex aspheric surfaces will be presented. Using convex surfaces reduces fabrication difficulty and allows for large apertures to suppress diffraction while the single element choice simplifies overall configuration and eases optical alignment. More importantly, this new design converts a Gaussian laser beam to a super-Gaussian beam which alleviates severe diffraction effects and increases the working distance of the shaped output laser beam.

## THEORETICAL ANALYSIS

Two conditions must be met for a shaping system: a) Output intensity equals to input intensity. i.e. consistent with energy conservation law; b) All rays must maintain the same optical path length (OPL). Combining condition b) and Snell's Law, an analytical expression for aspheric surfaces of bi-convex shaper in terms of lens length $S$, shaper refractive index $n$, entrance radius $r_{1}$ and exit radius $r_{2}$ may be deduced. A lucid analytical sag expression for both surfaces of a bi-convex single lens shaping system can be expressed as [2],

$$
\begin{align*}
& Z_{1}(r)=\int_{0}^{r} n\left[-\left(n^{2}-1\right)+\left(\frac{(n-1) s}{r_{2}-r_{1}}\right)^{2}\right]^{-1 / 2} d r_{1}  \tag{1}\\
& Z_{2}(r)=\int_{0}^{r} n\left[-\left(n^{2}-1\right)+\left(\frac{(n-1) s}{r_{2}-r_{1}}\right)^{2}\right]^{-1 / 2} d r_{2} \tag{2}
\end{align*}
$$

Similar sag expression holds for design of the other type shapers. The design of a concave-convex single lens

[^0]shaper could be accomplished with the aid of the foregoing equations by replacing $r_{2}$ by $-r_{2}$. Equations for Keplerian shaper will be obtained with replacing $r_{2}$ by $-r_{2}$ and replacing $n$ by $1 / n$. By replacing $n$ by $1 / n$, we get the sag expression for Galilean shaper which is the same as the equations in [2].

Assume the system is rotationally symmetric. The intensity of the input beam at radius $r$ is given by the function $f(r)$. Usually, the input profile of most interest is the Gaussian profile which is a good representation of the cross section of a TEM00 laser beam.

$$
\begin{equation*}
f(r)=\left(2 / \pi \omega_{0}^{2}\right) \exp \left(-2 r^{2} / \omega_{0}^{2}\right) \tag{3}
\end{equation*}
$$

The output profile was chose to be a super-Gaussian distribution of order $P$ :

$$
\begin{equation*}
g(r)=g_{0} \exp \left(-2(r / R)^{P}\right) \tag{4}
\end{equation*}
$$

With

$$
\begin{equation*}
g_{0}=\frac{2^{2 / P} P}{2 \pi R^{2} \Gamma(2 / P)} \tag{5}
\end{equation*}
$$

It follows from energy conservation law that,

$$
\begin{equation*}
2 \pi \int_{0}^{r_{1}} f(r) r d r=2 \pi \int_{0}^{r_{2}} g(r) r d r \tag{6}
\end{equation*}
$$

With (18), (13) and (14) could be solved numerically to define the two surfaces of shaper much easier than dealing with the complicated differential equations.

## GEOMETRIC ANALYSIS

To verify the theory illustrated above, a bullet-looking shaper with an aperture ( 5.7 mm ) about three times the input beam size is designed. At the edge, the intensity is reduced by a factor of $10^{-7}$ from its peak on axis. It redistributes a Gaussian beam with a 2 mm radius to a super-Gaussian beam with parameters, $R=6 \mathrm{~mm}, P=12$. The length of the shaper is chosen to be 30 mm , and BK7 glass with index of refraction $\mathrm{n}=1.51947$ at $\lambda=532 \mathrm{~nm}$ is used. Ray tracing with ZEMAX is displayed in Fig. 2. The shaper looks like a bullet where the front surface redistributes the beam profile and the rear surface recollimates the light rays. As expected, a super-Gaussian beam with a pretty wide ( 6 mm ) flat top in the center
comes out at the exit. Also presented are the input and output geometric images.


Figure 1: Ray tracing diagram using ZEMAX: (a) Schematic of the bullet-looking shaper; (b) Input beam profile; (c) Output beam profile; (d) Input 2D profile; (e) Output 2D profile.

## PHYSICAL OPTICAL PROPAGATION ANALYSIS

As we mentioned earlier, the main reason for designing a super-Gaussian shaper is that we want to minimize the destructive effects on the profile uniformity caused by diffraction as the beam propagates. This is the most important advantage of a super-Gaussian shaper over the pure flat top type with a sharp edge. To look into the issue, it is necessary to perform physical optical propagation instead of only using the geometric optical model as we did before.

Assume that the output beam from the shaper is a plane wave and we will calculate the effect of diffraction on the propagating beam for both super-Gaussian and flat-top. A cylindrically symmetrical beam shaper will be considered as a circular aperture here for simplicity. A circular aperture of diameter 2 a is located at the rear surface of beam shaper. The field amplitude of the beam shaper output is denoted as $u_{0}\left(r_{0}\right)$. After propagating a distance D, the field amplitude can be given by the FresnelKirchhoff integral [14] as

$$
\begin{align*}
u(r)= & i 2 \pi N e^{-i \pi N\left(\frac{r}{a}\right)^{2}} \int_{0}^{1} \frac{r_{0} u_{0}\left(r_{0}\right) e^{-i \pi N\left(\frac{r_{0}}{a}\right)^{2}}}{a} \\
& * J_{0}\left(\frac{2 \pi N r_{0} r}{a^{2}}\right) d\left(\frac{r_{0}}{a}\right) \tag{7}
\end{align*}
$$

where $J_{0}$ is the Bessel function of order 0 , the phase term doesn't affect the intensity calculation, and the Fresnel number N for the circular aperture is defined as

$$
\begin{equation*}
N=\frac{a^{2}}{D \lambda} \tag{8}
\end{equation*}
$$

In the case of the beam shaper, the radius a is in the vicinity of several mm , so the distance D is the dominating term in the Fresnel number for a certain wavelength. We care about near field diffraction pattern instead of the far field pattern since the Airy disk pattern of far field diffraction is not desirable for a beam shaper. The near field diffraction pattern of a circular aperture has approximately N large-amplitude Fresnel ripples across the full width of the beam, and these larger fringes are then modulated by many smaller-amplitude but higherfrequency Fresnel ripples on top of them [14]. In order to keep the quasi-flat profile, the Fresnel number N should not be too small, which means the distance $D$ the beam can propagate without substantial profile change is limited. For example, in our case $\lambda=532 \mathrm{~nm}$, $a=5.7 \mathrm{~mm}$, the distance D can not exceed 3 m in order to keep N bigger than 20.

It is straightforward to evaluate (19) numerically. Some of the diffracted patterns are displayed for our case, and results for flat top are shown as well for comparison. The diffraction patterns are the same as expected: a top-hat modulated by a series of circular rings. However, the amplitudes of the ripples in the super-Gaussian case are obviously smaller than those in the flat top cases. The quasi-flat top profile can be preserved much better along the propagation path. Diffraction can be further reduced by choosing a rounder output beam, which means smaller order $P$ for a super-Gaussian profile.


Figure 2: Diffraction pattern at 0.5 m after sharp-edge shaper.


Figure 3: Diffraction pattern at 0.5 m after supergaussian shaper.


Figure 4: Diffraction pattern at 1 m after sharp-edge shaper.


Figure 5: Diffraction pattern at 1 m after supergaussian shaper.


Figure 6: Diffraction pattern at 2 m after sharp-edge shaper.


Figure 7: Diffraction pattern at 2 m after supergaussian shaper.

## CONCLUSION

Using a new design procedure in which the output beam profile is chosen as super-Gaussian, we designed a bi-convex shaper capable of controlling diffraction effect. A design of a single-lens shaper is presented along with detailed performance analysis. Both geometrical and diffractive propagation simulations were performed. The results show clear suppression of destructive effects on profile uniformity caused by diffraction in case of superGaussian shaper compared with a flat-top shaper. In addition to its ease of alignment, this new shaper is a promising substitute in some high definition beam profile shaping application, especially for shaping the drive laser of photocathode electron gun.

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