# REDUCTION OF BEAM EMITTANCE OF PEP-X USING QUADRUPLE BEND ACHROMAT CELL

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### Abstract

SLAC National Accelerator Laboratory is studying an option of building a high brightness synchrotron light source machine, PEP-X, in the existing PEP-II tunnel [1, 2]. By replacing 6 arcs of FODO cells of PEPII High Energy Ring (HER) with two arcs of DBA and four arcs of TME and installation of 89.3 m long damping wiggler an ultra low beam emittance of 0.14 nm-rad (including intra-beam scattering) at 4.5 GeV is achieved. In this paper we study the possibility to further reduce the beam emittance by releasing the constraint of the dispersion free in the DBA straight. The QBA (Quadruple Bend Achromat) cell is used to replace the DBA. The ratio of outer and inner bending angle is optimized. The dispersion function in the non-dispersion straight is controlled to compromise with lower emittance and beam size at the dispersion straight. An undulator of period length 23 mm, maximum magnetic field of 1.053 T, and total periods of 150 is used to put in the 30 straights to simulate the effects of these IDs on the beam emittance and energy spread. The brightness including all the ID effects is calculated and compared to the original PEP-X design.

### **INTRODUCTION**

The horizontal (natural) emittance of an electron beam in an electron storage ring is determined by the equilibrium between quantum fluctuations and radiation damping. The resulting emittance of electron beams is

$$\varepsilon_x = C_q \gamma^2 \frac{I_5}{I_2 - I_4} \tag{1}$$

where  $\gamma$  is the relativistic energy factor. The radiation integrals are  $I_2 = \oint 1/\rho^2 ds$ ,  $I_4 = \oint D_x/\rho(1/\rho^2 + 2K)ds$  and  $I_5 = \oint H/|\rho|^3 ds$ , where *s* is the length coordinate along the accelerator,  $\rho$  is the bending radius of dipoles,  $D_x$  is the dispersion function, *K* is the focusing function, and the *H*function is

$$H = \frac{1}{\beta_x} \left\{ \left\{ D_x^2 + (\alpha_x D_x + \beta_x D_x')^2 \right\} \right\}$$
(2)

Here,  $D'_x$  is the derivative of the dispersion function with respect to *s*,  $\beta_x$  is the betatron amplitude function, and  $\alpha_x = -\frac{1}{2}(d\beta_x/ds)$ . The minimization of beam emittance

requires minimizing the average *H*-function through the dipoles. Using a small angle approximation, the minimum emittance for the DBA and the TME lattices are

$$\varepsilon_{\text{MEDBA}} = \frac{C_q \gamma^2}{4\sqrt{15}J_x} \theta^3 \tag{3}$$

$$\varepsilon_{\text{TME}} = \frac{1}{3} \varepsilon_{\text{MEDBA}} = \frac{C_q \gamma^2}{12\sqrt{15}J_x} \theta^3$$
(4)

Where 
$$C_q = 55\hbar/(32\sqrt{3}mc) = 3.83 \times 10^{-13}m$$
,  $J_x \sim 1$  is the

horizontal damping-partition number, and  $\theta$  is the bending angle of dipoles in one bending section. Relaxing the achromatic condition, the emittance can generally be reduced by a factor of 3.

This article studies the option of the quadruple-bend achromat (QBA) lattice, which is defined as a super cell composed of two TME and two DBA dipoles with unequal bending strengths for the outer and inner dipoles.

### **QBA CELL**

The modified lattice is based on the PEP-X baseline design [2]. Two DBA cells are replaced by one OBA cell. In order to match the H function between dispersion matching dipole (outer dipole) and non dispersion matching dipole (inner dipole) the bending angle of inner dipole should be 31/3 larger than the outer dipole. This can be achieved by lengthening the inner dipole (same  $\rho$ as outer dipole) or increasing the dipole strength of the inner dipole (different  $\rho$  from outer dipole). To save the space of the QBA cell the latter is used in the design. The dipole length is 1 m. The bending angle of inner dipole is 2.250 and outer is 1.50. The allowed dispersion function in the middle straight section is controlled to compromise with lower emittance and beam size at the dispersion straight. In this case the emittance is not optimized instead the brightness is tried to. The dispersion at middle straight is constrained to less or equal to 0.03 m to control the beam size increase due to the dispersion to less than one sigma beam size without dispersion. Figure 1 shows one of the QBA cells.



Figure 1: Optic function of QBA cell. The dispersion is 0.02 m at the middle straight.

The phase advance of this QBA cell is  $\phi_x/2\pi = 1.625$ ,  $\phi_y/2\pi = 0.75$ . The beam parameters at ID are:  $\beta_x / \beta_y = 8.9$  / 3.1 m,  $\eta_x = 0$  and  $\beta_x / \beta_y = 6.6$  / 4.5 m,  $\eta_x = 0.02$ . The

Light Sources and FELs A05 - Synchrotron Radiation Facilities ring emittance with 2 arcs of QBA and 4 arcs of TME cells and 89.3 m long damping wiggler is 0.0577 nm-rad without Intra Beam Scattering (IBS) and 0.1 nm-rad including IBS. It's 30% reduction compared to the design of 0.14 nm-rad ring emittance with IBS in reference [2]. There are more QBA examples shown in Table 1 with dispersion 0.03m, 0.02m and 0.015 m at the inner straight, and two sets of QBA cell phase advance ( $\phi_x$ ,  $\phi_y$ )/2 $\pi$  =(1.625, 0.75) (case 1,3,5) and ( $\phi_x$ ,  $\phi_y$ )/2 $\pi$  =(1.75, 0.75) (case 2,4,6).

Table 1: Summary of QBA cases with different dispersions in inner straight and phase advances. Cases 1, 3, 5 have QBA cell phase advance  $(\phi_x, \phi_y)/2\pi = (1.625, 0.75)$ . Cases 2, 4, 6 have QBA cell phase advance  $(\phi_x, \phi_y)/2\pi = (1.75, 0.75)$ . The effect of ID on beam emittance and energy spread is also shown. The emittance calculation does not include IBS.

Case	D <sub>x</sub> (m)	$ \begin{array}{c} \epsilon_0 \text{ w/o ID} \\ (1e^{-11}) \end{array} $	$\begin{array}{c} \epsilon_0 \ w \ 30 \ IDs \\ (1e^{-11}) \end{array}$	$ \sigma_{\rm E} \le 30 \text{ IDs} \\ (1e^{-3}) $
1	0.03	5.2362	8.5922	1.0713
2	0.03	5.1543	5.9692	1.0713
3	0.02	5.7693	5.7510	1.0709
4	0.02	5.7898	4.9110	1.0709
5	0.015	6.0959	5.1182	1.0706
6	0.015	6.1443	4.7366	1.0706

# EFFECTS ON EMITTANCE AND ENERGY SPREAD DUE TO IDS

Releasing dispersion in ID straight will reduce the emittance. However this dispersion function has possibility to increase the emittance reversely. Unlike the dispersion free straight when ID is added the emittance is reduced. In dispersive straight the ID may blow up the emittance depending on the field strength of ID. The energy spread is also changed due to IDs. The change of emittance and energy spread due to ID can be calculated by [3]:

$$\varepsilon_{x} = C_{q} \gamma^{2} \frac{I_{50} + \sum I_{5w}}{I_{20} + \sum I_{2w} - I_{40} - \sum I_{4w}}$$
(5)

$$\left(\frac{\sigma_E}{E}\right)^2 = \left(\frac{\sigma_E}{E}\right)_0^2 \frac{I_{30} + \sum I_{3w}}{2I_{20} + I_{40} + 2\sum I_{2w} + \sum I_{4w}}$$
(6)

The synchrotron integrals can be simplified as

$$I_{20} = \frac{2\pi}{\rho_0}, \quad I_{30} = \frac{2\pi}{\rho_0^2}$$
(7*a*)

$$I_{50} = \langle H \rangle \int \frac{1}{\left| \rho^3 \right|} ds = f_h \hat{H} \frac{2\pi}{\rho_0^2}$$

$$\tag{7b}$$

$$I_{2w} = \frac{N_{p}\lambda_{w}}{2\rho_{w}^{2}}, \quad I_{3w} = \frac{4}{3\pi} \frac{N_{p}\lambda_{w}}{\rho_{w}^{3}}$$
(7c)

$$I_{5w} = \hat{H} \int_{w} \frac{1}{\left|\rho^{3}\right|} ds \approx \frac{4\hat{H}L_{w}}{3\pi\rho_{w}^{3}}, L_{w} = N_{p}\lambda_{w}$$
(7d)

Light Sources and FELs A05 - Synchrotron Radiation Facilities  $\hat{H}$  is the *H* function in the long straight, which is constant and equal to the *H* value at the end of the dipole,  $f_h$  the ratio of *H* average in the dipole and  $\hat{H}$ . The contribution is mainly from the dipole radiation therefore is related to the bending radius and total length of the dipole. Table 2 lists the bending radius of different dipoles, including damping wiggler and undulator used in the ring. The total length and radiation loss U<sub>0</sub> per turn of different dipoles are also listed in Table 2. The period of damping wiggler is 10 cm. The period length of undulator is 23 mm. There are total 30 straight sections.

Table 2: List of bending radius, magnetic field and total length of different types of dipole magnets including damping wiggler and undulator. Synchrotron radiation per turn of every type of dipole is also shown.

dipole	ρ	$B_0$	L <sub>tot</sub>	$U_0$
	(m)	(T)	(m)	(MeV)
TME	85.85	0.1748	2.7*128	0.2816
QBA(I)	25.46	0.5895	1*32	0.285
QBA(O)	38.20	0.393	1*32	0.127
DW	10.01	1.5	4.96*18	2.5945
IDs	14.25	1.053	3.45*30	0.78

In general 
$$I_{40} \ll I_{20}, I_{4w} \ll I_{2w}$$

$$\begin{split} \frac{I_{2w}}{I_{20}} &= \frac{U_w}{U_0}, \frac{I_{5w}}{I_{50}} \approx \frac{8B_w}{3\pi f_h B_0} \frac{U_w}{U_0}, \\ U_0 &= C_\gamma E^4 / \rho_0, U_w = C_\gamma E^4 L_w / (4\pi \rho_w^2) \\ \text{Equations(5),(6) become} \\ \frac{\varepsilon_x}{\varepsilon_{x,0}} &= (1 + \sum_w \frac{8}{3\pi f_h} \frac{B_w}{B_0} \frac{U_w}{U_0})(1 + \sum_w \frac{U_w}{U_0})^{-1} \\ (\frac{\sigma_E}{E})^2 &= (\frac{\sigma_E}{E})_0^2 \times (1 + \sum_w \frac{8}{3\pi} \frac{B_w}{B_0} \frac{U_w}{U_0})(1 + \sum_w \frac{U_w}{U_0})^{-1} \\ \text{If the magnetic field } B_w \text{ of insertion device,} \\ B_w &\leq \frac{3\pi f_h}{8} B_0, \text{ where } B_0 \text{ the magnetic field of ring dipole,} \\ \text{emittance will decrease. The energy spread will decrease} \\ \text{if } B_w &\leq \frac{3\pi}{8} B_0. \text{ Table 1 shows ring emittance without} \\ \text{undulator and with total of 30 undulators calculated by} \end{split}$$

MAD8. In all cases 89.3 m damping wiggler is included. In straight with larger dispersion the  $\hat{H}$  is larger and  $f_h$  is smaller the IDs effect tends to increase the emittance. The higher horizontal phase advance can achieve higher  $f_h$ and therefore less emittance growth or even damping the emittance when ID is added as shown in Table 1.

The energy spread is 6.3490E-04 without damping wiggler and undulators, 1.1254E-03 with damping wiggler and 8.1458E-04 with 30 undulators. The last column of Table 1 shows the energy spread with both damping wiggler and 30 IDs. The increase of energy spread is mainly from damping wiggler. The disadvantage of QBA is the beam size increasing in the dispersion straight due to energy spread. It weakens the gain of

lower emittance. However in the non dispersion straight the gain of lower emittance is straight forward.

# **DYNAMIC APERTURE**

The dynamic aperture tracking of ring with two values of phase advance in QBA cell is shown in Figure 2. The higher phase advance means stronger sextupole to correct the natural chromaticity and smaller dynamic aperture is expected.



Figure 2: Dynamic aperture tracking of ring with QBA cells case 3 and case 4 in Table 1.

Considering the dynamic aperture and emittance gain, case 3 in Table 1 is chosen to further investigate. The on and off momentum dynamic aperture tracking is shown in Figure 3. The on momentum aperture is comparable to the base line design [2]. The 1.5% off momentum aperture is smaller than the baseline design. The off momentum dynamic aperture needs to be improved.



Figure 3: Dynamic aperture for on and off momentum tracking.

### BRIGHTNESS

The brightness of undulator photon beam in non dispersive and dispersive straight is calculated. The beam current is 1.5 Amp and beam emittance is 0.1 nm-rad (with IBS). The result is shown in Figure 4. The brightness of same ID photon beam of baseline lattice is also shown for comparison. The brightness of both photon beams in QBA straights is larger than the baseline brightness.



Figure 4: Brightness of ID photon beam in QBA straights. The undulator period length is 23 mm, magnetic field 1.053 T. Beam current is 1.5 Amp. Ring emittance is 0.1 nm-rad (with IBS).

## DISCUSSION

The modified lattice is based on the PEP-X baseline design [2]. The QBA cell is proved that it can be used to further reduce the beam emittance. By proper controlling the dispersion released in the straight section, the beam size increase due to energy spread in dispersion region can be manipulated and both photon beams in non dispersive and dispersive straight can gain on brightness. The same study can be applied to the updated PEP-X design presented in this conference [4].

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