TOLERANCE STUDY FOR THE ECHO-ENABLED HARMONIC GENERATION FREE ELECTRON LASER*

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Abstract

The echo-enabled harmonic generation free electron laser (EEHG FEL) holds great promise in generation of coherent soft x-ray directly from a UV seed laser within one stage. The density modulation in the harmonic generation process is affected by the smearing effect caused by the fluctuations of energy and current along the beam, as well as the field error of the dispersive elements. In this paper we study the tolerance of the EEHG FEL on beam quality and field quality. The diffusion effect from incoherent synchrotron radiation (ISR) in the dispersion sections and the second modulator are also studied.

INTRODUCTION

There has been continually growing interest in generating fully coherent (both longitudinally and transversely) and powerful short wavelength radiation using the harmonic generation free electron laser (FEL) scheme, as reflected by the many proposals and funded projects worldwide [1-3]. In the classic HGHG scheme [4], the up-frequency conversion efficiency is relatively low, so that multiple stages are generally needed to generate coherent soft x-rays starting with a UV seed laser with the wavelength ~ 200 nm [5].

Recently a new method entitled echo-enabled harmonic generation (EEHG) was proposed for generation of high harmonics using the beam echo effect [6, 7]. In the EEHG FEL the beam is energy modulated in the first modulator and then sent through a dispersion section with strong dispersion strength after which the modulation obtained in the first modulator is macroscopically washed out while simultaneously complicated fine structures (separated energy bands) are introduced into the phase space of the beam. A second laser is used to further modulate the beam energy in the second modulator. After passing through the second dispersion section the separated energy bands will be converted into current modulation and the echo signal then occurs as a recoherence effect caused by the mixing of the correlations between the modulation in the second modulator and the fine structures.

The EEHG scheme has a remarkable up-frequency conversion efficiency. It has been shown in [7] that the Fermi@Elettra FEL may operate in a single stage to achieve 10 nm soft x-rays from the 240 nm seed laser. Our recent time-dependent simulation [8] also confirmed the good performance of EEHG FEL where 3.8 nm radiation in the water window is generated from 190 nm seed laser and the bandwidth is very close to the Fourier transform limit. In this paper we will focus on some practical considerations of an EEHG FEL and the degradation effects from unperfect beam and field qualities. We will use the typical parameters of the Fermi@Elettra FEL: $E = 1.2$ GeV, $\sigma_E = 150$ keV, and $\epsilon_n = 1.5$ mm mrad.

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CHOICE OF ENERGY MODULATION AMPLITUDES

According to Ref. [7], the maximized bunching factor for the nth harmonic may be written as,

$$b_n = \frac{0.67}{(n+1)^{1/3}} F(A_1)\,, \tag{1}$$

where $A_1 = \Delta E_1/\sigma_E$ is the dimensionless modulation amplitude in the first modulator and $F(A_1) = \left[J_1(A_1 x)e^{-A_1 x^2/2}\right]_{max}$. As shown in Fig. 2 of Ref. [7], the maximal value of $F(A_1)$ increases with A_1, but the growth slows down when $A_1 > 3$. In order to get sufficient bunching while still keeping the slice energy spread within a small level, it's desirable to choose $A_1 = 3$. The value of A_2 does not affect the the bunching factor, but it is related to the final slice energy spread and determines the strength of the dispersion sections. The slice energy spread of the beam at the entrance to the radiator is found to be,

$$\sigma_E^{'} = \sigma_E \sqrt{1 + \frac{A_1^2}{2} + \frac{A_2^2}{2}}\,, \tag{2}$$

Generally speaking, using a large A_2 could decrease the required dispersion strength, which is helpful to reduce the space for the dispersion sections. As we will show below that the small dispersion strength also mitigates the diffusion from ISR and enhances the tolerance of the field quality of the chicanes. But if A_2 is too large, it may result in a large slice energy spread which if beyond the tolerance will greatly degrade the FEL lasing. Thus the choice of A_2 should be made based on specific parameters of the FEL projects. Take the Fermi@Elettra FEL project as an example, using Xie's formulae [9], it's found that the FEL performance will not be degraded if the slice energy spread of the beam at the entrance of the radiator is less than 500 keV. So increasing A_2 to 3 will not degrade the FEL performance as compared to the case when $A_2 = 1$. The corresponding optimized dispersion strength are $R_{56}^{(1)} = 8.198$ mm and $R_{56}^{(1)} = 2.625$ mm for $A_2 = 1$ and $A_2 = 3$, respectively.

TOLERANCE ON QUANTUM DIFFUSION

Quantum fluctuations in the process of ISR lead to diffusion in energy. If the rms value of the energy spread caused by this diffusion exceeds the spacing of two adjacent energy bands, it may result in the overlapping of the bands, which will smear the fine structures of the longitudinal phase space and thus degrade the EEHG performances. Analysis shows that the spacing of the adjacent energy bands is in the order of $(\pi/B_1)\sigma_E$. So one can increase A_2 to reduce the optimized value of B_1, which further increases the spacing, as can be seen in Fig. 1. The corresponding spacing between adjacent energy bands is about 18 keV and 55 keV, respectively. The dispersion sections should be properly designed to make sure that the ISR induced energy spread growth is much smaller than the spacing between adjacent energy bands.

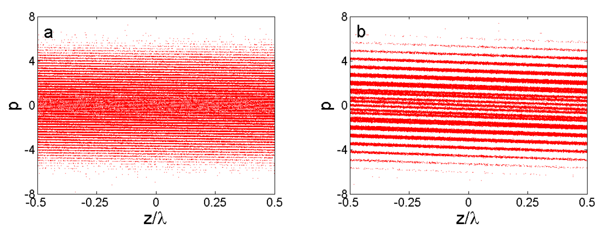

Figure 1: Longitudinal phase space at the exit from the first dispersion section for the case $A_2 = 1$ (a) and $A_2 = 3$ (b).

Quantum Diffusion in Bend

The energy spread caused by ISR when beam passes a length L in a bend with the bending radius ρ can be calculated as

$$\Delta\sigma_E^2\big|_{ISR} = \frac{55e^2\hbar c}{48\sqrt{3}}\frac{L}{\rho^3}\gamma^7 \,. \tag{3}$$

We consider a compact 4-dipole symmetric chicane. The length of the dipole and the distance between dipoles are assumed to be 0.2 m and 0.25 m. The energy spread caused by ISR for the two cases are found to be 2.9 keV and 1.3 keV, respectively. The bunching factor drops by about 20% for the $A_2 = 1$ case while that for $A_2 = 3$ is negligible.

It is worth pointing out that the diffusion caused by ISR strongly depends on the design of the dispersion section. In the example above we used a very compact dispersion section (< 1.5 m) to be able to demonstrate the ISR effect for the $A_2 = 1$ case. In practical design, for some given dispersion strength, one can increase the spacing between the dipoles and thus increase the bending radius to mitigate the ISR effect. This might not be effective, if there is not enough space for the chicane. Another option is to increase the value of A_2 (for example, $A_2 = 3$) to reduce the optimized strength of the dispersion section which simultaneously also increases the spacing between adjacent energy bands and makes the fine structures more robust.

Quantum Diffusion in Undulator

The separated energy bands must survive during the passage through the second modulator. For a planar undulator, the slice energy spread growth from quantum diffusion can be written as [10]

$$(\Delta\gamma)^2 = \frac{7}{15}\frac{\hbar}{m_0 c}L_u r_e \gamma^4 \kappa_\omega^3 K^2 F(K) \,, \tag{4}$$

where r_e is the classical radius of the electron, $\kappa_\omega = 2\pi/\lambda_\omega$, λ_ω is the undulator period length, L_u is the total length of the undulator, K is the undulator strength and $F(K) \approx 1.42K$ when $K \gg 1$. For a modulator with 9 periods of 15 cm, the energy spread growth caused by the quantum diffusion in the modulator is found to be about 0.33 keV. So it will not affect the EEHG FEL performance.

TOLERANCE ON FIELD QUALITY

The performance of an EEHG FEL depends on the field quality of the magnetic elements, including the dispersion sections, undulators, etc. In this paper we will confine our studies to the field quality of the dispersion sections. The field error may result in that the value of R_{51} becomes not equal to zero at the exit from the chicane. The rms value for R_{51} may be written as

$$R_{51} = \frac{2L_B}{\rho}\frac{\Delta B}{B} \,, \tag{5}$$

where L_B is the magnet lengths, and ΔB is the rms error of the magnetic field in the bends. The longitudinal phase space for various values of R_{51} is shown in Fig. 2 where we can clearly see the smearing effect. In order not to wash out the bunching, one may require the condition $R_{51}\sigma_x < \lambda_r/20$ to be satisfied, where λ_r is the wavelength of the harmonic.

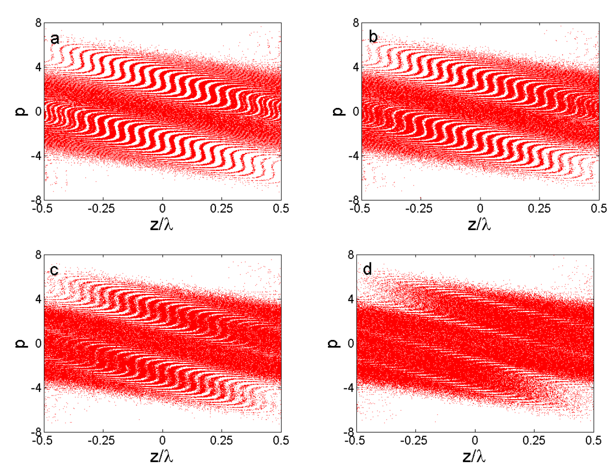

Figure 2: Longitudinal phase space at the exit of the second dispersion section for various R_{51}: *(a)* $R_{51}\sigma_x = 0.05\lambda_r$; *(b)* $R_{51}\sigma_x = 0.1\lambda_r$; *(c)* $R_{51}\sigma_x = 0.2\lambda_r$; *(d)* $R_{51}\sigma_x = 0.5\lambda_r$.

Assuming the beam size at the entrance of the second dispersion section to be 40 μm and $\lambda_r = 10$ nm, the field error may need to be smaller than $1.25 \cdot 10^{-3}$. We simulated the R_{51} and the bunching factor by introducing errors in the magnetic field of the dipoles. The errors have a Gaussian distribution. The results are shown in Fig. 3 where we see that the field errors must be controlled to be smaller than 10^{-3} to maintain a large bunching factor.

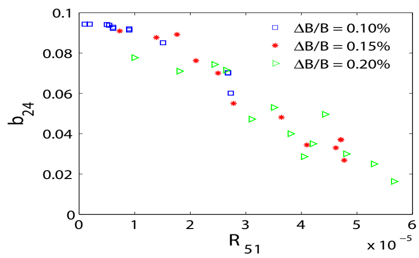

Figure 3: Bunching factor vs R_{51} for various field errors. The standard deviation of $\Delta B/B$ indicated in the plot.

TOLERANCE ON BEAM QUALITY

Energy Chirp

Similar to the classic HGHG scheme, the central wavelength and bunching efficiency in the EEHG scheme are also affected by the energy chirp. The dimensionless chirp factor is defined as $h = dp/d\zeta$. An analytical estimation shows the presence of the energy chirp shifts the harmonic wave number nk to [11]

$$k_E = \frac{(n+1)(1+hB_1) - 1}{1 + h(B_1 + B_2)} k\,, \tag{6}$$

with the corresponding bunching factor

$$\begin{aligned} b_{k_E} &= J_{n+1}\left(\frac{k_E}{k} A_2 B_2\right) J_1\left(A_1 \frac{nB_2 - B_1}{1 + h(B_1 + B_2)}\right) \\ & \exp\left[-\frac{1}{2}\left(\frac{nB_2 - B_1}{1 + h(B_1 + B_2)}\right)^2\right]. \end{aligned} \tag{7}$$

The bunching factor for the 24th harmonic is shown in Fig. 4 for various chirp factors. The change of the central wavelength of the echo signal due to the energy chirp is similar for $A_2 = 1$ and $A_2 = 3$, but the value of the bunching factor is less sensitive to the energy chirp for the latter case. For example, for $A_2 = 3$ even for a relatively large chirp of $h = 0.015$, which corresponds to 18 MeV/ps, the degradation in bunching factor is still negligible.

Energy Modulation

Due to microbunching instability, the beam after bunch compressor is likely to have some energy modulation. We assume that the beam has an energy modulation with a wavelength of 5 μm and amplitude δE. The residual energy modulation may result in the broadening of the FEL bandwidth which is found to be about $h/2n$ [11]. For the Fermi@Elettra project, the FEL bandwidth at 10 nm is

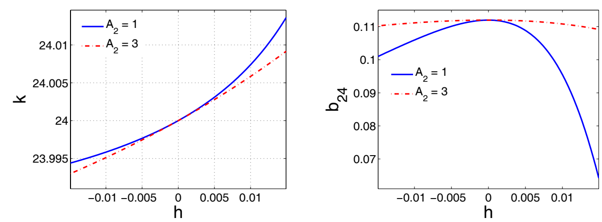

Figure 4: (a) Harmonic number at various energy chirp; (b) Bunching factor at various energy chirp.

about $6 \cdot 10^{-5}$. To assure that the bandwidth broadening due to the residual energy modulation is smaller than the expected FEL bandwidth, the residual energy modulation δE needs to be smaller than 60 keV. It should be pointed out that the sensitivity of the spectrum broadening to residual energy modulation for the EEHG FEL is comparable to that for the classic HGHG FEL. But for the EEHG FEL, since the modulation amplitudes are smaller, the tolerance on the initial slice energy spread could be enhanced. So one may increase the laser heater induced energy spread to more effectively damp the residual energy modulation caused by microbunching instability [12]. This might be helpful to obtain a transform limited FEL pulse.

CONCLUSIONS

We addressed several practical concerns in the path to realization of EEHG FEL. It seems to us that with practical accelerator and magnet technologies, the EEHG FEL is able to provide high power soft x-ray radiation with narrow bandwidth from a UV seed laser in a single stage. After taking into account the FEL performance, field tolerance, and the desire of a compact chicane with negligible ISR effect, we would suggest following parameters for the Fermi@Elettra FEL: $A_1 = A_2 = 3$, $R_{56}^{(1)} = 2.625$ mm and $R_{56}^{(2)} = 0.116$ mm.

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