# MULTIPASS ARC LATTICE DESIGN FOR RECIRCULATING LINAC MUON ACCELERATORS*

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*Abstract*

Recirculating linear accelerators (RLA) are the most likely means to achieve rapid acceleration of short-lived muons to multi-GeV energies required for Neutrino Factories and TeV energies required for Muon Colliders. A drawback of this scheme is that a separate return arc is required for each passage of the muons through the linac. In the work described here, a novel arc optics based on a Non-Scaling Fixed Field Alternating Gradient (NS-FFAG) lattice is developed, which would provide sufficient momentum acceptance to allow multiple passes (two or more consecutive energies) to be transported in one string of magnets. An RLA with significantly fewer arcs will reduce the cost. We will develop the optics and technical requirements to allow the maximum number of passes by using an adjustable path length to accurately control the returned beam to synchronize with the linac RF phase.

## INTRODUCTION

In a companion paper [1], we proposed a muon RLA consisting of a single linac with pulsed quads and separate teardrop return arcs, as shown in Figure 1. That pulsed linac dogbone-shaped RLA increases the number of passes; from 8 passes to 12, leading to cost savings. However, in that scheme, one needs to separate different energy beams coming out of a linac and to direct them into appropriate droplet-shaped arcs for recirculation. Each pass through the linac would call for a separate fixed energy droplet arc, increasing the complexity of the RLA. Here, we propose to employ a novel arc optics based on a NS-FFAG [2] lattice which would provide sufficient momentum acceptance to allow multiple passes (two or more consecutive energies) to be transported in each string of magnets (single beam-line). Studies show that this arc structure is very compact and the momentum acceptance could be from -30% to +90%.

## DROPLET ARC REQUIREMENTS

The new concept of a large momentum acceptance Non-Scaling FFAG-like arc in a dogbone RLA is to maximize the number of passes that $\mu^{\pm}$ can be accelerated through a single linac. The arc layout is similar to the separated arcs structure in Figure 1. It is composed of a large number of two types of unit cells, $B_p$ and $B_n$, to form a closed 180 degree arc. The dipoles in unit cells $B_p$ and $B_n$ bend in opposite directions. The numbers of unit cells are $N_p$ and $N_n$, which satisfy the requirements that the displacement in the transverse plane is $x_{dis}=0$ and the total bending angle is $\left(N_p-N_n\right)\cdot\theta_{unit}=180$.

In the linac, $\mu^+$ and $\mu^-$ experience the same transport optics. After the linac, they are bent in opposite directions by the spreader bending magnet. To transport $\mu^+$ and $\mu^-$ with the same arc structure, the arc should have these basic characteristics:

1) The arc must be achromatic and mirror symmetric, so that $\mu^+$ and $\mu^-$ pass through the same lattice in opposite directions $\beta_s=\beta_e$, $\alpha_s=-\alpha_e$.

2) The phase advance per cell should be 90 degrees (especially in the bend plane), so that the variation of Twiss functions is minimized and it is easier to match the linac optics.

3) The beam phase relative to the SRF cavity must be accurately controlled due to the fixed RF frequency, so the path length in the arc needs to be adjustable.

Besides the above requirements for each pass, this arc also has the ability to accommodate multi-pass beam transport, i.e. large momentum acceptance. In each pass, all the above conditions should be fulfilled and the beam orbit offset should be small to make the vacuum aperture

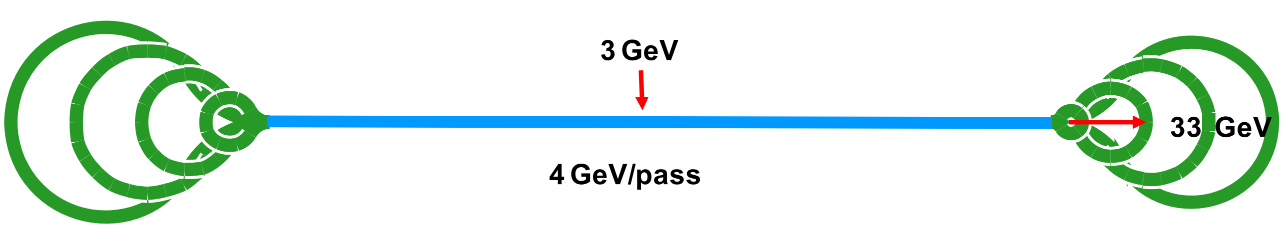


Figure 1: Layout of an 8-pass 'Dogbone' RLA with the top-to-injected energy ratio of 11.

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size acceptable. The aperture for the low-momentum muon beams is determined by the muon emittances, beta functions, and dispersion.

The NS-FFAG lattice provides extremely strong focusing, resulting in very small beta functions, very small dispersion, and very small transverse aperture.

## NS-FFAG BASIC CELL STUDY

The NS-FFAG basic cell [2] (assuming that the beam bends in the horizontal plane) contains a triplet magnet arrangement composed of an inward bending magnet at the center with negative gradient ("combined function" magnet, horizontally defocusing), and two outward bending magnets located at each side with positive gradient. To simplify the structure, it is symmetric with respect to the center of the middle combined function dipole (we will later see the advantage of this symmetric structure). By using a combined function magnet, the arc structure is very compact.

Assuming that the combined function magnet field is linear, we can express it as $B_y = B_0 + Gx$, $B_x = Gy$

Where, $B_0$ is the central field and $G$ is the field gradient. By optimizing $B_0$ and magnet length, the total bend angle through one cell is

$$\theta_{unit} = \frac{L_{QD}}{\rho_{QD}} - \frac{2L_{QF}}{\rho_{QF}}$$

Once the geometric structure is set, the central field and magnet length cannot be changed, but gradients are still adjustable. By adjusting the gradients, the betatron oscillation frequencies and the dispersion can be changed.

In a periodic lattice, we can express the transverse transport matrix as

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{16} \\ M_{21} & M_{22} & M_{26} \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos\phi + \alpha\sin\phi & \beta\sin\phi & (1-\cos\phi-\alpha\sin\phi)D - \beta D'\sin\phi \\ -\gamma\sin\phi & \cos\phi - \alpha\sin\phi & \gamma D\sin\phi + (1-\cos\phi+\alpha\sin\phi)D' \\ 0 & 0 & 1 \end{bmatrix}$$

where $\beta$, $\alpha$, $\gamma$ are periodic Courant-Snyder functions, $D$, $D'$ are periodic dispersion, and $\phi$ is the phase advance per period. The dispersion can be expressed as

$$D = \frac{M_{16}(1-\cos\phi+\alpha\sin\phi) + M_{26}\beta\sin\phi}{2(1-\cos\phi)}$$

$$D' = \frac{-M_{16}\gamma\sin\phi + M_{26}(1-\cos\phi-\alpha\sin\phi)}{2(1-\cos\phi)}$$

According to the arc requirements, we have $\phi = 90$. To simplify the dispersion match between the $B_p$ and $B_n$ unit cells, the achromatic lattice with $D = 0, D' = 0$ is one of the best choices. An advantage of an achromatic unit cell is that it is easy to insert dispersion-free matching cells.

A symmetric system can be treated as two mirror systems, $N$ and $N_{mirror}$. The matrix $M$ can be expressed with $N$ as

$$M = \begin{bmatrix} 1+2N_{12}N_{21} & 2N_{12}N_{22} & 2N_{12}N_{26} \\ 2N_{11}N_{21} & 1+2N_{12}N_{21} & 2N_{11}N_{26} \\ 0 & 0 & 1 \end{bmatrix}$$

We get that $M_{11} = M_{22}$ is automatically satisfied, so $\alpha = 0$. For $D = 0, D' = 0$, we get $M_{16} = 0, M_{26} = 0$. $M_{16}$ and $M_{26}$ have a common item $N_{26}$, which lowers the number of conditions from two to one. With this symmetric structure, 90 phase advance and the achromatic condition corresponds to $N_{26} = 0$ at the middle plane and $M_{11} = 0$ for the basic cell, giving the magnet gradients as two adjustable knobs.

OPTIM has been used to optimize the lattice, where table 1 lists the magnet properties of the two basic cells designed for a 6.2 GeV muon beam. The layouts are BD-o-BF-o-BD and BDre-o-BFre-o-BDre. The drift distance o is 40cm. Comparing the focusing from bends $1/\rho^2$ and quads $G/(B\rho)$, we find that the focusing from the bends can be ignored compared to that from quads. So the quad focusing strengths are totally dominant in the optics. In the basic cells $B_p$ and $B_n$, the quad strengths are the same but the bending angles are opposite.

Table 1: Combined Function Magnet Properties

| Mag. | L(cm) | B(kG) | G(kG/cm) | θ(deg) |
|---|---|---|---|---|
| BD | 0.5233 | 35.08 | -2.28 | 5 |
| BF | 0.5233 | -35.08 | 5.60 | -5 |
| BDre | 0.5233 | -35.08 | -2.28 | 5 |
| BFre | 0.5233 | 35.08 | 5.60 | -5 |

Due to the very strong focusing magnets, the beta functions and dispersion are small, as shown in Figure 2. The maximum beta functions in the x and y planes are 4.92 m and 3.39 m. The maximum and minimum dispersions are 7.21 cm and -7.21 cm. The beta functions and dispersion naturally match well for the two basic cells that bend in opposite directions.

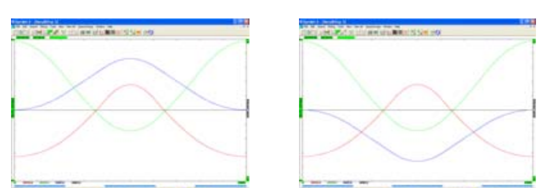

Figure 2: Beta functions and dispersion in unit cells.

To study the momentum acceptance and optics of the NS-FFAG structure for different energies, the code MADXP- Polymorphic Tracking Code (PTC) [3] is used. Since the beam energy change is large for successive passes, a perturbation method such as OPTIM for beam optics study does not work well. PTC allows symplectic integration through all elements with user control over the precision (with full or extended Hamiltonian).

Figure 3 shows beta functions, orbit displacement, dispersion, and path length of the basic cell Bp at different energies. The energy acceptance ranges from -30% to +90% or from 4 GeV to 10.8 GeV. Based on the linac optics design in [1], this arc can accommodate at least two passes of the beam. Scaling the reference energy to 26 GeV, then the energy accommodated ranges from 18 GeV to 53 GeV. So the total number of separate arcs is reduced from 6 to 3. It can be seen that as the energy increases, the beta function changes in a small range, less than 20 m, the maximum orbit offset is about 4 cm, and the maximum dispersion is around 10 cm, which is acceptable for a large momentum muon beam. The path length changes as a parabolic function of energy rather than linearly, which allows the relative RF phase change to be limited to a small range. (Minus ds means the path length is longer than that of the reference particle.)

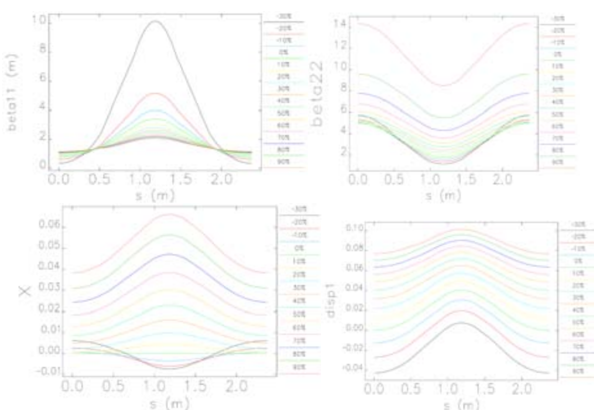


Figure 3: Beam optics for different energies at $B_p$ cell.

For the optics of the basic cell $B_n$, the beta functions have the same property as those of $B_p$, but the orbit offset and dispersion have the opposite value, as expected since they have opposite bending. To match the dispersion, beta functions and orbit offset between unit cells $B_p$ and $B_n$, the –I section should be used. Here, taking advantage of the 90 degree phase advance per cell and the very small focusing from the bend, it is easy to see that the –I section can be achieved by inserting two periods of a unit cell without bending. Here, it should be noticed that this –I section is referred to the reference particle.

Figure 4 illustrates the droplet arc optics. At the bottom the quadrupoles are depicted in red and the combined-function bending magnets in blue. Between the outward bend and inward bend section, there is a –I section. The total circumference is about 200 m with 2*60 degrees outward bend and 300 degree inward bend. This arc is very compact compared with a general FODO arc lattice, which is very important for the short lived muon beam.

To control the returned beam phase to synchronize with the RF, there are usually three ways to change path length; using an achromatic magnet structure, mechanically moving the arc, or changing the RF wavelength. Considering this RLA structure is to accelerate muon beams in two directions in a large arc, the best choice to adjust path length is using an achromatic structure. A chicane structure [4] is very simple, with the property that the higher energy, the shorter the path length. It is widely used in various facilities. Figure 3 shows that the path length change as a function of energy is a parabolic curve. For one arc to transport two passes, it is best that the energy of the two passes is located at two sides of the curve, symmetric about the peak, so that the path length change due to the chicane can be minimized.

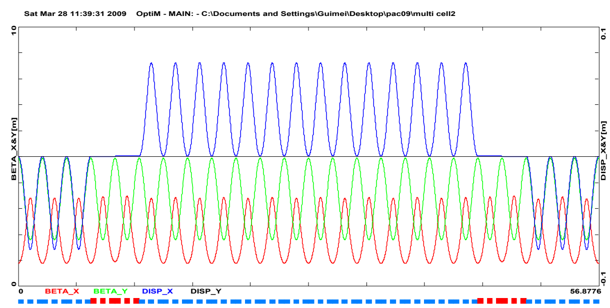


Figure 4: Droplet arc optics.

## CONCLUSION

A droplet arc based on a NS-FFAG lattice has been designed to transport multipass beams, lowering the number and hence the total cost of the arcs for a muon RLA. Studies show that such a lattice can accommodate beam energies over a very large range and have the property of small beta functions and small dispersion due to the strong focusing of a NS-FFAG. The special optics match required inside the droplet structure is solved by inserting –I section. To synchronize the beam phase with RF, a chicane structure is adopted to adjust beam path length.

More studies will be investigated to scale this design to higher energy and to improve the matching of the beam optics between the arc and linac.

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