TAPERED SIX-DIMENSIONAL COOLING CHANNEL FOR A MUON COLLIDER [*]

Robert B. Palmer, Richard C. Fernow, Brookhaven National Laboratory, Upton, NY, 11973

Abstract

A high-luminosity muon collider requires a reduction of the six-dimensional emittance of the captured muon beam by a factor of $\approx 10^6$. Most of this cooling takes place in a dispersive channel that simultaneously reduces all six phase space dimensions. We describe a tapered 6D cooling channel that should meet the requirements of a muon collider. The parameters of the channel are given and preliminary simulations are shown of the expected performance.

INTRODUCTION

A complete scheme for cooling a muon beam sufficiently for use in a muon collider has been previously described[1]. This scheme uses separate 6D ionization cooling channels for the two signs of the particle charge. In each, a channel first reduces the emittance of a train of muon bunches until they can be injected into a bunch-merging system. The single muon bunches, one of each sign, are then sent through a second tapered 6D cooling channel where the transverse emittance is reduced as much as possible and the longitudinal emittance is cooled to a value below that needed for the collider. The beam can then be recombined and sent through a final cooling channel using high-field solenoids that cools the transverse emittance to the required values for the collider while allowing the longitudinal emittance to grow. This paper mainly describes the design of the 6D cooling channel before bunch merging.

Cooling efficiency is conveniently measured using a parameter Q, which is defined as the rate of change of 6D emittance divided by the rate of change of the number of muons in the beam[2]. In a given lattice Q starts off small due to losses from initial matching, then rises to a large value ($Q \approx 15$ is typical for the channels discussed here), and finally falls as the emittance of the beam approaches its equilibrium value.

The idea for the 6D cooling channel described here originated with the RFOFO cooling ring[2]. This design evolved into a helical channel referred to as a "Guggenheim" in order to avoid serious problems with injection of large emittance beams. We found that good cooling efficiency requires that the channel be tapered. In that case when Q starts to fall off the lattice is modified to reduce the beta function. This ensures that the beam emittance is always large compared with the equilibrium emittance.

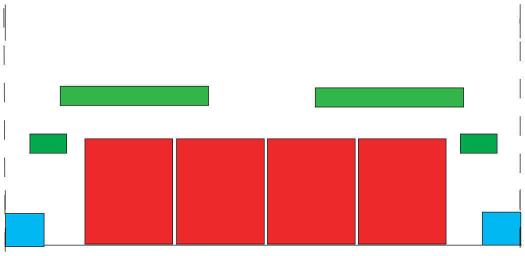

Figure 1: Layout (r versus z) of cell for stage 6. The cell length is 2.36 m; blue: absorbers, red: RF cavities, green: coils.

INITIAL DESIGN

A simplified model of the channel was used for the initial design[3]. This step was useful in order to give guidance on the parameters and performance expected in every stage. The cooling was simulated in a series of straight lattices. The layout of a typical lattice cell is shown in Fig. 1. This arrangement only produces transverse cooling. Longitudinal cooling requires emittance exchange between the transverse and longitudinal phase space. This was modeled by applying a matrix to the beam coming out of the straight channel. The amount of exchange was controlled using a parameter δ, which reduced the momentum spread and increased the angular divergence of the beam.

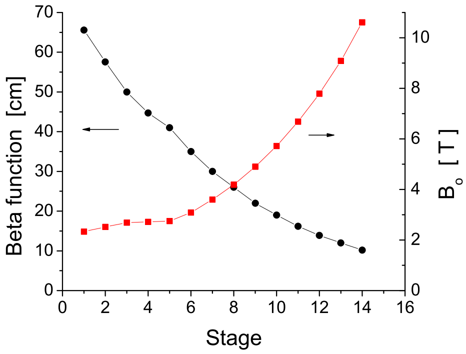

Figure 2: (left) Design value of beta function; (right) maximum value of on-axis solenoid field.

The beta function was reduced by scaling down all dimensions and raising the on-axis solenoid field, as shown in Fig. 2. The desired beta function varies from 66 to 10 cm in the pre-merge channel, while the on-axis field increases from 2.3 to 10.6 T. The beta function determined from these straight lattices was very similar to those found in the more realistic simulations described later since the required

---

[*] Work supported by US Department of Energy under contract DE-AC02-98CH10886

dipole field strength is much smaller than the solenoid field. The RF frequency in the stages increased monitonically from 201 to 805 MHz. A gradient of 15.5 MV/m was used for all cavities. This procedure produced a reference 14-stage pre-merge 6D cooling channel design. Ideal tapering requires that the rms angular divergence in the channel ($\approx$ 130 mrad) stay approximately constant. The rms momentum spread was slowly reduced down the channel. The first five stages were used to match the beam from a phase-rotation system into the 6D cooling channel.

Simulations of the channel performance were done using the ICOOL code[4]. The performance obtained for the pre-merge channel is shown in Fig. 3. The reference channel design reduced the transverse emittance to 1.3 mm and the longitudinal emittance to 1.9 mm with a transmission of 45%. All emittances quoted in this paper are rms normalized values.

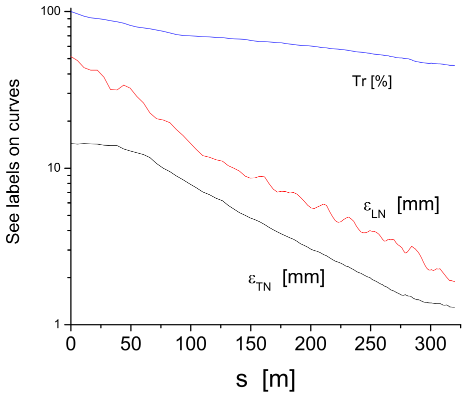

Figure 3: Emittances and transmission as a function of distance along the pre-merge channel for the reference design.

The design of the 6D cooling channel after bunch merging is similar in concept. However, near the end of the post-merge channel the beta function becomes very small ($\approx$ 2.4 cm). Since good cooling requires that the absorber half-length be comparable with the minimum value of the beta function, this becomes impractical with liquid hydrogen and we switched to using LiH as the absorber material. The performance obtained in the reference design after bunch merging is shown in Fig. 4. The reference post-merge channel reduced the transverse emittance to 0.4 mm and the longitudinal emittance to 1.1 mm with a transmission of 48%.

REALISTIC EMITTANCE EXCHANGE

We next made a more realistic design of emittance exchange using a dispersive channel with wedges, so that higher momentum particles go through the thicker part of the wedge. The cells from the straight lattices in the reference design were laid out along the arc of a circle. The dispersion D comes from slightly tipping the solenoids. We generated 3D cylindrical field maps for each of the stages by superimposing the fields from all the solenoids in the cell and its neighbor cells. Tracking in each cell of a given stage used the same field map. Closed orbits were found for each of the stages by tracking a reference particle using the field maps. The wedge material was liquid hydrogen. The number of cells for each stage was chosen so that the Q variable remained moderately large. We required that the RF frequencies be restricted to harmonics of 201 MHz since the pre-merge beam is actually a bunch train.

The beam for these studies was prepared by sending a large-amplitude gaussian beam through stage 5 of the channel. The particles that successfully reached the end of stage 5 were used as the initial beam for studying the 6D cooling performance of the channel. This procedure introduces correlations among the phase space parameters, e.g. a dependence of longitudinal momentum on transverse amplitude. Once the channel design is optimized we will simulate the performance using the matched beam coming from the charge separation system upstream of this channel.

The relative amount of transverse and longitudinal cooling can be adjusted by changing the opening angle α_W and the transverse location of the wedge. These parameters are related to the matrix parameter δ through the formula

$$\delta = \frac{\eta \beta D}{p} \tan\left(\frac{\alpha_W}{2}\right) \qquad (1)$$

where η is the energy loss dE/dx in the absorber, β is the relativistic velocity, and p is the particle momentum. For a wedge angle of 90^o the dispersions corresponding to the reference δ values vary from 6 to 20 cm. The dispersion is shown as a function of distance for stage 6 in Fig. 5. The dispersion at the absorber (s=0) is predominantly along the vertical direction.

We found that using a constant radius of curvature in the channel, similar to our initial Guggenheim concept, did not produce adequate dispersion to give the required longitudinal cooling. As a result we adjusted the radius of curvature

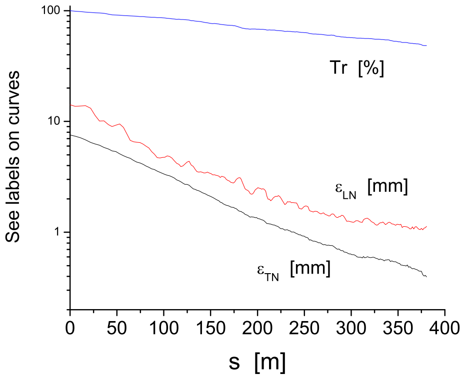

Figure 4: Emittances and transmission as a function of distance along the post-merge channel for the reference design.

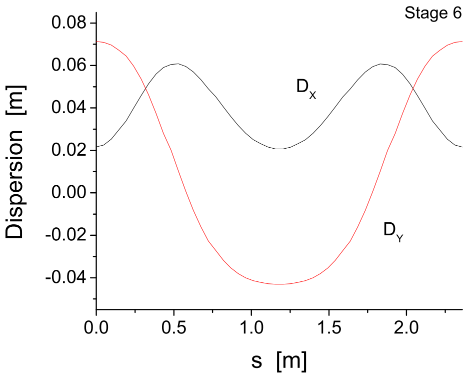

Figure 5: Dispersion as a function of distance for stage 6

of each stage to be proportional to the value of the beta function. This produces a piecewise-constant ”ziggurat” channel. The dispersion for these channels is compared in Fig. 6. The ziggurat dispersion in the later stages is too small for 90^o wedges, so we had to gradually increase the wedge angle to 130^o.

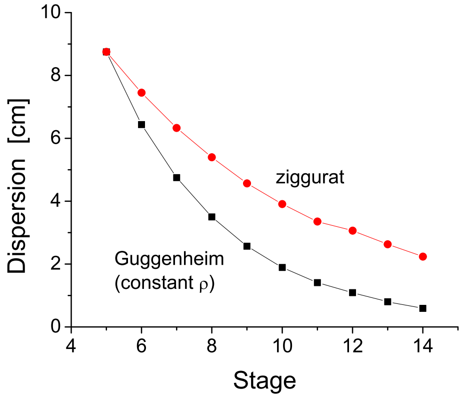

Figure 6: Dispersion for the Guggenheim and ziggurat channels.

We assumed the liquid hydrogen was contained in a vessel with aluminum beam windows and safety windows similar to those used in the MICE experiment[5]. For now the absorber has a ”house” shape with the wedge corresponding to the roof of the house. The absorber windows were taken as flat planes on either side of the absorber. We assumed the safety windows were made from AlBeMet and that the thicknesses scaled with the radius of the absorber. We assumed the RF cavities had radially-stepped beryllium windows similar to those in Study 2 for a neutrino factory[6]. The thickness was scaled by the RF frequency and by the ratio of our pulse repetition rate (15 Hz) to that used in Study 2 (50 Hz).

The transverse and longitudinal emittances and the transmission including decays are shown as a function of distance along the channel in Fig. 7. The results shown in the figure only represent stages 6-14. Even so, this channel is significantly longer than the reference channel. The simulation produced a transverse emittance of 1.5 mm and a longitudinal emittance of 1.7 mm with a transmission of 53% including decays, which is close to the desired values.

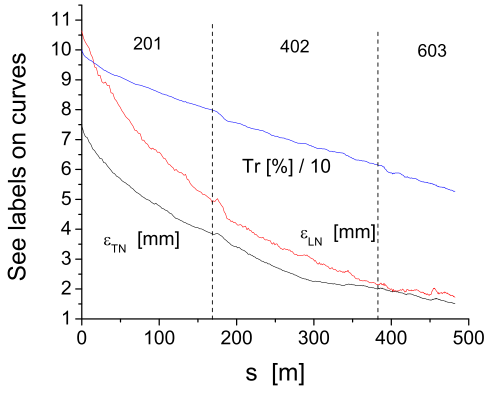

Figure 7: Emittances and transmission as a function of distance along the channel for the more realistic simulations. The vertical lines show the parts of the channel that used 201, 402, and 603 MHz cavities.

PROSPECTS

The ziggurat channel described here is one of the possible choices for 6D cooling in a muon collider. The small dispersion in the later stages of the realistic channel clearly hurt its performance. A larger dispersion would allow more efficient emittance exchange and more freedom in optimizing the amount of transverse and longitudinal cooling in each stage. Several ideas are being investigated now to increase the dispersion, including adding a small dipole field in each cell. Eventually the modeling also needs to include additional details, including bending out of the horizontal plane and modifying the shape of the absorber vessels and the absorber windows.

We would like to thank Pavel Snopok, Juan Gallardo and David Neuffer for useful discussions.

REFERENCES

[1] R.B. Palmer, Proc. 2009 Part. Acc. Conf., p. 652.

[2] R. Palmer et al., Phys. Rev. ST-AB 8, 061003 (2005).

[3] R. Palmer, https://mctf.fnal.gov/meetings/2010/0 _28/ palmer_1004-v3.pdf, April 2010.

[4] R. Fernow, Proc. 2005 Part. Acc. Conf., p. 2651.

[5] W. Lau et al., The development of a 6061-aluminum window for the MICE liquid absorber, LBNL-59001, August 2005.

[6] S. Ozaki, R. Palmer, M. Zisman (eds), Feasibility Study II of a muon-based neutrino factory, BNL-52623, June 2001.