SIX-DIMENSIONAL BUNCH MERGING FOR MUON COLLIDER COOLING*

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Abstract

A muon collider requires single, intense, muon bunches with small emittances in all six dimensions. It is most efficient to initally phase-rotate the muons into many separate bunches, cool these bunches in six dimensions (6D), and, when cool enough, merge them into single bunches (one of each sign). Previous studies only merged in longitudinal phase space (2D). In this paper we describe merging in all six dimensions (6D). The scheme uses rf for longitudinal merging, and kickers and transports with differing lengths (trombones[1]) for transverse merging. Preliminary simulations, including incorporation in 6D cooling, is described.

INTRODUCTION

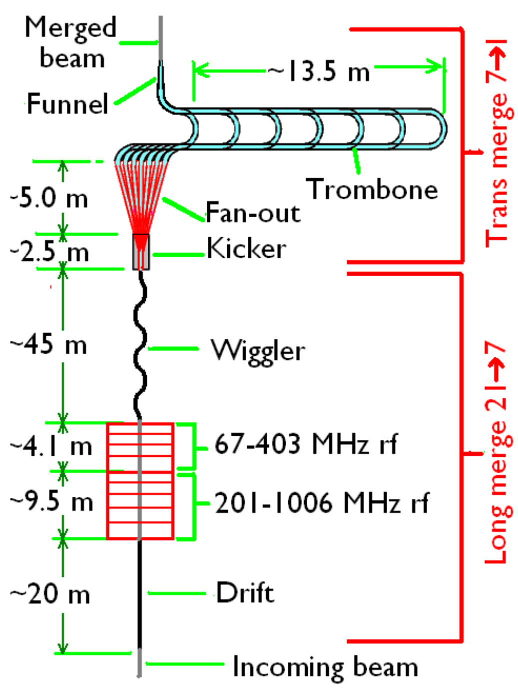

Figure 1: Schematic of 6D merge.

Muons are efficiently generated by pion decay, but they then have very large emittances. A muon collider requires low emittances, which can be achieved using transverse ionization cooling[2], combined with emittance exchange using dispersion and shaped absorbers. For efficient capture, muons are first phase-rotated[3] by rf into a train of many bunches. But for high luminosity, we need just one bunch of each sign, so after some initial cooling, these bunches should be merged.

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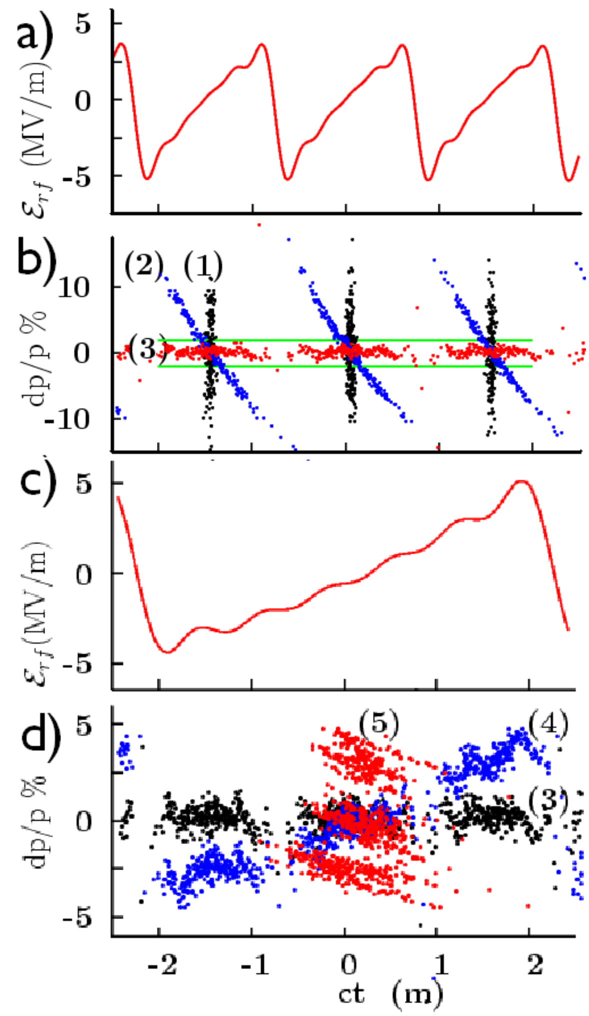

Figure 2: Longitudinal Merge: a) 201 MHz + harmonics; b) Initial Rotation (1)=inital, (2)=drifted, (3)=after rf $_1$; c) 67 MHz + harmonics; d) Merge (3)=from above, (4)=rf $_2$, (5)=drifted).

Earlier studies merged only in the longitudinal dimension (2D). One of these[4] stacked 21 bunches in momentum space, using a negative k FFAG and an arbitrarily good induction linac pulse shape. Its transmission was only 70%, and it was of questionable practicality. A more recent study[5] appears to be more realistic, simpler and more efficient, but it only merged 12 bunches.

A problem with any 2D merge is that the resulting bunches, with large longitudinal, but still small transverse emittances, do not match well into the current designs of 6D cooling. A 6D merge, in contrast, can match very well into a 6D cooling system identical to that used before the merge (see Fig. 5b and discussion in the final section).

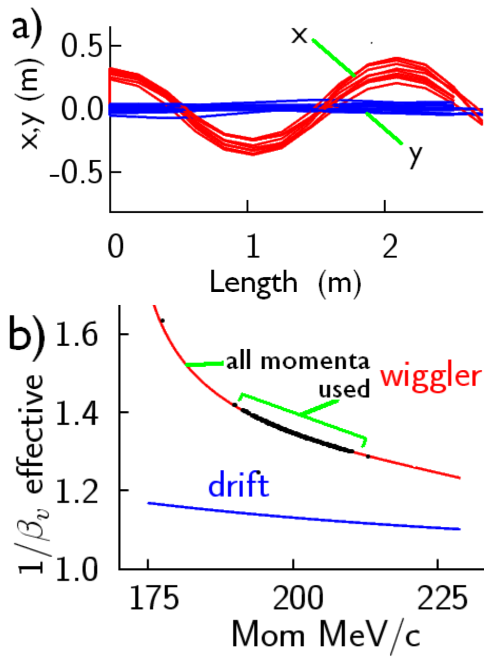

Figure 3: Wiggler: a) Typical tracks x blue & y red, vs. z; b) $\beta_v(\text{effective}) = \frac{dz}{cdt}$ vs. momentum for the wiggler. The red line is theoretical, the black dots those used in this simulation. The lower blue line is for a straight channel.

6D MERGE

Figure 1 shows a schematic of the system. Of the 21 bunches accepted, the initial longitudinal merge combines groups of three into 7 combined bunches. This is followed by a transverse merge that combines the seven into one.

Longitudinal Merge

Two phase rotations are used to achieve the longitudinal merge:

1) The first acts on all 21 bunches simultaneously, lowering their momentum spreads and making them longer. It uses an initial drift of 20 m, followed by a system of 201 MHz rf, plus the 4 higher harmonics given in Table 1. Fig. 2a shows the effective voltage waveform of the 5 rf systems. Fig. 2b shows the phase spaces from a 1D simulation: the points (1) show the three bunches prior to the drift, (2) after the drift, and (3) after the 201 MHz rf system, ending this first rotation.

2) The second rotation acts on each group of three and stacks them above one another in momentum. It uses 67 MHz rf, plus the 5 higher harmonics given in table 1. Fig. 2c shows the effective voltage waveform from the 6 rf frequencies. Fig. 2d shows phase plots from the 1D simulation: points (3) are of the tracks after the first rf, (4) after the 67 MHz rf, and (5) those after a wiggler, ending the second rotation.

Table 1 lists the harmonic rf as a realistic sequence of separate rf sections with lengths L and gradients $\mathcal{E}$, but the simulations used the approximation of superimposed harmonics extending over the full lengths (9.5 & 4.15 m), with average gradients $<\mathcal{E}>$.

Table 1: Parameters of Rf for Longitudinal Merge

	L	f	$<\mathcal{E}>$	$\mathcal{E}$	ϕ
	m	MHz	MV/m	MV/m	deg
Rot 1	3.0	201	1.6	5.06	81
	2.5	403	0.72	2.74	85
	2.0	604	0.45	2.14	87
	1.25	805	0.33	2.51	88
	1.0	1006	0.15	1.42	90
Total	**9.5**				
Rot 2	2.0	67	1.16	3.34	-81
	1.0	134	0.52	2.99	-85
	0.4	201	0.33	4.67	-87
	0.3	268	0.24	4.56	-88
	0.25	335	0.18	4.15	-88
	0.2	403	0.15	4.36	-89
Total	**4.15**				

The wiggler, used in the second rotation, reduces the required length, and thus decays. It has sinusoidal transverse fields with amplitude 1.138 T and wavelength 2.085 m, and a superimposed quadrupole field of 0.65 T/m to give equal focusing in the two directions. Orbits from an ICOOL[6] 3D simulation are shown in figure 3a. Fig. 3b shows the resulting inverse forward velocities vs. momentum, together with the same quantity in a straight transport. The average slope is 4.8 times greater for the wiggler than for a straight channel. This wiggler plays a role similar to that of the Helical Transport Channels used in Neuffer's 2D merge[5].

Transverse Merge

To merge in transverse phase space, the 7 bunches are first separated using a linearly rising kicker field (from -518 G to $+518$ kG in 90 nsec). The kicker is 2.5 m long, 0.9×0.45 m transversely, with maximum stored energy of 1050 J. Following the kicker there is a 5 m 'fanout', followed by seven 30 cm diameter apertures feeding 7 'trombone'[1] transport channels, with lengths set to bring all 7 bunches to a 'funnel' at the same time. Conceptually, the transports could be long bent solenoids, but the longer part of them would probably use quadrupoles.

Fig. 4a shows simulated tracks at the entrance apertures, and Fig. 4b their locations in the funnel at the end of the trombone transport lines. These tracks are then captured into a single channel.

A final longitudinal phase rotation, not yet simulated, is required to match the now combined beams into the next stage of 6D cooling.

Performance

The inital and final emittances are given in Table 2. The transmission without decay is 95% and 86% with decay included.

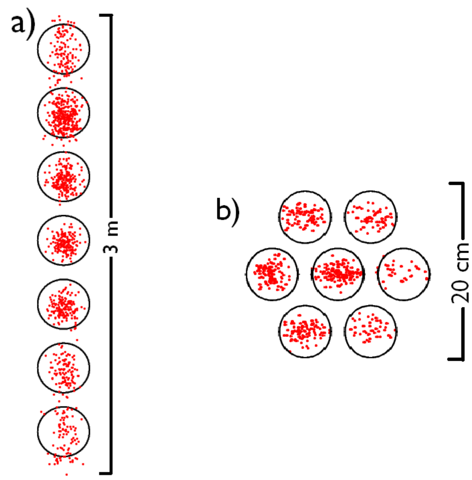

Figure 4: $x-y$ distributions in transverse merging: a) after fan-out for capture into 7 transports; b) at end of transports, before capture as one beam.

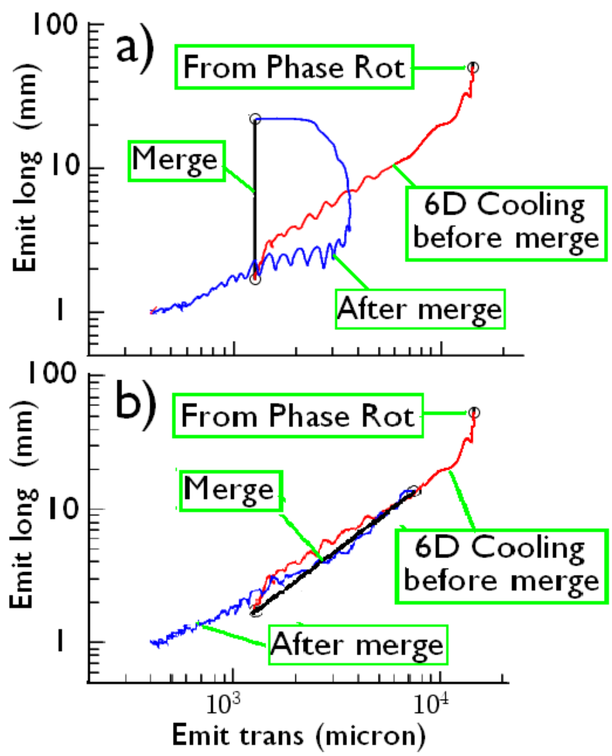

Figure 5: Longitudinal vs transverse emittances from phase rotation to 6D cooling, including bunch merges: a) with 2D merge; b) with 6D merge.

Table 2: Performance

	ϵ_{in} mm	ϵ_{out} mm	n_{merged}	dilution
Transverse x	1.37	7.5	$\sqrt{7}$	2.07
Transverse y	1.37	7.7	$\sqrt{7}$	2.13
Longitudinal z	2.25	13.4	3	1.98

APPLICATION AND PROSPECTS

Having a merge in 6D leads to more efficient use of our current 6D cooling systems, because the merge generates a beam with emittances essentially the same as those earlier in the cooling before the merge, as shown in Fig. 5b, leading to a near perfect match. In contrast, a 2D merge generates emittances that when fed into the same 6D cooling channel, yield (Fig. 5a) an initial rapid heating of the transverse emittance, that can only be re-cooled after the longitudinal emittance has been reduced. This result could be peculiar to the Guggenheim RFOFO 6D cooling[4] used, but may well be more general.

With an average length of approximately 100 m, decay losses are significant, and the dilutions of all emittances ($\approx 2\times$) are uncomfortably high. Much of the longitudinal dilution comes from the finite number of harmonic frequencies used, and the resulting wiggles in the rotated phase spaces. Breaking the first rotation into two parts would greatly reduce this. The length of the system could also be reduced: a 4.2 m wiggler could be used instead of the 20 m initial drift, and the length of the 45 m wiggler could be reduced to 25 m by increasing the wiggler fields by 7%. The use of kickers in two directions, instead of one, would reduce their voltage and stored energy, and would reduce dilution from the energy spreads. If a shorter phase rotation channel allowed the capture of 12, instead of 21 bunches, then a nice option would be to use x and y kickers followed by simple septa to feed a 4 channel trombone.

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