EPICYCLIC TWIN-HELIX IONIZATION COOLING SIMULATIONS*

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Abstract

Parametric-resonance Ionization Cooling (PIC) is proposed as the final 6D cooling stage of a high-luminosity muon collider. For the implementation of PIC, we earlier developed an epicyclic twin-helix channel with correlated behavior of the horizontal and vertical betatron motions and dispersion. We now insert absorber plates with short energy-recovering units located next to them at the appropriate locations in the twin-helix channel. We first demonstrate conventional ionization cooling in such a system with the optics uncorrelated. We then adjust the correlated optics state and induce a parametric resonance to study ionization cooling under the resonant condition.

INTRODUCTION

Combining muon ionization cooling with parametric resonant dynamics should allow much smaller final transverse muon beam sizes than ionization cooling alone [1-2]. Thus, high luminosity would be achieved in a collider with fewer muons. Parametric-resonance Ionization Cooling (PIC) is accomplished by inducing a parametric ½-integer resonance in a muon cooling channel. The beam is then naturally focused with a period of the channel's free oscillations. Absorber plates for ionization cooling together with energy-restoring RF cavities are placed at the beam focal points. At the absorbers, ionization cooling limits the angular spread while the parametric resonance causes a strong reduction of the beam spot size. This resonant cooling scheme should provide equilibrium transverse emittances that are at least an order of magnitude smaller than those achievable with conventional ionization cooling [2].

IONIZATION COOLING SIMULATIONS

Twin-Helix Channel with Correlated Optics

The PIC concept requires [1-4] that, in the cooling channel, the horizontal free oscillation period λ_x is a low-integer multiple of the vertical free oscillation period λ_y while both λ_x and λ_y are low integer multiples of the dispersion oscillation period λ_D. We earlier developed the twin-helix channel [3, 4], which is compatible with the PIC requirements and can provide the following correlated values of λ_x, λ_y and λ_D:

$$\lambda_x = 2\lambda_y = 4\lambda_D . \qquad (1)$$

The twin-helix channel is a superposition of two opposite-helicity equal-period and equal-strength helical dipole harmonics [5, 6] and a straight normal quadrupole. The magnetic field strength at the center of each dipole harmonic B_d and the quadrupole field gradient $\partial B_y/\partial x$ corresponding to the correlated optics are given by

$$\begin{aligned} B_d &= 6.515\cdot 10^{-3}\, p/\lambda \quad [T], \\ \partial B_y/\partial x &= 2.883\cdot 10^{-3}\, p/\lambda^2 \quad [T/m], \end{aligned} \qquad (2)$$

where p is the muon momentum in MeV/c and λ is the helix period in m. The optical properties of the twin-helix channel are well-understood [3, 4]. The periodic orbit amplitude x_{max}, dispersion amplitude $D_{x\,max}$ and horizontal and vertical chromaticities ξ_x and ξ_y corresponding to the correlated optics are given by

$$\begin{aligned} &x_{\max} = 0.121\,\lambda \quad [m], \quad D_{x\max} = 0.196\,\lambda \quad [m], \\ &\xi_x = -0.646, \quad \xi_y = -0.798. \end{aligned} \qquad (3)$$

Equations (2) and (3) are used to obtain the twin helix correlated optics parameters for given muon momentum and helix period. Note that the twin-helix channel was earlier demonstrated [3, 4] to posses a large dynamic aperture even under the correlated optics state.

Simulation Setup

In a system with equally distributed partial cooling decrements of the three emittances and equal damping decrements of the beam size and angle spread at the absorber plates, the dispersion value at the absorber locations is given by [2]

$$D_{abs} = \frac{\Delta E_{abs}}{\partial \Delta E_{abs}/\partial x}\left(2 - \frac{4}{3}\beta^2\right), \qquad (4)$$

where ΔE_{abs} is the energy loss in the absorber, $\partial \Delta E_{abs}/\partial x$ is the energy loss gradient due to the absorber thickness gradient, and β is the muon relativistic factor. For instance, for a 2 cm thick Be wedge with a thickness gradient of 0.3, Eq. (4) gives a dispersion value of 3 cm.

The absorbers are placed every two periods of the twin helix, which is the shortest possible separation corresponding to a half of the period of the horizontal betatron oscillations and one period of the vertical betatron oscillations. The absorbers are centered on the reference trajectory and their axes are aligned with the reference momentum.

In the initial simulations without an induced parametric resonance, we used the parameters given above, where each absorber was immediately followed by a 2 cm long energy recovery region centered on and aligned with the

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absorber axis. First, to avoid having to time RF cavities, we modeled energy recovery using regions of static electric field. We later replaced them with same-length RF cavities. The strength of the static electric field and the voltage gradient of the RF cavities were adjusted to completely compensate energy loss in the absorbers. The length of the energy recovery region was chosen to be short compared to the helix period in order to reduce the area where the muon momentum is different from its reference value and thus minimize perturbation of the reference orbit. Having short RF cavities also allows one to decouple from transit time effects. However, about 6.5 MeV energy loss from a 200 MeV/c muon in 2 cm of Be can be compensated with a realistic RF cavity. Optimizing the length of the cooling channel is not one of the goals of this conceptual study but, in a realistic channel, as large a fraction of the channel space as practically possible would be filled with absorbers and RF cavities.

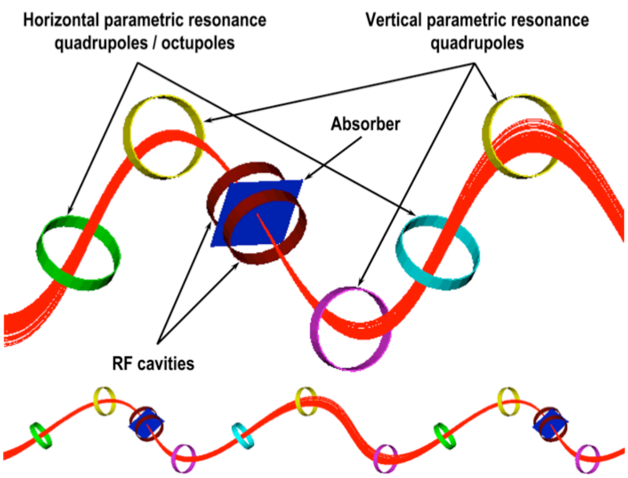

Figure 1: G4beamline event display showing an energy absorber, RF cavities, resonance driving quads, and octupole aberration correction magnets.

The half-integer parametric resonances in the later simulations were induced in the two planes by two sets of 2 cm long quadrupoles placed with half the periodicity of the horizontal and vertical betatron oscillations every $\lambda_x/2 = 2\lambda$ and $\lambda_y/2 = \lambda$ along the channel. The horizontal and vertical resonance driving lenses were positioned at the $\lambda_x/8$ and $\lambda_y/8$ distances in front of the focal point at the absorber for the x and y resonances, respectively. Each set consisted of alternating positive and negative polarity quadrupoles to compensate their betatron tune shift. In order to further reduce the energy modulation caused by the absorbers and energy recovery, each absorber was closely surrounded by two RF cavities. We also reduced the absorber thickness to 2 mm to first better understand the resonant dynamics. The horizontal resonance driving lenses included an octupole field harmonic for the study of non-linear effect compensation. The resulting system configuration is illustrated in Fig. 1.

Study without Induced Parametric Resonance

We used the GEANT-based G4beamline program [7] to track 10^3 200 MeV/c muons through 500 helix periods of the cooling channel containing the wedge absorbers and static electric field regions. The resulting phase-space diagrams at different distances along the channel are shown in Fig. 2. Note that introduction of the absorbers and energy recovery changes the periodic orbit and periodic momentum. This leads to a mismatch between the initial beam and the new periodic trajectory causing the initial increase of the horizontal phase space in Fig. 2 due to filamentation.

We next tuned the periodic momentum to correspond to the correlated optics value. Both with the static field and RF and at both 200 MeV/c and 250 MeV/c, we observed stable oscillations with two rather than one attractor points in the horizontal phase space. Our interpretation of this effect is that it is a manifestation of a parasitic parametric resonance caused by energy kicks from the absorbers and electric field every $\lambda_x/2$.

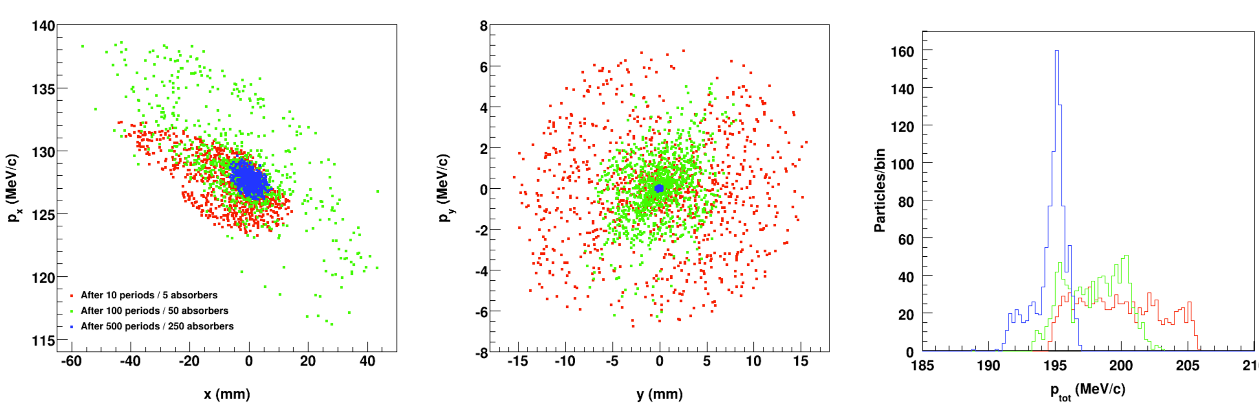

Figure 2: Horizontal (left) and vertical (middle) phase-space distributions and momentum distribution histogram (right) at different distances along the cooling channel containing the wedge absorbers and static electric field regions. The initial beam had uniform spatial $\sigma_x = \sigma_y = \pm 2\ mm$, angular $\theta_x = \theta_y = \pm 50\ mrad$ and momentum $\Delta p/p = \pm 2.7\%$ distributions and was launched along the reference orbit. The stochastic processes and muon decay were off.

Study with Induced Parametric Resonance

We induced a parametric resonance in the horizontal plane using the resonance driving quadrupole lenses discussed above. Figure 3 illustrates the particle horizontal phase-space trajectory at the absorber. The trajectory demonstrates the expected hyperbolic behavior corresponding to a half-integer resonance where the angle increases as the amplitude shrinks. The hyperbolic dependence becomes distorted for large amplitudes and angles due to aberrations. We introduced an octupole magnetic field harmonic in the horizontal resonance driving lenses to study its effect on the non-linear dynamics as shown in Fig. 3.

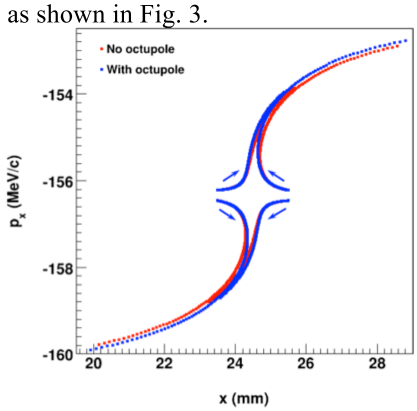

Figure 3: G4beamline simulations of half-integer parametric resonant dynamics in horizontal phase space without absorbers and RF. A first attempt to adjust the distortion of the separatrix using octupoles is shown.

Note that introduction of the parametric resonance in the horizontal plane causes an instability in the vertical plane because the resonance driving quadrupoles are placed with the periodicity of the vertical betatron oscillations. We believe that it can be either compensated by appropriate optics or the parametric lens system can be modified so that a single quadrupole set drives the required parametric resonances in both planes taking advantage of the correlated optics design.

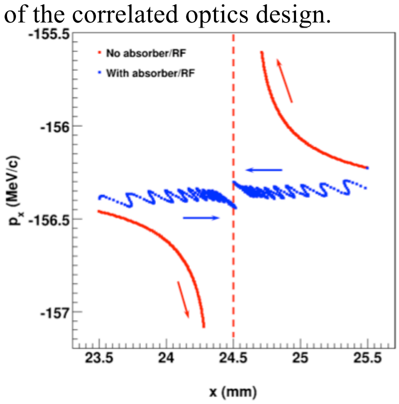

Figure 4: G4beamline simulations of half-integer parametric resonant dynamics in horizontal phase space without and with absorbers and RF. Ionization cooling limits the angular spread while the resonance reduces the beam size. Stochastic processes are not included.

Thin 2 mm absorber plates closely surrounded by two energy-restoring RF cavities were then installed at the focal points. The RF cavities were synchronized using an unperturbed reference particle, which ignored ionization energy loss and electric fields. The synchronous phase was set at 30°. The average beam energy in this case is determined by the timing of the RF cavities rather than their voltage gradient and is equal to the reference particle energy. The stabilizing effect of the absorbers and RF is illustrated in Fig. 4. The absorbers and RF prevent the angle from growing while the size shrinks.

We next tracked a beam of fifty 250 MeV/c muons in the horizontal plane under the parametric resonant condition through 1000 helix periods of the cooling channel containing absorber plates and RF cavities. Figure 5 shows the resulting phase-space diagrams at the absorbers at different distances along the channel. Note that the beam evolves into a state with a large angular spread and a small size, which is consistent with the parametric resonant picture.

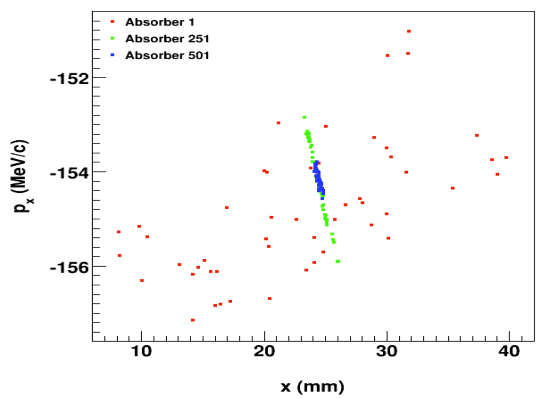

Figure 5: Horizontal phase-space distribution in the middle of the absorbers at different distances along the cooling channel with parametric resonance. The stochastic processes and muon decay were off.

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