EPIC MUON COOLING SIMULATIONS USING COSY INFINITY*

James Anthony Maloney, Bela Erdelyi (Northern Illinois University, DeKalb, Illinois), Alex Bogacz, Yaroslav Serg Derbenev (JLAB, Newport News, Virginia), Andrei Afanasev, Rolland Paul Johnson (Muons, Inc, Batavia), Vasiliy Morozov (ODU, Norfolk, Virginia).

Abstract

Next generation magnet systems needed for cooling channels in both neutrino factories and muon colliders will be innovative and complicated. Designing, simulating and optimizing these systems is a challenge. Using COSY INFINITY, a differential algebra-based code, to simulate complicated elements can allow the computation and correction of a variety of higher order effects, such as spherical and chromatic aberrations, that are difficult to address with other simulation tools. As an example, a helical dipole magnet has been implemented and simulated, and the performance of an epicyclic parametric ionization cooling system for muons is studied and compared to simulations made using G4Beamline, a GEANT4 toolkit.

INTRODUCTION

One of the key technical challenges to proposed neutrino factories and muon colliders is the design and implementation of an effective beam cooling system [1, 2]. Muons produced for such machines have an extremely large phase space volume requiring cooling and transporting within a relatively short decay time. The specifications for a high-luminosity muon collider, in particular, are challenging. A variety of competing schemes have been proposed to address this challenge [2], often involving complicated magnetic fields which are not easily simulated. Furthermore, these systems are often more sensitive to the effects of fringe fields and higher order aberrations within the beam. The complicated geometries of the next-generation magnet systems used in these cooling channels make it difficult to address these effects using conventional methods. COSY INFINITY [3] can be used to compute and help correct the higher order effects in these next-generation magnet systems.

SIMULATION OF THE EPICYCLIC TWIN HELIX CHANNEL IN COSY INFINITY

The twin helix channel model [4] for Epicyclic Parametric Resonance Ionization Cooling (EPIC) [5] presents a promising proposal for the cooling challenges of a muon collider. Such a channel can be simulated and studied with COSY INFINITY (COSY).

Epicyclic Parametric Resonance Ionization Cooling

Epicyclic Parametric Resonance Ionization Cooling (EPIC) theory uses a combination of induced parametric resonances to cause periodic beam size reductions. Ionization cooling is then achieved with absorbing wedges. Emittance exchange allows for longitudinal cooling while energy restoring RF is used to maintain particle momentum. The fundamentals for such a design are illustrated in Figure 1. A varying dispersion function (in blue) necessary for aberration correction is related to the superimposed harmonic magnetic fields (horizontal plane in red and two fields for the vertical plane in green) via correlated optics.

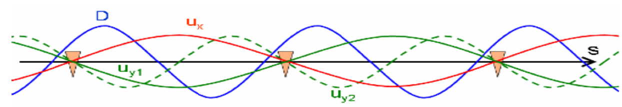

Figure 1: Example of EPIC layout.

The Twin Helix Channel

One interesting case for trying to implement the EPIC theory is the twin helix channel [4]. This channel utilizes two helical harmonics with equal scalar field components and equal but opposite wave numbers.

The helical harmonics, in the horizontal plane, are given by:

$$B_x = -\left(\frac{2}{nk}\right)^{n-1} b_n[I_{n-1}(nkx) + I_{n+1}(nkx)]sin(nkz) \quad (1)$$

$$B_y = \left(\frac{2}{nk}\right)^{n-1} b_n[I_{n-1}(nkx) - I_{n+1}(nkx)]cos(nkz) \quad (2)$$

$$B_z = -2\left(\frac{2}{nk}\right)^{n-1} b_n I_n(nkx)cos(nkz) \quad (3)$$

Where n = 1, and 2 represent the dipole and quadrupole terms, b_n is the n-1 order derivative of the vertical field component, and I_n are modified Bessel functions of the 1st kind. When the two helical harmonics with identical moments (b_{n1}=b_{n2}), and equal but opposite wave numbers (k_1=-k_2) are superimposed, the resulting compound field leaves only a vertical field component in the horizontal plane:

$$B_x = 0 \quad (4)$$

$$B_y = 2\left(\frac{2}{nk}\right)^{n-1} b_n[I_{n-1}(nkx) - I_{n+1}(nkx)]cos(nkz) \quad (5)$$

$$B_z = 0 \quad (6)$$

The result is to create a periodic orbit in the horizontal plane, and regions of stable transverse motion in both planes. Absorbing wedges followed by energy recovering cells are inserted at points of low dispersion to create effective 6D ionization cooling.

Implementation of the Twin Helix in COSY INFINITY

COSY INFINITY (COSY) is a DA based simulation code that allows the calculation of transfer maps of beam systems to arbitrary order. [3] For systems where a

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#physics_maloney@yahoo.com

reference particle that orbits in a plane, such as involved in the twin helix channel previously mentioned, implementation in COSY can be accomplished by modifying the existing base code to include a new helical magnet element. The new magnetic element is defined by specifying the magnetic field in the x-z plane, and allowing COSY to calculate the potential in all space from Maxwell's equation. The element allows the user to specify parameters for the length of the cell, the wave number for the harmonic fields, the half gap of the magnet, and the dipole and quadrupole field component at the center of the field.

COSY includes a customizable absorbing wedge element that is parameterized by the Bethe-Bloch formula. This element is placed inside the helix along with a DC electric field for energy recovery. Ultimately, the DC field can be replaced with an RF cell once a time structure has been fixed. The result is a complete simulation of a twin helix with both dipole and quadrupole moments, absorbing wedge and energy recovering cell.

COSY can now be used to calculate the transfer and aberration maps for the element for any number of orbital periods. This allows for statistical studies to tune system performance, examination of optical aberrations, and error analysis of the system.

EMITTANCE STUDIES OF THE TWIN HELIX IN COSY INFINITY

Using studies of the twin helix in a modified version of G4Beamline (a GEANT4 toolkit) [6] as a baseline for comparison, the COSY simulation examine the performance of the basic twin helix cell. Table 1 summarizes the parameters utilized in the standard cell. In this configuration the standard cell is twice the helix period and the absorber wedge is placed where the value of the dispersion function is 3cm, and immediately followed by the energy restoring DC electric field.

Table 1: Twin Helix Parameters

Parameter	Value
Reference momentum	200 MeV/c
Helix period	1 m
Dipole Strength	1.3031406 T
Quadrupole Strength	0.57640625 T/m
Absorber Material	Beryllium
Absorber Thickness	2 cm
Absorber Gradient	.3 (symmetric at surfaces)
DC Field Length	2 cm
DC Field Strength	322.806156 MV/m

As a consistency check, a sample distribution of particles was sent through a single cell without either the absorbing wedge or DC field included. Figure 2 demonstrates how the cell manipulates horizontal phase space. The transfer matrix, extracted using COSY, has a determinant of 1, verifying that there is no change in emittance, as expected in absence of any wedge absorber.

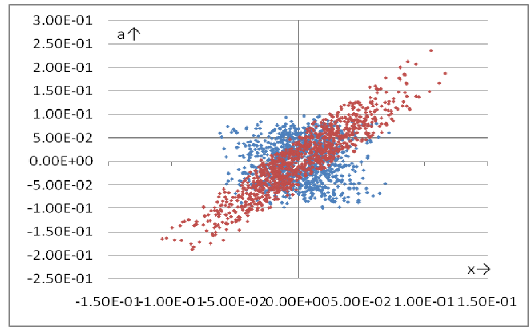

Figure 2: Initial (blue) and final (red) distribution in horizontal phase space for single Twin Helix cell without absorbing wedge.

Study of Emittance Change from Transfer Matrices in Twin Helix Systems

COSY generates an array containing an initial distribution of particles, represented as vectors with particle optical coordinates of the form:

$$\{r_k\} = (x_k, a_k, y_k, b_k, t_k, \delta_k) \quad (7)$$

Where a and b are the dimensionless horizontal and vertical momentum, t is time of flight and δ is the change in total energy of the particle. COSY calculates the function, *M*, known the transfer map (or taylor map) for the system, which described the evolution of particles in the system. The linear terms of the transfer map function comprise the matrix (transfer matrix) of the system. [7]

$$\{\vec{r_f}\} = M\{\vec{r_i}\} = \begin{pmatrix} (x|x) & \cdots & (x|\delta) \\ \vdots & \ddots & \vdots \\ (\delta|x) & \cdots & \{\delta|\delta\} \end{pmatrix}\{\vec{r_i}\} \quad (8)$$

Using the transfer matrix of the system and a randomly generated array of particles, emittance of the twin helix system can also be evaluated. The transverse (4D) and 6D emittance can be calculated from the determinant of the covariance matrix derived from a distribution of particles. [8]

$$\epsilon^{4D} = \sqrt[4]{det(\mathcal{M}_{ij})_{4D}} \qquad \epsilon^{6D} = \sqrt[6]{det(\mathcal{M}_{ij})_{6D}} \quad (9)$$

The covariance matrix is a function of the particle distribution, and the transfer matrix relates changes in the initial and final beam emittance by the following relation:

$$\epsilon_f = \int_{beam} dx_f dp_{x_f} \quad (10)$$

$$\epsilon_f = \int_{beam} |\det M| dx_i dp_{x_i} = |detM|\epsilon_i \quad (11)$$

$$|detM| = \frac{\epsilon_f}{\epsilon_i} \quad (12)$$

The determinants of the 4D and 6D components for this matrix, and the transfer matrices of systems with 10 and 50 continuous cells with wedges and absorbers, is provided in Table 2.

Table 2: Transfer Matrix Emittance Calculations

System	4D	6D
1 cell	0.10056	0.0432475
10 cells	0.04156	0.00281985
50 cells	4.69456E-05	6.75281E-07

Study of Particle Distributions in Twin Helix Systems

An array containing the vectors of an initial particle distribution was composed with the 3rd order transfer map of the twin helix system. The emittance of the distribution is given as

$$\varepsilon_{tr,rms} = \sqrt{\langle x^2\rangle\langle a^2\rangle - \langle xa\rangle^2}\sqrt{\langle y^2\rangle\langle b^2\rangle - \langle yb\rangle^2} \quad (12)$$

The emittance calculations for both the initial and final vector arrays are summarized in Table 3, and can be analyzed to see change as the beam is transported through the twin helix channel. Figures 3, 4 and 5 show the horizontal phase space of the initial and final distributions.

Table 3: Particle Distribution Emittance Calculations

System	ϵ(4D)
Initial	9.96835E-07
1cell	1.00325E-07
10 cells	1.8326E-08
50 cells	9.62192E-10

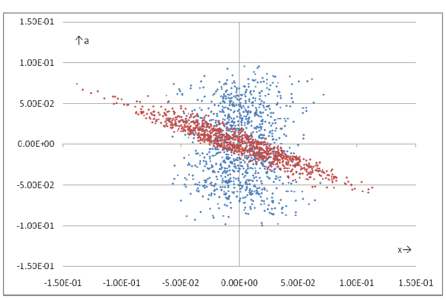

Figure 3 Initial (blue) and final (red) distribution in horizontal phase space for 1 Twin Helix cell.

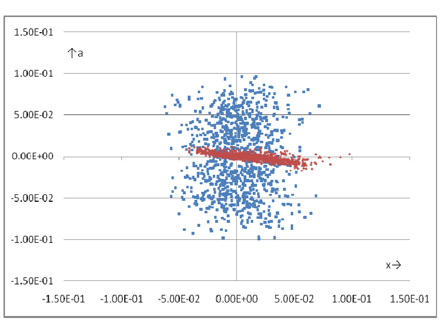

Figure 4: Initial (blue) and final (red) distribution in horizontal phase space for a series of 10 Twin Helix cells.

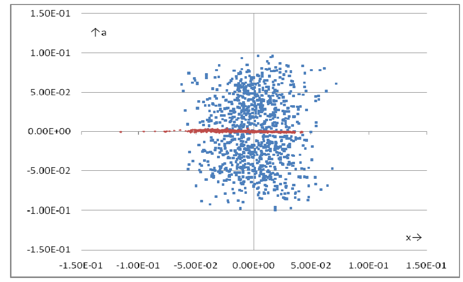

Figure 5: Initial (blue) and final (red) distribution in horizontal phase space for a series of 50 Twin Helix cells.

CONCLUSION

The twin helix channel model for an EPIC muon cooling system has been implemented in COSY, and preliminary studies offer supporting evidence of cooling. Using the same structure, higher order harmonic multipoles (e.g. sextupole and octupole) will need to be added to correct existing chromatic and spherical aberrations in the system. Using the transfer and aberration map output from COSY will aid in the design of such corrective optical systems. Furthermore, the methods developed in implementing this innovative magnet system can be used to simulate similarly novel magnet systems in other proposed muon cooling channels being developed for a next generation neutrino factory or muon collider.

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