EFFECTS OF E-BEAM PARAMETERS ON COHERENT ELECTRON COOLING

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Abstract

Coherent Electron Cooling (CeC) requires detailed control of the phase between the hadron an the FEL-amplified wave packet. This phase depends on local electron beam parameters such as the energy spread and the peak current. In this paper, we examine the effects of local density variations on the cooling rates for CeC.

INTRODUCTION

Coherent Electron Cooling (CeC) [1] is a new concept in intense, high energy hadron beam cooling, in which the Debye screened charge perturbation calculated in [2] is used to seed a high-gain free electron laser (FEL). Using delays to give the perturbing hadron an energy-dependent longitudinal displacement relative to its frequency modulated charge perturbation, the hadron receives an energy-dependent kick which reduces its energy variation from the design energy.

The equations of motion in [1] assume that the electron bunch is the same physical size as the hadron bunch, and has a homogeneous charge density across the entire bunch. In practice, the electron bunches will be much shorter than the hadron bunch, and this local spacial inhomogeneity in the charge distribution will alter the gain length of the FEL, resulting in both a change in the amplification of the initial signal and a phase shift. In this paper we consider these inhomogeneity effects, determining cooling equations for bunched beam CeC consistent with these effects and determining thresholds for the cooling parameters.

EQUATIONS OF MOTION

To analytically study the effects of the bunch inhomogeneity, we consider a set of equations for which the energy spread of the hadron bunch is small compared to the size of the RF buckets, and introduce a phenomenological cooling term into the hamiltonian equations of motion for the RF bucket harmonic oscillator, to obtain

$$\epsilon' = -\xi(\phi)\sin(k_r D_\ell \epsilon + \Psi(\phi)) + V_0\phi \tag{1a}$$

$$\phi' = -\eta\,\epsilon \tag{1b}$$

where ξ is the cooling parameter and Ψ is a phase shift due to the FEL. For a one-dimensional FEL the phase shift is $\hat{z}/2$ where $\hat{z} = z/L_G^0$ is the normalized length of the FEL. All effects of inhomogeneity arise from the variation of the gain length L_G as a function of density. It is now necessary to determine the scaling of each of the individual parameters with the density of the electron bunch. From here on, we assume that

$$n_0(\phi) = N_0 f(\phi) \tag{2}$$

where $f(0) = 1$ and $\int d\phi\, f(\phi) = 1$.

From existing theoretical treatments of the FEL problem using an initial phase space perturbation ([3] - [5]) we know that the resulting current and phase space density of the FEL amplifier is proportional to ρ, the Pierce parameter, which in turn is proportional to $n_0^{1/3}$. As the cooling kick is proportional to the longitudinal space charge of the FEL modulated density perturbation, it is reasonable to suspect that $\xi(\phi) = \xi_0 f(\phi)$ where ξ_0 is the cooling parameter at the middle of the electron bunch. From one-dimensional FEL theory ([4], [5]) the gain length scales as $L_G \propto n_0^{-1/3}$, and so we can rewrite $\hat{z} = f(\phi)\hat{z}^0$ where $\hat{z}^0$ is the normalized FEL undulator length in units of the gain length as determined by the electron bunch parameters at the center of the bunch (*i.e.* peak current).

Because all scaling of density goes with $n_0^{1/3}$, it is convenient to use for $f(\phi)$ a bounded support density given by, for example,

$$f(\phi) = \left(1 - \frac{\phi^2}{\phi_0^2}\right)^3 \Theta(1 - |\phi/\phi_0|) \tag{3}$$

where Θ is the step function. This removes the pesky fractional powers and provides a reasonable physical description of an electron bunch.

In practice, the e-bunch length will be much shorter than the hadron bunch length, so the hadron bunch length must be painted by a series of e-bunches many times in a single synchrotron oscillation to get effective cooling. We therefore take the synchrotron angular coordinate of the e-bunch to be given by $\phi \mapsto \phi + f(\omega_0 t)$ where a hierarchy of time scales is given by $\tau_c^{-1} \ll \omega_0 \ll \omega_s$. where τ_c is the cooling time and $\omega_s^2 = V_0\eta$ is the synchrotron frequency.

Defining $\tau = \omega_s t$, $\hat{\epsilon} = k_r D_\ell \epsilon$ and $T = \hat{\xi}_0 t$, and averaging over a single ω_s period, we employ the two-timing method [6] to remove the rapid synchrotron oscillations and obtain the envelope equation from the definitions of the envelope and phase functions

$$\hat{\epsilon} = A(T)\sin(\tau + \Psi(T)) \tag{4a}$$

$$\phi = A(T)\hat{\eta}\cos(\tau + \Psi(T)) \tag{4b}$$

gives an envelope equation from the integral

$$\frac{dA}{dT} = -\int_{-\pi}^{\pi} d\psi\ \sin\psi\ f(\hat{\epsilon}, \theta) \tag{5}$$

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where

$$f(\hat{\epsilon},\theta) = \left(1-\theta^2/\phi_0^2\right)\exp\left\{-\frac{\theta^2}{\phi_0^2}\hat{z}_0\right\}\times \left[\sin(\hat{\epsilon})\cos\left(\frac{\theta^2}{\sqrt{3}\phi_0^2}\hat{z}_0\right)+\cos(\hat{\epsilon})\sin\left(\frac{\theta^2}{\sqrt{3}\phi_0^2}\hat{z}_0\right)\right] \quad (6)$$

Carrying out this integral and dropping rapidly oscillating terms yields the envelope equation

$$\frac{dA}{dT} \approx -\exp\left\{-\frac{A^2\hat{\eta}^2}{2\phi_0^2}\hat{z}_0\right\} J_0\left(\frac{A^2\hat{\eta}^2}{2\phi_0^2}\hat{z}_0\right) J_0\left(\frac{A^2\hat{\eta}^2}{2\sqrt{3}\phi_0^2}\hat{z}_0\right) J_1(A)\times \left[\cos\left(\frac{A^2\hat{\eta}^2}{2\sqrt{3}\phi_0^2}\hat{z}_0\right)\cos\left(\frac{f^2\hat{z}_0}{\sqrt{3}\phi_0^2}\right) - \sin\left(\frac{A^2\hat{\eta}^2}{2\sqrt{3}\phi_0^2}\hat{z}_0\right)\sin\left(\frac{f^2\hat{z}_0}{\sqrt{3}\phi_0^2}\right)\right]\times J_0\left(\frac{2A\hat{\eta}f\hat{z}_0}{\sqrt{3}\phi_0^2}\right)\left(1-f^2/\phi_0^2\right)\exp\left\{-\frac{f^2}{\phi_0^2}\hat{z}_0\right\} \quad (7)$$

The origins of each term are insightful and intuitive. The gaussian envelope with the first Bessel function corresponds to the finite size of the individual electron bunches, while the second Bessel function that appears comes from the changing phase shift due to the electron bunch inhomogeneity. $J_1(A)$ comes directly from the cooling term, $\sin(\hat{\epsilon})$, while the painting terms arise from a combination of the gaussian envelope and phase shifts during the painting. Assuming a single painting is given by $f = u_0 T$ for $T \in (-T_p, T_p)$, and averaging over a single painting with $u_0 T_p/\phi_0 \gg 1$ gives the approximate cooling equation as

$$\frac{dA}{dT} \approx -\exp\left\{-\frac{A^2\hat{\eta}^2}{2\phi_0^2}\hat{z}_0\right\} J_0\left(\frac{A^2\hat{\eta}^2}{2\phi_0^2}\hat{z}_0\right) J_0\left(\frac{A^2\hat{\eta}^2}{2\sqrt{3}\phi_0^2}\hat{z}_0\right) J_1(A)\times \frac{\phi_0}{u_0 T_p\sqrt{\hat{z}_0}}\left[(1.47+0.447\hat{z}_0^{-1})\cos\left(\frac{A^2\hat{\eta}^2}{2\sqrt{3}\phi_0^2}\hat{z}_0\right) - (0.31+0.31\hat{z}_0^{-1})\sin\left(\frac{A^2\hat{\eta}^2}{2\sqrt{3}\phi_0^2}\hat{z}_0\right)\right] \quad (8)$$

where the numerical terms arise from taking dimensionless integrals over the single painting. In practice, the numerical integrations converge within $u_0 T_p/\phi_0 \sim 2$, so in this case $\infty \approx 2$.

NUMERICAL RESULTS

We used the anticipated parameters of operation for the Proof of Principle configuration for CeC at RHIC (see Table [1]) to develop a set of solutions for the cooling. The numerical solution gives the envelope function as a function of time, normalized to the cooling rate. This gives an amplitude-dependent cooling rate given in Fig. (1).

Table 1: Proposed parameters of a Proof of Principle CeC system.

Ion energy	203 MeV/nucleon
Dispersion Parameter ($k_r D_\ell$)	≈ 1
Ion bunch length (σ_i)	5 mm
Electron bunch length (σ_e)	3.3 mm
Phase slip (η)	.0014
RF frequency (ω_r)	$28.15/2\pi$ MHz
FEL Normalized Length (z/L_G)	~ 3

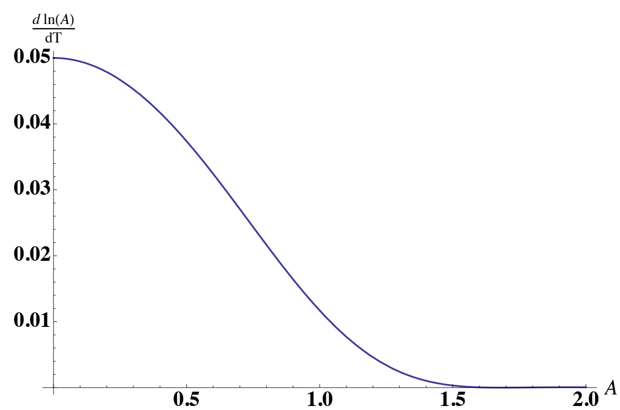

Figure 1: Cooling rate for the given set of parameters with painting at width $3\sigma_\imath$

We take for $L = 3\times\sigma_i$ for the purposes of calculating the cooling rate, thereby covering almost all of the ions in the bunch in a single sweep. Using this set of parameters, a numerical solution of equation (8) is possible and given in Fig. (3).

As can clearly be seen, cooling becomes less efficacious at larger amplitudes in this painting scheme, although different painting schemes may result in different cooling rates, and this is a subject for study in future work.

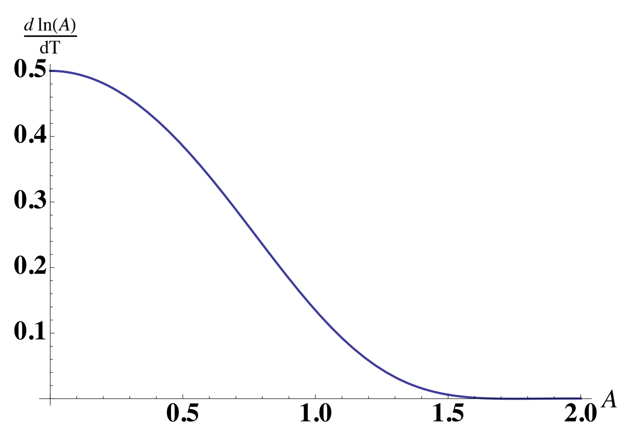

Figure 2: Cooling rate for the given set of parameters with no painting.

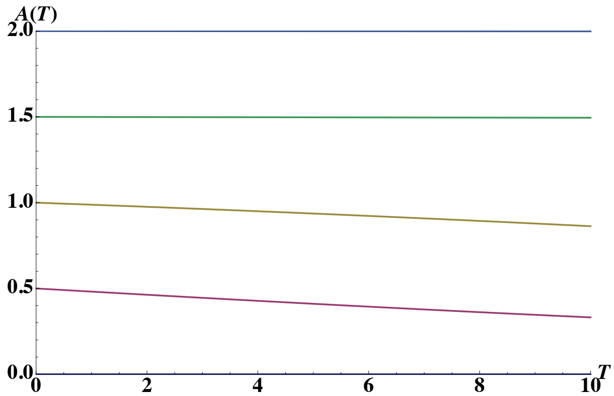

Figure 3: Envelope functions for the given set of parameters with painting at width $3\sigma_\imath$

CONCLUSION

We have presented a dynamical description of Coherent Electron Cooling which incorporates the electron bunch inhomogeneity, as well as the scanning action of the CeC system to effectively cool an ion bunch which is longer than the electron bunches being considered. This leads to an approximate envelope equation which contains the inhomogeneities and painting parameters of the electron bunches as nonlinearities affecting the cooling decrement parameter. Numerical results were obtained for these equations.

The cooling rate is best optimized by analyzing the interplay of the parameter $1.47\phi_0/u_0T_p\sqrt{\tilde{z}_0}$ and optimizing that with the interplay with the effects of ϕ_0 within the gaussian envelope. Furthermore, it is worthwhile for future consideration to look at alternative painting schemes and what effects they may have on the functional forms. Finally, we note that the specifics of the averaging break down if $u_0T_p/\phi_0 < 2$ and in this regime a different approximation would be necessary that would yield analytically different results. This would correspond to having large overlap between each painting sweep. Further investigation of this regime is warranted in future work.

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