MODELING A 10 GeV LASER-PLASMA ACCELERATOR WITH INF&RNO*

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Abstract

The numerical modeling code INF&RNO (INtegrated Fluid & paRticle simulatioN cOde, pronounced "inferno") is an efficient 2D cylindrical code to model the interaction of a short laser pulse with an underdense plasma. The code is based on an envelope model for the laser while either a particle-in-cell (PIC) or a fluid description can be used for the plasma. The effect of the laser pulse on the plasma is modeled with the time-averaged ponderomotive force. These and other features allow for a significant speedup compared to standard full PIC simulations while still retaining physical fidelity. A boosted Lorentz frame (BLF) modeling capability has been introduced within the fluid framework enhancing the performance of the code. An example of a 10 GeV laser-plasma accelerator modeled using INF&RNO in the BLF is presented.

INTRODUCTION

Reliable numerical modeling in 3D of a laser-plasma accelerator (LPA) [1], where a short and intense laser pulse interacts with an underdense plasma over distances ranging from a few millimeters/centimeters (yielding $\sim 0.1/1$ GeV electron energy [2, 3]) up to a meter (as in the Berkeley Lab Laser Accelerator project (BELLA) [4] where ~ 10 GeV electrons are expected), is a challenging task. A 3D "full" (*i.e.*, where we take into account the fastest time scale represented by the oscillations of the laser field) PIC simulation requires $10^4 - 10^5$ CPU hours in today's supercomputers for a millimeter-scale plasma and $\sim 10^6$ CPU hours for a centimeter-scale plasma. The simulation of a meter-scale plasma as required by a typical BELLA-like run necessitates tens of millions of CPU hours and so becomes impractical with standard simulation tools [5]. However, simulations are required since the physics involved in the laser-plasma interaction is highly nonlinear and, consequently, analytical solutions are lacking. Numerical modeling plays a central role in understanding of the physics. Two solutions have been proposed to overcome this limitation and allow for the simulation of multi-GeV LPA stages: *i.* use reduced models; *ii.* run the simulation in an optimal BLF [6] instead of in the laboratory frame. Codes based on reduced models allow for a significant speedup compared to full PIC simulations either because of dimensionality reduction (*e.g.*, 2D cylindrical instead of full 3D cartesian) or because of approximations in the physical description of the system (*e.g.*, quasi-static instead of fully dynamic plasma response, ponderomotive approximation instead of full Lorenz force, etc.). Even if they may lack important elements of the physics (*e.g.*, a quasi-static code can not describe self-injection), their use has been proven to be successful in several relevant scenarios [7, 8, 9]. The use of a BLF has been strongly pursued by several groups [10, 11, 12, 13]. The advantage of running a simulation in a BLF relies on the fact that, if backward propagating waves (*e.g.*, Raman backscattering) can be neglected, and this is usually true given the phenomenology of LPAs, then it has been shown [6] that the unbalance between the maximum and minimum physical scales involved in a simulation, which contribute to set the computational complexity of the problem, is not invariant under Lorentz transformation. It turns out that, in general, the laboratory frame is not the optimal choice to run the simulation, while running it in a boosted frame can considerably reduce the scale unbalance, shortening (also by several orders of magnitude) the simulation length. The INF&RNO computational framework [15], currently under development at LBNL, is a 2D cylindrical (r-z) code that adopts an envelope model for the laser pulse and makes use of the ponderomotive force approximation to describe the interaction of the laser pulse with the plasma. The plasma can be modeled using either a PIC or a fluid description. Both PIC and fluid modalities are integrated in the same computational framework allowing for staged simulations (*e.g.*, PIC-mode for injection and fluid-mode for acceleration). For the PIC part, a dynamical resampling of the phase space distribution is implemented in order to reduce on-axis noise. It is also possible to load and track self consistently externally injected bunches. The code has been validated and benchmarked against analytical solutions and other codes (*e.g.*, fully 3D PIC [11, 14]), details can be found in [15]. Recently a BLF modeling capability has been introduced within the (noiseless) fluid framework. The employment of the BLF by the user is almost transparent since a set of "wrapper" routines take care of all the necessary data transformations between the laboratory frame and the BLF, where the simulation is performed, during initialization and output dumps. The ultimate goal of the INF&RNO project is obtaining a fast code suitable for modeling the relevant features of a LPA where, for a given problem, it is possible to switch between several physical descriptions/levels of approximations in order to clearly identify in each situation the relevant physics involved. In these proceedings we provide an overview of the new features of the INF&RNO framework focusing in particular on the implementation and testing of the BLF modeling capability. An application of the code to a 10 GeV LPA is also presented.

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FEATURES OF THE CODE

INF&RNO is a 2D-cylindrical (r-z) code which adopts non-dimensional, "comoving" variables defined as $\xi = k_p(z-ct)$ (longitudinal) and $\rho = k_p r$ (transverse), where $k_p = \omega_p/c$, ω_p is the plasma frequency corresponding to the chosen reference density n_0, and c is the speed of light. The time is also rescaled with $1/\omega_p$, that is $\tau = \omega_p t$. The laser pulse is described using an envelope model [1]. Denoting by $a_\perp = eA_\perp/mc^2$ the normalized vector potential of the laser, the (slowly varying) envelope $\hat{a}$ is defined by $a_\perp = \frac{\hat{a}(\xi,\rho)}{2} e^{i(k_0/k_p)\xi} + c.c.$ The envelope evolves according to

$$\left(\nabla_\perp^2 + 2i\frac{k_0}{k_p}\frac{\partial}{\partial\tau} + 2\frac{\partial^2}{\partial\xi\partial\tau} - \frac{\partial^2}{\partial\tau^2}\right)\hat{a} = \frac{\delta}{\overline{\gamma}}\hat{a}\,, \quad (1)$$

where $2\pi/k_0$ is the central laser wavelength, $\delta = n/n_0$ is the (normalized) plasma density and $\overline{\gamma}$ is the relativistic factor associated with the local plasma fluid velocity. Typically, the second order time derivative can be neglected for forward going light waves but must be retained when running simulations in the BLF. Keeping this term ensures the exact Lorentz invariance of the equation. The fully electromagnetic wakefield, described by the fields E_z, E_r, B_ϕ, evolves according to Ampère-Maxwell laws. The background plasma can be modeled using either a PIC or a fluid description while for external injected bunches only the PIC description is currently available. Laser-matter coupling is described via the ponderomotive approximation [1]. The PIC and fluid modalities are integrated in the same computational framework, enabling an easy switch from one description to the other (combined simulations). Concerning numerical aspects, all the fields are discretized into the same 2D mesh (no staggering is adopted). Longitudinal derivatives are computed using a second-order finite difference upwind scheme: $(\partial_\xi f)_{i,j} = (-3f_{i,j} + 4f_{i+1,j} - f_{i+2,j})/(2\Delta_\xi)$, where $f_{i,j}$ is the field value at the (i,j) node and Δ_ξ the longitudinal cell size. Radial derivatives are computed using a standard centered second-order accurate scheme. Second and fourth order Runge-Kutta integrators (RK2/RK4) are available for fluid quantities and wakefield evolution while plasma particles and externally injected bunches can be pushed with either RK4 or the standard Boris pusher [5]. The laser envelope is updated in time solving Eq. 1 using a second-order Crank-Nicolson scheme. Compact low-pass filters [17] are available for field and current smoothing. In the PIC modality force interpolation and charge/current deposition are performed using quadratic shape functions. The user has large freedom in loading numerical particles over the computational domain (the numerical particle distribution is controlled by a simple user-defined routine) and this freedom can be used to selectively provide a better sampling of the plasma phase space distribution within the dynamically interesting zones without greatly increasing the overall number of simulated particles. Because of the cylindrical symmetry, particles loaded at large radii carry generally more charge than particles loaded on-axis. If/when these "heavy" particles approach the $r = 0$ axis, they may induce "spikes" in density and currents increasing the noise level in the fields. This detrimental effects can be partially mitigated via dynamical particle splitting of the high charge particles approaching the axis [15].

MODELING A 10 GeV LPA IN THE BLF

Implementation of the BLF

A BLF modeling capability has been recently introduced within the INF&RNO/fluid framework. This simulation modality is almost transparent for the user which only has to set the velocity of the boosted frame (*i.e.*, $\gamma_{\rm BLF}$) and initialize the system as he/she would be in the standard laboratory frame (LF). A set of "wrapper" functions automatically perform all the necessary operations to set properly the physical parameter (laser, plasma, externally injected bunch[es], *etc.*) and the numerical ones (grid settings) in the BLF. The swiping plane technique [13] is used to initialize the simulation in the BLF and to reconstruct the output data in the LF at a fixed laboratory time. In all the tests performed to validate the implementation of the BLF in the fluid framework, no evidence of the violent numerical instability which usually affects PIC simulations in the BLF when $\gamma_{\rm BLF} \gtrsim 50$ in 3D (or $\gamma_{\rm BLF} \gtrsim 100$ in 2D) has been observed. This supports the conjecture [16] that the instability may be caused by some grid-particle interaction (*e.g.* numerical Cerenkov effect). The theoretical speedup $S(\gamma_{\rm BLF})$ for a LPA simulation in a BLF and the optimal boost velocity ($\gamma_{\rm BLF}^{opt}$) have been computed in [13]. Denoting by γ_w the relativistic factor of the wake we have: $S \simeq (1+\beta_{\rm BLF})^2\gamma_{\rm BLF}^2$ for $\gamma_{\rm BLF} \ll \gamma_w$, $S \simeq \gamma_w^2$ for $\gamma_{\rm BLF} \gtrsim \gamma_w$ and, for practical purposes, $\gamma_{\rm BLF}^{opt} \simeq \gamma_w$ can be assumed. However in our case the maximum value for $\gamma_{\rm BLF}$ (and so the maximum speedup) is more likely to be limited by the validity of the envelope approximation in the BLF. The envelope approximation relies on a time scale separation between the fast laser oscillations and the slowly evolving envelope. Due to the Doppler redshift of the laser light in the BLF the fulfillment of the time scale separation condition becomes more difficult to achieve the higher is $\gamma_{\rm BLF}$. For instance, in a case with $\gamma_w \simeq 100$, we found that the maximum acceptable value for $\gamma_{\rm BLF}$ is around 40. For higher values of $\gamma_{\rm BLF}$ the simulation results start to deviate significantly (a few percent) from the ones obtained in the LF.

Simulation of a 10 GeV Stage

We consider the interaction of a 40 fs, 40 J, linearly polarized, bi-Gaussian laser laser pulse with a 75 cm long preformed plasma channel. The on-axis density is $n_0 = 10^{17}$ cm^{-3} ($k_0/k_p = 131$) at the entrance of the channel and increases linearly along the plasma column (+35 % after 75 cm). The laser pulse is focused at the entrance of the channel down to a spot of $w_0 = 90$ μm ($k_p w_0 = 5.5$), yielding a focused normalized intensity of $a_0 = 1.1$. The

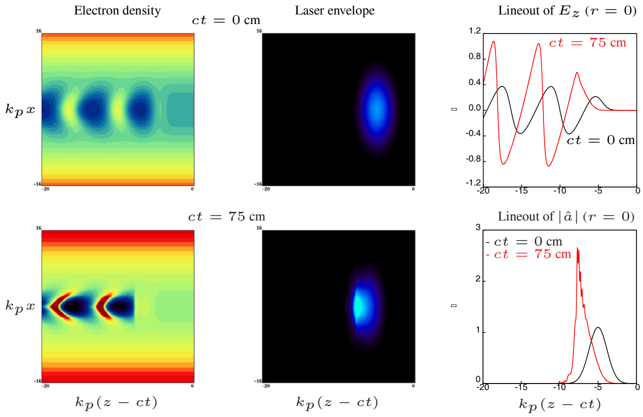

Figure 1: Plasma density (left), laser envelope (center) and the lineouts of the longitudinal accelerating field / laser envelope (right) at different times in the simulation.

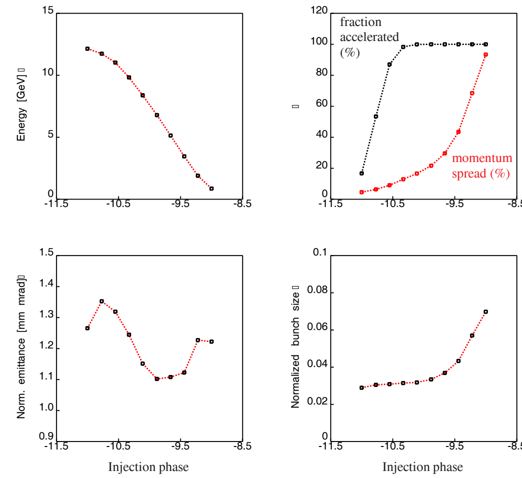

Figure 2: Properties of the accelerated bunch at the end of the plasma channel as a function of the injection phase.

laser pulse is (linearly) matched in the channel. The numerical parameters (fluid simulation) were $k_p\Delta\xi = 1/200$, $k_p\Delta r = 1/12$, and $\Delta\tau/\Delta\xi = 0.2$. Running this simulation using INF&RNO in the LF would require approximately 10^4 CPU hours, while running it using the BLF with $\gamma_{\rm BLF} = 12$ requires only ~ 24 CPU hours. In Fig. 1 we show the evolution of the plasma electron density (left), the laser envelope (center), and the lineouts of the longitudinal accelerating field / laser envelope (right) at different times. Significant self steepening has occurred at the end of the plasma but the structure of the wake is still regular.

An electron bunch with an initial energy of a few tens of MeV and percent level momentum spread was externally injected in the second plasma period behind the laser pulse. In order to characterize the performances of the LPA we explored injection phases in the range $-11 < \varphi_{inj} < -9$, where $\varphi_{inj} = k_p(z_{inj} - z_{laser})$, z_{inj} being the injection position and z_{laser} the laser centroid position. The initial phase space distribution for the injected bunch is gaussian with $k_p\sigma_x = k_p\sigma_y = k_p\sigma_z = 0.2$ and $\varepsilon_{n,x} = \varepsilon_{n,y} = 1$ mm mrad. Beam loading was neglected in the simulation. The properties of the bunch at the end of the acceleration length (75 cm) are summarized in Fig. 2, where we show final bunch energy, relative momentum spread, fraction of the accelerated charge (compared to the initial one), bunch size and emittance as a function of the injection position. At the exit of the plasma ~ 10 GeV electrons with 10% time-projected momentum spread and ~ 1.3 mm mrad projected normalized emittance were observed.

CONCLUSIONS

We have presented the INF&RNO computational framework: a 2D cylindrical, envelope, ponderomotive, PIC / fluid simulation code, and we have reported the progress on the implementation of a BLF modeling capability within the INF&RNO/fluid framework. INF&RNO is an efficient (several order of magnitude faster compared to standard simulation codes) and versatile numerical tool to explore the physics of LPAs and design experiments for the new generation of lasers such as BELLA, which is expected to deliver ~ 10 GeV electrons with good emittance and momentum spread from a meter scale LPA.

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REFERENCES

[1] E. Esarey, *et al.*, Rev. Mod. Phys. **81** (2009) 1229.

[2] S.P.D. Mangles, *et al.*, Nature **431** (2004) 535 (2004); C.G.R. Geddes, *et al.*, *ibid.* 538; J. Faure, *et al.*, *ibid.* 541.

[3] W.P. Leemans, *et al.*, Nature Physics **2** (2006) 696.

[4] W.P. Leemans, *et al.*, in: Proc. 2010 AAC Workshop, Annapolis, MD, p. 3, AIP (2010).

[5] C.K. Birdsall, A.B. Langdon, Plasma Physics Via Computer Simulation, Adam Hilger, (1991).

[6] J.-L. Vay, Phys. Rev. Lett. **98** (2007) 130405 .

[7] P. Mora, T.M. Antonsen, Phys. Plasmas **4**, (1997) 217.

[8] C. Huang, *et al.*, J. Comp. Phys. **217** (2006) 658.

[9] A.F. Lifshitz, *et al.*, J. Comp. Phys. **228** (2009) 1803.

[10] D.L. Bruhwiler, *et al.*, in: Proc. 2008 AAC Workshop, Santa Cruz, CA, p. 29 (2008).

[11] C. Benedetti, *et al.*, IEEE-TPS **36** (2008) 1790.

[12] S.F. Martins, *et al.*, Nature Physics **6** (2010) 311.

[13] J.-L. Vay, *et al.*, in: Proc. 2010 AAC Workshop, Annapolis, MD, p. 244 (2010).

[14] C. Benedetti, *et al.*, Proc. of EPAC08, Genoa, June 2008, WEPP127, p. 2794.

[15] C. Benedetti, *et al.*, in: Proc. 2010 AAC Workshop, Annapolis, MD, p. 250 (2010).

[16] S.F. Martins, *et al.*, Comput. Phys. Comm. **182** (2010) 869.

[17] J.S. Shang, J. Comp. Phys. **153** (1999) 312.