NUMERICAL STUDY OF PLASMA WAKEFIELDS EXCITED BY A TRAIN OF ELECTRON BUNCHES*

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Abstract

We study numerically the excitation of plasma wakefields by a train of electron bunches using the UCLA particle-in-cell code Quickpic. We aim to find an optimal regime that combines both the advantages of linear and non-linear plasma wakefield accelerator. On one hand, the longitudinal electric field excited by individual bunches add as in the linear regime, and the transformer ratio can be maximized (i.e. much larger than 2) by adjusting the number of particles in the bunches as well as their distance. On the other hand, the bunches create large wakefield independent of transverse sizes evolution while propagating through the plasma as in the non-linear regime. In principle, such a scheme can multiply the energy of the witness bunch following the drive bunch train in a single plasma wakefield accelerating stage. The parameters for electron bunches are chosen based on the current experiment at the Brookhaven National Laboratory Accelerator Test Facility (ATF), where this scheme can in principle be tested. Detailed simulation results are presented.

INTRODUCTION

In this paper we investigated the possibility of reaching large transformer ratio in nonlinear regime of PWFA. The transformer ratio is an important indication of energy transfer efficiency from driving bunch to witness bunch by the means of plasma oscillations. Previous simulations[1] have shown that large transformer ratio (larger than 2) is achievable in linear regime with a train of ramped bunches with appropriate charge ratio and distances in linear regime [2]. In this case, the energy transfer efficiency becomes optimal because those bunches experience the same decelerating wakefield. However, due to the fact that the later bunches experience stronger transverse focusing force than the earlier ones, the delicate balance that the decelerating wakefield under each bunch is equal cannot be kept while the bunches propagates in the plasma. Consequently, it is not practical to produce high-energy beams with the ramped bunch scheme in the linear regime.

As in nonlinear regime, the core of whole beam propagates in pure, uniform ion column, and the beam electrons are focused by a linear focusing force. Its emittance is preserved. If the beam with a matched emittance is focused at the plasma entrance, its transverse spot size will remain constant during the acceleration.

Our goal is to develop a ramped bunch train scheme in nonlinear regime, which allows for the obtaining a high transformer ratio as well as emittance preservation. Recently, the masking technique at the BNL-ATF has been developed to partially block sections of a long bunch with a corrected energy-spread [3]. It is capable of generating 1 to 5 microbunches with subpicosecond spacing. When we apply this technology to high-energy beams, the train of microbunches can be focused to a transverse size of about 5um. Therefore it is possible to produce a ramped bunch train that can excite nonlinear wakefields in plasmas Simulations using UCLA code Quickpic are performed to find the optimal beam-plasma parameters. To better describe the experimental parameters, in the simulation we use tri-guassian bunches instead of the longitudinally square bunches, although the exact shape does not matter as long as the plasma wavelength is longer than the bunch characteristic length.

MAXIMIZING TRANSFORMER RATIO IN LINEAR REGIME USING RAMPED BUNCH TRAIN

Discussion in this section reviews the ramped bunch scheme in linear regime, so the bunches are longitudinally square as in other literature. For a train of equidistant bunches, the maximum transformer ratio can be achieved when the following bunch is placed in the previous bunchs accelerating phase, and also the charge ratio of the bunches are tuned that retarding wakefield inside each bunch is equal to that of the first bunch. Suppose that the first bunch has charge Q_0, its transformer ratio is R (2 for symmetric long bunch), and the position to be ξ_1 . Mathematically illustration is as following:

$$\xi_{n\geq 2} = \xi_1 + n \cdot 1.5\lambda_p \qquad (1)$$

and

$$Q_1 : Q_2 : Q_3 : \cdots : Q_n = 1 : (R+1) : (R^2+1) : \cdots : (R-1)^{n-1} + 2\sum_{k=1}^{n-1}(R-1)^{k-1} \qquad (2)$$

Fig.1 is an example to illustrate the scheme. Four bunches with the width of 125um ($0.5\lambda_{pe}$) are placed $1.5\lambda_{pe}$ apart to drive the wakefield. Their charge is: $30 : 90 : 150 : 210pC$. The final transformer ratio reaches up to 7.96, which is four times that of a single bunch ($R \approx 2$).

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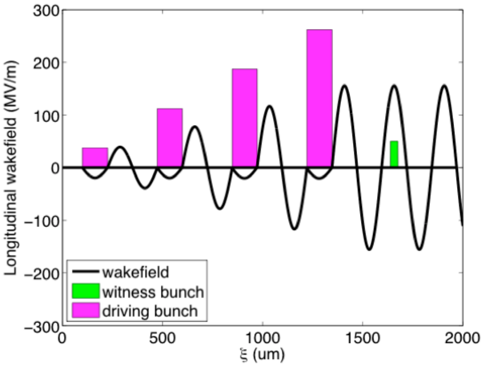

Figure 1: Ramped bunch train scheme. The bunches are transversely bi-Gaussian and longitudinally rectangular.

In addition, the transverse wakefield inside the n-th bunch are given by [2]:

$$E_{r,n}(r,\xi_n) = -\frac{en_{b1}}{\epsilon_0 k_p^2}[(n-1)^2 + \sin(k_p(\xi-\xi_n))]\frac{dR(r)}{dr} \tag{3}$$

In this equation, R(r) is the transverse component of the longitudinal wakefield. It can be seen that as a result of increasing the charge along the bunch train, the amplitude of transverse wakefield scales quadratically with the number of bunches. Larger focusing force of the later bunches leads to decrease of their transverse size, thus increase of the beam density and subsequently the wakefields. Therefore the initial purpose of reaching high transformer ratio by equalizing the retarding wakefields leads to increase in focusing force that drives the wakefields into the nonlinear regime. In addition, the focusing field is not linear with radius, which leads to emittance growth.

PRESERVED BEAM TRANSVERSE SIZE IN NONLINEAR REGIME

However, in nonlinear regime ($n_b \gg n_p, k_p\sigma_z \ll 1$), the nearby plasma electrons are expelled by the bunch head and a pure ion column is formed around the core of the beam. The transverse wakefield is linearly proportional to the radius as $E_r = (en_e/2\epsilon_0)\cdot r$, and E_r is constant along the propagating direction. The focusing strength K, defined as: $K = F_r/(r\cdot\gamma mc^2) = n_e e^2/2\epsilon_0\gamma mc^2$, is also a constant for a given plasma density. The envelope equation for the transverse size ($\sigma_{x,y}$) of the Gaussian bunch is:

$$\frac{d^2\sigma_{x,y}}{dz^2} + K\sigma_{x,y} = \frac{\epsilon_{Nx,y}^2}{\gamma^2\sigma_{x,y}^3} \tag{4}$$

where ϵ_N is the normalized emittance of the bunch. The variation of the spot size is minimized so that: $d^2\sigma_{x,y}/d\xi^2 = 0$, which yields a matched beam inside the ion column such that $\epsilon_N = (\gamma/2)^{1/2}k_p\sigma_{x,y}^2$. If we assume that at the plasma entrance, the beam is focused so that $d\sigma_{x,y}/dz = 0\, at\, z = 0$, with the matched plasma emittance, the beam size remains σ_{x_0,y_0} along the plasma.

MAXIMIZING TRANSFORMER RATIO IN NONLINEAR REGIME BASED ON ATF PARAMETERS

Muggli et al. have successfully generated trains of electron microbunches with adjustable subpicosecond spacing in ATF experiments [4], which can be applied in generation of wakefields for PWFA. The masking technique employed in the experiment allows easily for the tailoring of the bunch width, relative positioning and the charge ratio of the bunches. In the experiment, five equidistant drive bunches are produced with the transverse size of 100um and the energy of 59MeV. The plasma density is adjusted so that individual drive bunches are one plasma wavelength ($\Delta z = \lambda_{pe}$) apart, and they are followed by a witness bunch with the separation of one and half plasma wavelength so that to experience the peak accelerating field excited by the drive bunches. This resonant excitation can maximize the accelerating wakefield of the witness bunch. However, the ramped bunch train can be used to maximize the transformer ratio and hence the energy gain by the witness bunch. For simulation purposes, we assume the transverse size of bunches can be focused to $\sigma_{x,y} = 5um$. We adjust the bunch spacing and charge ratio of to investigate whether high transformer ratio is reachable in the nonlinear regime.

Single Bunch

Using the parameter listed in the right hand side column of Table 1, the left panel of Fig. 2 shows the simulation results after one time step. The four rows of the figure are normalized beam density, normalized plasma electron density, normalized plasma electron density and longitudinal wakefield on the beam axis, respectively. The plasma density figure and profile show that the nonlinear regime is reached, the plasma electron density changes from -1 to 0 in third row. The electric field profile shows that the transformer ratio is $R_1 = (E_{1+})/(E_{1-}) = 0.458/0.270 = 1.7$. This is almost the maximum value in the linear regime.

Table 1: Bunch train parameters

	ATF	**Simulation**
Current Typicol	“Gaussian”	
σ_z	25 um	
Energy	59 MeV	
n_e	$1.24\cdot 10^{16}\ cm^{-3}$	
σ_z	100 um	5 um
Charge	40pC/bunch	40pC for 1st bunch
Spacing	λ_{pe}	$\approx 1.5\lambda_{pe}$
n_b/n_p	0.005 ($n_b \ll n_p$)	$2(n_b > n_p)$
Regime	**linear**	**nonlinear**

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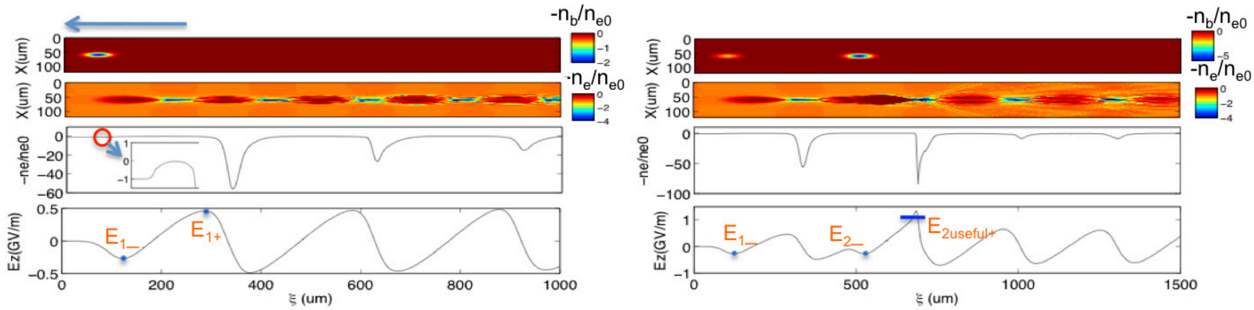

Figure 2: One bunch (left panel) and two bunches (right panel) propagated for one time step. Parameters are listed in Table 1.

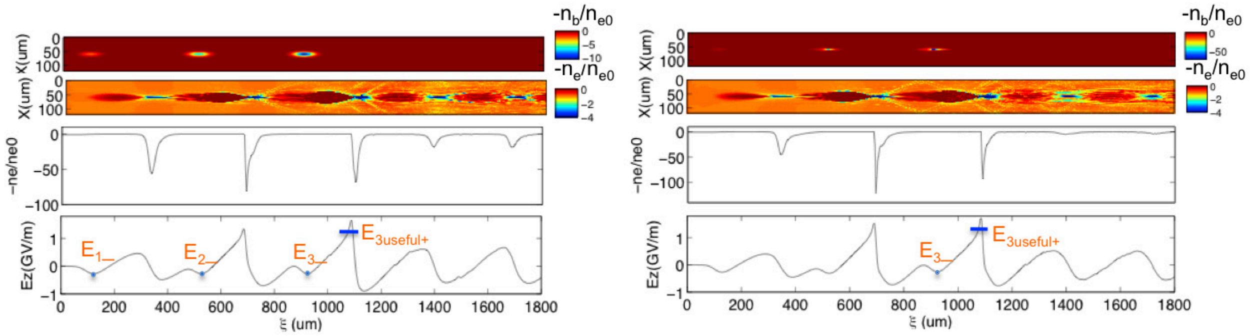

Figure 3: Three bunches propagated for one step (left panel), and for 2cm plasma length, which is 4.4 betatron oscillations (right panel). Parameters are listed in Table 1.

Two Bunches

The second bunch with same bunch shape and size is added to drive the wakefield (refer to the right panel of Fig. 2 for the image after one time step simulation). Optimization of the spacing and charge ratio shows that when the bunches are separated by $1.35\lambda_{pe}$(406um), and the charge bunch ratio Q_2/Q_1 is 2.8, the maximum transformer ratio is obtained as: $R2 = (E_{2useful+})/(E_{2-}) = 1.14/0.27 = 4.2$. Note that we used the $E_{2useful+}$ instead of the peak accelerating wakefield due to the appearance of spike. Also the decelerating wake under the two bunches has only 0.3% difference, an indication that the two bunches are losing energy at the same rate. This result is comparable with the ramped bunch scheme in linear regime, in which the spacing is $1.5\lambda_{pe}$, the charge ratio Q_2/Q_1 is 2.7, and the transformer ratio is 2.89 (obtained from Eqn. (2)).

Three Bunches

The third bunch is added to investigate whether the transformer ratio keeps increasing. As shown in the left panel of Fig. 3, the maximum transformer ratio (R_3) is 5.0 after one single step simulation, compared to 3.89 in linear theory. The third bunch is placed $1.32\lambda_{pe}$ behind the second one, and the charge ratio $Q_3/Q_1 = 4.2$. While the ramped bunch scheme in linear theory uses $1.5\lambda_{pe}$ as the spacing and 3.89 as the charge ratio. Further simulations were performed to verify that this high transformer ratio can be maintained through the propagation in the plasma, as shown in the right panel of Fig. 3. The transformer ratio increases from 5.0 to 5.2, i.e., stays essentially constant during the acceleration and propagation over $2cm$ of plasma or 4.4 betatron period of the beam envelope. However, simulations performed so far show that when adding the fourth bunch, the transformer ratio does not increase, because the system becomes very nonlinear.

CONCLUSION

We have showed in initial simulations that a transformer ratio larger than 2 can be reached in the nonlinear regime of the PWFA. In this regime the transformer ratio can be maintained over a plasma length appropriate for large energy gain and the witness bunch emittance can in principle be preserved. Further simulations will look at longer plasma lengths and at the evolution of the witness bunch.

REFERENCES

[1] Kallos, E., et al., Particle Accelerator Conference, pp.3070-3072. (2007).

[2] Kallos, E., et al., 13th AAC Workshop, pp. 580-585. (2009)).

[3] P. Muggli, et al., Physical Review Letters 101, 054801 (2008).

[4] P. Muggli and B. Allen, Phys. Rev. ST Accel. Beams 13, 05280 (2010).

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