AN APPLICATION FOR TUNES AND COUPLING EVALUATION FROM TURN-BY-TURN DATA AT THE FERMILAB BOOSTER*

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Abstract

A console application using the phasing of Turn-by-Turn signals from the different BPMs has been tested at the Fermilab Booster. This technique allows the on-line detection of the beam tunes during the fast Booster ramp in conditions where other algorithms were unsuccessful.
The application has been recently expanded to include the computation of the linear coupling coefficients. Algorithm and measurement results are presented.

INTRODUCTION

Turn-by-turn beam position measurements still remain the most reliable tool for tune determination. The standard FFT method for tune evaluation has resolution of about $1/N$, N being the number of turns, which is insufficient in the case of rapid changes of the tunes and/or fast decoherence of the betatron oscillations. Much better precision can be achieved with the so-called Continuous Fourier Transform (CFT) method which consists in evaluating

$$Z(\nu) = \frac{1}{N}\Sigma_{n=1}^{N}\, e^{-i2\pi\nu(n-1)}\; z_n$$

(ν continuous variable) z_n being the beam position at n-th turn following a single kick ($z \equiv x, y$), and finding the largest $|X(\nu)|$.

In absence of random noise, CFT provides precision of about $1/N^2$; the precision may be improved with the addition of some other techniques, but they fail in the presence of noise.

In the case of white noise the r.m.s. error on the tune evaluation provided by CFT is

$$\sigma_\nu \simeq \frac{\sqrt{6}\sigma}{\pi N^{3/2} A}$$

σ being the BPM r.m.s. error and A the betatron oscillation amplitude. This is better than with the simple FFT but may still be insufficient when the noise level is high and only a small number of turns is available. We have shown [1] that the *a priori* knowledge of machine optics may help to drastically improve the precision of tune determination by applying the CFT-analysis to the *"phased"* sum

$$\tilde{z}_n = \Sigma_{k=1}^{M} z_n^{(k)}\; e^{-i\mu_z^{(k)}}$$

where $\mu_z^{(k)}$ is the (nominal) phase advance at the k-th BPM

The signal to noise ratio is improved by a factor $\sqrt{M}$, M being the number of BPMs.

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The algorithm was implemented in an already existing ACNET control system application of the Fermilab Booster. The new algorithm gives operators *on-line* feedback on the tunes where the previous approach analyzing individual BPMs failed, namely for the most part of the ramp, and simplified optics measurements by TBT data analysis as well as by tune measurement [2].

COUPLING MEASUREMENT

The actual nominal Booster tunes are Q_x=6.75 and Q_y=6.85. Correction of the difference linear coupling is necessary for setting the horizontal tune closer to the vertical one, thus providing more room for space charge detuning in presence of large beam intensity.

The sum linear coupling resonance is believed to be source of vertical emittance growth in the Fermilab Booster [4].

For these reasons is desirable to have a reliable tool for *on-line* measuring and possibly compensating the linear coupling resonance coefficients.

The Booster ACNET application for the tune measurement has been recently expanded to include linear coupling coefficients measurement. The algorithm is the same used, for instance, at Tevatron[3].

In the presence of coupling, the excitation of one of the two modes will excite an oscillation in the other mode too; if for instance the beam is kicked in the horizontal plane, the resulting vertical motion is in first order approximation described by

$$y_n^j = \Big[\sqrt{\beta_y^j}\Big(\mathrm{e}^{-i\Phi_y^j} w_+^j - \mathrm{e}^{i\Phi_y^j} w_-^j\Big) - \sqrt{\beta_x^j}\mathrm{e}^{i\Phi_x^j}\sin\chi_j\Big] A_x \mathrm{e}^{iQ_x(\theta_j+2\pi n)} + c.c. \quad (1)$$

where χ_j is the tilt of the j-th BPM and $\Phi_z \equiv \int_0^\theta d\theta' R/\beta_z - Q_z\theta$ (periodic phase function). When instead the vertical mode is excited, the resulting horizontal motion is

$$x_n^j = \Big[\sqrt{\beta_x^j}\Big(\mathrm{e}^{-i\Phi_x^j} w_+^j + \mathrm{e}^{i\Phi_x^j} w_-^{*j}\Big) + \sqrt{\beta_y^j}\mathrm{e}^{i\Phi_y^j}\sin\chi_j\Big] A_y \mathrm{e}^{iQ_y(\theta_j+2\pi n)} + c.c. \quad (2)$$

The (periodic) functions $w_\pm$ are related to the distribution of coupling elements by

$$w_\pm(\theta) = -\int_0^{2\pi} d\theta' \frac{C^\pm(\theta')}{4\sin\pi Q_\pm}\mathrm{e}^{-iQ_\pm[\theta-\theta'-\pi\mathrm{sign}(\theta-\theta')]}$$

with $Q_\pm \equiv Q_x \pm Q_y$ and

$$C^\pm(\theta) \equiv \frac{R\sqrt{\beta_x\beta_y}}{2B\rho}\Big\{\Big(\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}\Big) + B_\theta\Big[\Big(\frac{\alpha_x}{\beta_x} - \frac{\alpha_y}{\beta_y}\Big) - i\Big(\frac{1}{\beta_x} \mp \frac{1}{\beta_y}\Big)\Big]\Big\}\mathrm{e}^{i(\Phi_x \pm \Phi_y)}$$

R being the machine radius. The functions $\tilde{w}^\pm \equiv w_\pm \mathrm{e}^{iQ_\pm\theta}$ are constant in coupler free regions and experience a discontinuity $-iC^\pm\ell/2R$ at coupler locations, with ℓ coupler length. On the resonances $Q_x \pm Q_y = int$ the functions $\tilde{w}^\pm$ are constant. Reconstruction of $w_\pm$ functions from the measurement data allows us to obtain information on the coupling strength and sources. If the kick occurs in the horizontal plane (x and y must be exchanged otherwise) the Fourier component $Y_j(Q_x)$ of $y_j(\theta)$ is related to the values of $w_\pm$ at the j-th BPM according to Eq.(1). When the BPM tilt is negligible or already known, the number of unknown quantities per BPM reduces to two and, under the assumption that between two consecutive BPM's there are no strong sources of coupling, we can retrieve the (constant) value of $\tilde{w}^\pm(\theta)$ in the region between them by knowing $Y_j(Q_x)$ and $Y_{j+1}(Q_x)$.

Of course the value of $w_\pm$ does not depend upon whether the beam is kicked in the horizontal or vertical plane. Therefore at the Booster, where each BPM measures the horizontal and vertical beam position, one could argue that by kicking in both planes simultaneously, the real and imaginary part of $w_\pm$ could be computed at each single BPM with no need of pairing. Actually, a closer look at Eqs.(1) and (2) reveals that, owing to the fact that -w_- and w_-^* have identical imaginary part, the corresponding set of equations is under-constrained.

With $w_\pm$ functions known, we can compute the strength of coupling resonances (coupling coefficients) as

$$\bar{C}^\pm = \frac{n_\pm - Q_\pm}{\pi}\int_0^{2\pi} d\theta \; w_\pm \mathrm{e}^{in_\pm\theta} \tag{3}$$

where $n_\pm \equiv \mathrm{Round}(Q_x \pm Q_y)$. The difference resonance coefficients provides the minimum reachable tune distance, $\Delta = |C^-|$, whereas the sum resonance coefficient, C^+, gives the stopband of the linear sum resonance which reduces the available tune space. Eq.(3) is obtained under assumption of weak coupling. If coupling is strong then higher order terms should be taken into account as discussed in [5].

IMPLEMENTATION

The algorithm was implemented in an ACNET control system application to give operators on-line feedback on the Booster coupling parameters. The application, B38, is the same application used to give on line feedback for Booster tunes.

The application works as follows. A kicker, horizontal or vertical, is set up to kick the beam every 500 turns. On completion of the ramp the application reads out the turn

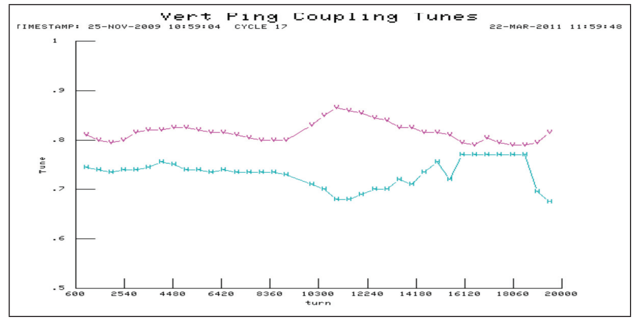

Figure 1: Horizontal (red) and vertical (green) tune vs. turn number with vertical pinger excitation.

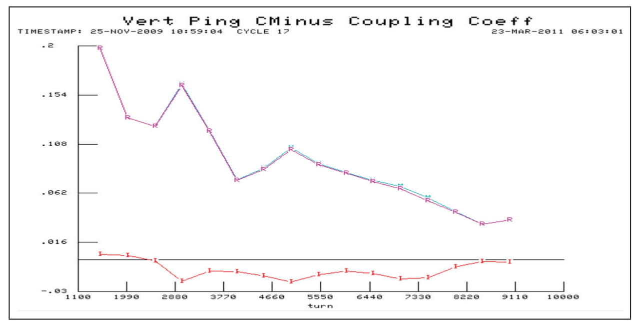

Figure 2: $\mathcal{R}(C^-)$ (green), $\mathcal{I}(C^-)$ (cyan) and $|(C^-)|$ (red) vs. turn number with vertical pinger excitation.

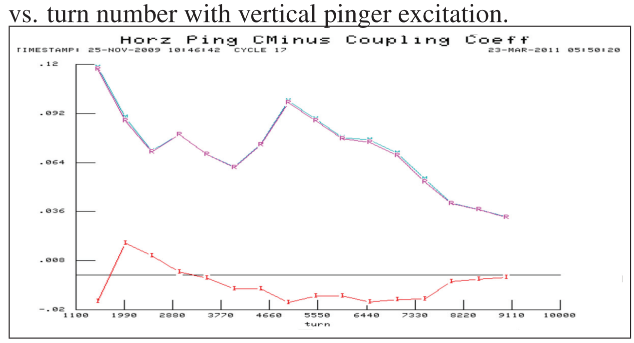

Figure 3: $\mathcal{R}(C^-)$ (green), $\mathcal{I}(C^-)$ (cyan) and $|(C^-)|$ (red) vs. turn number with horizontal pinger excitation.

by turn BPM data for all turns and all BPMs. Both horizontally pinged data and vertically pinged data are taken. The CFT spectra of the phased sum BPM data are then displayed so that an operator can select the tunes. The coupling coefficients are then computed from the individual BPM turn by turn data. Fig. 1 shows tunes along a typical Booster ramp as measured by B38. Figs. 2 and 4 show the C^- and C^+ coefficients respectively, obtained exciting the beam with the vertical pinger, while Figs. 3 and 5 show the same quantities when the beam is excited through the horizontal pinger. As expected, the values measured either with horizontal or vertical pinger are very close. Comparison of Figs. 4 and 5 for C^+ suggests that the precision of the measurement of the sum resonance strength is somewhat worse. Still, the resonance appears not large enough to pose a real threat for beam stability. Finally Figs. 6, 7, 8 and 9 show $\tilde{w}^-$ and $\tilde{w}^+$ at turn 4505 obtained by horizontal and vertical excitation. Large difference in values between horizontal and vertical pinger results may be due to BPM tilts which we have not yet tried to determine.

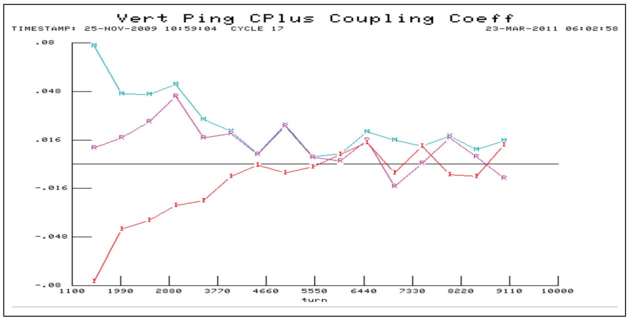

Figure 4: $\mathcal{R}(C^+)$ (green), $\mathcal{I}(C^+)$ (cyan) and $|(C^+)|$ (red) vs. turn number with vertical pinger excitation.

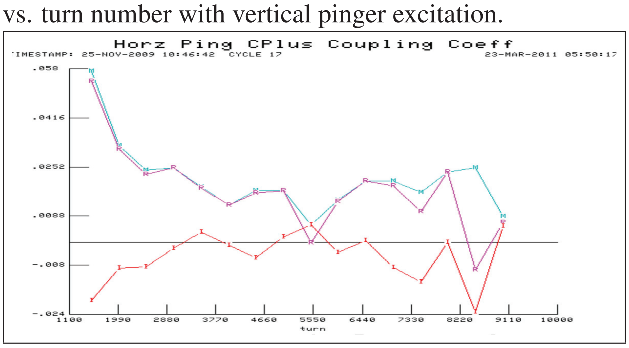

Figure 5: $\mathcal{R}(C^+)$ (green), $\mathcal{I}(C^+)$ (cyan) and $|(C^+)|$ (red) vs. turn number with horizontal pinger excitation.

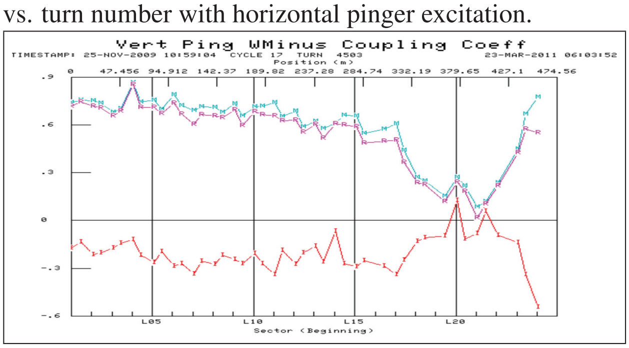

Figure 6: $\mathcal{R}(\tilde{w}^-)$ (green), $\mathcal{I}(\tilde{w}^-)$ (cyan) and $|(\tilde{w}^-)|$ (red) vs. longitudinal position with vertical pinger excitation.

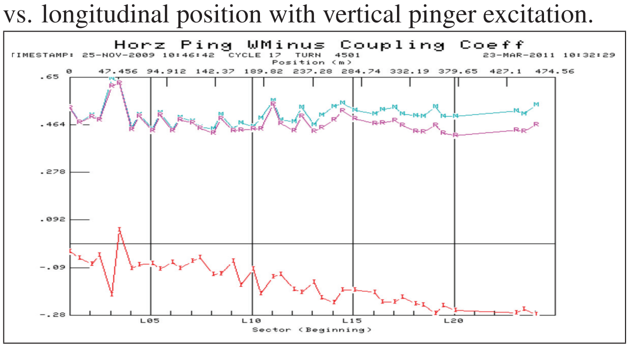

Figure 7: $\mathcal{R}(\tilde{w}^-)$ (green), $\mathcal{I}(\tilde{w}^-)$ (cyan) and $|(\tilde{w}^-)|$ (red) vs. longitudinal position with horizontal pinger excitation.

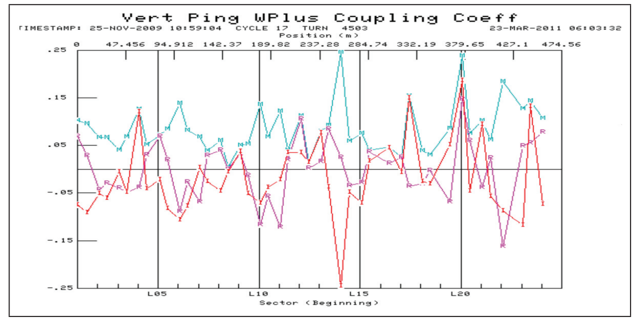

Figure 8: $\mathcal{R}(\tilde{w}^+)$ (green), $\mathcal{I}(\tilde{w}^+)$ (cyan) and $|(\tilde{w}^+)|$ (red) vs. longitudinal position with vertical pinger excitation.

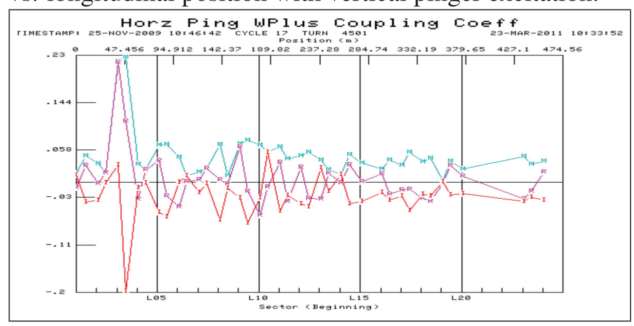

Figure 9: $\mathcal{R}(\tilde{w}^+)$ (green), $\mathcal{I}(\tilde{w}^+)$ (cyan) and $|(\tilde{w}^+)|$ (red) vs. longitudinal position with horizontal pinger excitation.

SUMMARY AND OUTLOOK

Although improved by the phased sum technique the automatic identification of the tunes is not always successful. This makes the use of the on-line application difficult. Ideas for further improvements are under investigation.

Measurements have indicated that the effect of the skew quadrupoles is by a factor 3 weaker than expected from the nominal optics. A calibration of the skew quadrupole circuits using the TBT data is planned.

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