DESIGN OF AN ACHROMATIC AND UNCOUPLED MEDICAL GANTRY FOR RADIATION THERAPY *

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Abstract

We are presenting the layout and the optics of a beam line to be used as a medical gantry in radiation therapy. The optical properties of the gantry's beam line are such as to make the beam line achromatic and uncoupled. These two properties make the beam spot size, which is delivered and focused by the gantry, on the tumor of the patient, independent of the angular orientation of the gantry. In this paper we present the layout of the magnetic elements of the gantry, and also present the theoretical basis for the optics design of such a gantry [1].

INTRODUCTION

A medical gantry, as it is used in the radiation treatment of cancer patients, is the last part of the beam optical system, of the accelerator complex, which delivers and focuses the beam on the tumor. The curved line shown in figure 1 is a schematic diagram of a gantry which can rotate about a horizontal axis. The particle beam (green arrow in fig. 1) enters the gantry, and is guided by the gantry on the tumor (red spot in fig. 1). As the gantry rotates about the axis shown in figure 1, the beam exiting the gantry always lies on a plane normal to the rotation axis at the point of the isocenter. Thus the gantry facilitates the ability of the beam delivery system, to deliver the beam at the tumor, which is placed at the isocenter, from any angle on this vertical plane, which is normal to the rotation angle of the gantry as stated earlier.

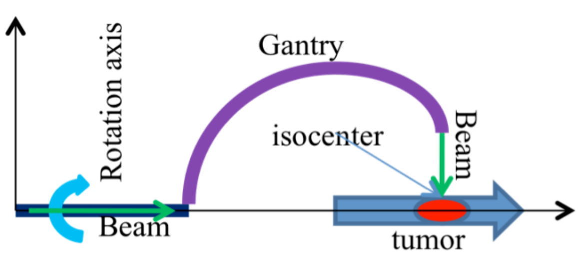

Figure 1: A schematic diagram of a medical gantry irradiating a tumor (red spot). The green arrow indicates the beam direction. The gantry which is the curved line shown in this figure may rotate about a horizontal axis. Thus the gantry can direct the beam from different angles on to the tumor.

The gantry consists of dipoles and quadrupoles elements whose median symmetry plane lies on a plane which contains the rotation axis of the gantry. In this paper we define this plane as the "plane of the gantry". As the beam is transported along the axis of rotation of the gantry and before it enters the gantry, it is focused by "normal" quadrupoles and experiences no linear beam coupling. Subsequently the beam enters the gantry, and is transported to the delivery point which is the tumor. The transported beam at the tumor is still linearly uncoupled as long as the plane of the gantry lies either, on the horizontal, or the vertical plane. In these two orientations of the gantry the dipoles and the quadrupoles are "normal" therefore the linear R-transport-matrix appears linearly uncoupled. As the gantry rotates about this horizontal axis, to irradiate the tumor at a different angle, the linear beam coupling is unavoidable since the coordinate system of the gantry's quadrupoles is rotated with respect to the coordinate system of the quadrupoles at the entrance of the gantry which are located along the axis of rotation. As a result, in this rotated coordinate system, the beam will be linearly coupled as it is transported by the gantry to the tumor and the beam size at the location of the tumor will vary in size depending on the orientation of the gantry. In this paper we present a method which makes the beam transport R-matrix linearly uncoupled achromatic and independent of the orientation angle of the gantry as is proven in the following section.

THEORETICAL BACKGROUND

In this section we discuss the theoretical basis for the optical design of the gantry's beam line. The general form of the first order matrix (R-Matrix) of a beam line which consists of quadrupoles is given by the matrix below.

$$R = \begin{pmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} & \begin{pmatrix} R_{13} & R_{14} \\ R_{23} & R_{24} \end{pmatrix} \\ \begin{pmatrix} R_{31} & R_{32} \\ R_{41} & R_{42} \end{pmatrix} & \begin{pmatrix} R_{33} & R_{34} \\ R_{43} & R_{44} \end{pmatrix} \end{pmatrix} =$$

$$= \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & R_{22} & R_{23} & R_{24} \\ R_{31} & R_{32} & R_{33} & R_{34} \\ R_{41} & R_{42} & R_{43} & R_{44} \end{pmatrix} \quad (1)$$

If the quadrupoles of the beam line are "normal" the 2x2 sub-matrices R_{xy}, and R_{yx} appearing in equation (1) above are zero and the R-matrix becomes linearly uncoupled. In this paper we will refer to this uncoupled matrix with the

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symbol R(0°). Now suppose that we rotate the quadrupoles of this beam line by an angle θ, about their optical axis. The new R-matrix R(θ) is given by [2]:

$$R(\theta) = M(\theta)R(0^o)M^T(\theta) \qquad (2)$$

In equation (2) M(θ) is a matrix given by,

$$M(\theta) = \begin{pmatrix} I\cos\theta & I\sin\theta \\ -I\sin\theta & I\cos\theta \end{pmatrix} \qquad (3)$$

and $M^T(\theta)$ is its transpose. The symbol I in equation (3) is the 2x2 identity matrix. Using (1) and (3) and carrying out the matrix multiplication in (2) we obtain.

$$R(\theta) = \begin{pmatrix} R_{xx}\cos^2\theta + R_{yy}\sin^2\theta & -R_{xx}\cos\theta\sin\theta + R_{yy}\sin\theta\cos\theta \\ -R_{xx}\cos\theta\sin\theta + R_{yy}\sin\theta\cos\theta & R_{xx}\sin^2\theta + R_{yy}\cos^2\theta \end{pmatrix} \qquad (4)$$

It is easy to see that the relation (4) is reduced to the identity $R(\theta) \equiv R(0^o)$ if the condition $R_{xx}(0^o)=R_{yy}(0^o)$ is satisfied, and we also assume that the $R(0^o)$ matrix is linearly uncoupled ($R_{xy}(0^o)=R_{yx}(0^o)=0$). Thus the rotation matrix R(θ) in equation (2) becomes:

$$R(\theta) = \begin{pmatrix} R_{xx}(0^o) & 0 \\ 0 & R_{xx}(0^o) \end{pmatrix} = R(0^o) \qquad (5)$$

This R matrix in equation (5) is independent of the rotation angle θ and also the matrix remains uncoupled. Now suppose that the beam line consists of a set of dipoles and quadrupoles which may form the gantry's beam line which is represented schematically by the curved line shown in Fig. 1. It is possible to design the optics of the gantry's beam line such that the R-matrix $R(0^o)$ satisfies the following condition:

a) $R_{xx}(0^o)=R_{yy}(0^o)$, $R_{xy}(0^o)=0$, and $R_{yx}(0^o)=0$ as mentioned above and also the condition,

b) $R_{16}(0^o)=0$, and $R_{26}(0^o)=0$, which makes the beam line achromatic. With these two conditions (a) and (b) being satisfied, it is obvious that the R-matrix R(θ) of the gantry is independent of the orientation of the gantry, $\{R(\theta) \equiv R(0^o)\}$, therefore the beam size at the location of the tumor, is independent of the angle θ, of the orientation of the gantry's plane.

DESIGN OF AN UNCOUPLED AND ACHROMATIC GANTRY

In this section we present the layout of the gantry's magnets and its optics.

Layout of the Gantry's Magnets

Fig. 2 shows the placement of the dipole and quadrupole magnetic elements which make up the gantry which rotates about a horizontal axis. The beam enters the gantry along the rotation axis, and exits the gantry on a plane which is normal to the axis of rotation at the isocenter point. As we proved in the previous section the beam size at the isocenter is independent of the angle of rotation of the gantry, because the beam optics of the gantry is designed to be uncoupled and achromatic. In this design of the gantry we have allowed one meter of drift space between the exit of the gantry and the placement of the tumor. This drift space may be utilized for beam diagnostics and beam control instrumentation.

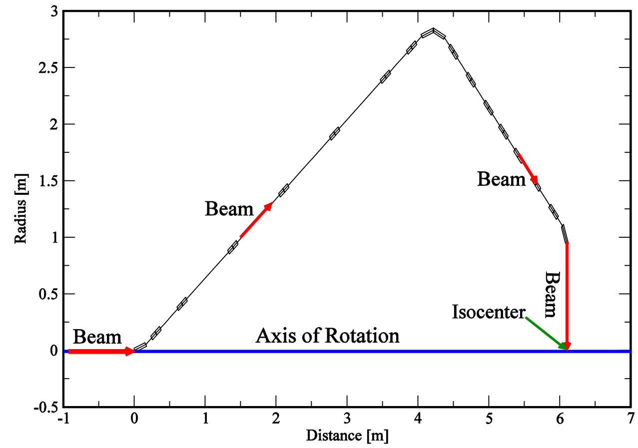

Figure 2: The location of the main magnetic elements of the gantry. The gantry may rotate about the horizontal axis.

Beam Optics of the Gantry

In this section we present the beam optics of the gantry's beam line which should satisfy the "uncoupled" and achromatic conditions discussed in the earlier sections. The design of this beam line is a "spin-off" of a beam line [1] which bends the beam in the horizontal and vertical direction simultaneously. Although in that paper [1] we used six quadupoles in each straight section of the beam line to satisfy the "uncoupled" and achromatic conditions, in this paper which is devoted exclusively in the design of a medical gantry we use seven quadrupoles in each straight section of the gantry as shown in Fig. 2. The additional quadrupole provides us with the flexibility to accept various beam sizes in the entrance of the gantry, with the "uncoupled" and "achromatic" conditions still satisfied. Also in this design, we adopted a symmetric placement of the quadrupoles in each straight section. This symmetric arrangement although it may require one additional quadrupole per straight section it reduces the number of independent power supplies to power the magnets, and this may be advantageous for beam control. The beam optics computer codes TRANSPORT [3] and MAD [4] were used in the optimization of the gantry's beam line, to satisfy both, the "uncoupled" and "achromatic" conditions. In particular the twiss parameters (a_x, β_x, a_y, β_y) along the beam line as shown in figure 3, and figure 4 were calculated by the MAD computer code with the "twiss" command set in the "coupled" mode. The twiss parameters and the dispersion functions along the gantry as calculated by the MAD computer code are shown in Fig. 3 for the case where the "plane of the gantry" is horizontal, therefore the beam line is uncoupled because of the placement of the magnets. Fig. 4 shows the same quantities as Fig. 3 but the "plane of the gantry" is rotated by 45° with respect to that of the gantry's corresponding to Fig. 3.

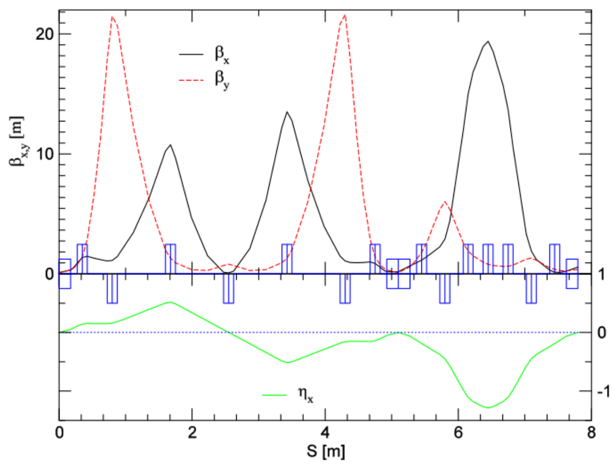

Figure 3: The horizontal and vertical beta functions and the dispersion function of the gantry's beam line when the "gantry's plane" is horizontal.

At this orientation of the gantry's plane, although the beam is coupled inside the gantry, it is uncoupled when the beam exits the gantry, and remains uncoupled afterwards. The beta functions which are shown in figure 4 have been calculated with the computer code MAD with the "twiss" command of the code set in the "couple" mode.

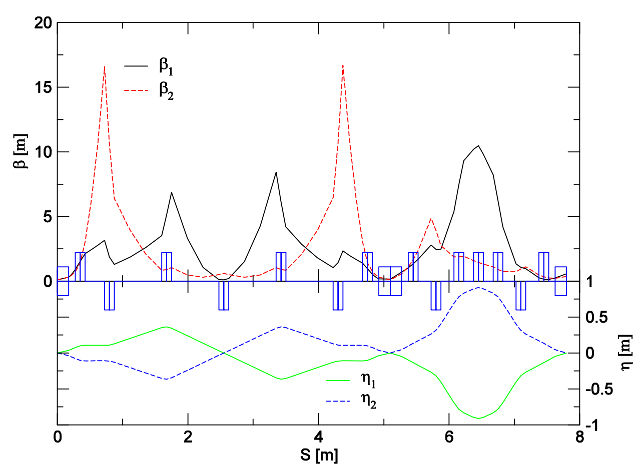

Figure 4: The horizontal and vertical beta functions and the dispersion function of the gantry. The "plane of the gantry" is rotated by 45° with respect to the horizontal plane.

The first order transport R-matrix which corresponds to this beam line of figure 3 and figure 4 is shown below.

$$R(\theta)=\begin{pmatrix} 1.544505 & -0.065595 & 0 & 0 & 0 & 0 \\ 6.736658 & 0.361353 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.544505 & -0.065595 & 0 & 0 \\ 0 & 0 & 6.736658 & 0.361353 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & -0.056103 \\ 0 & 0 & 0 & 0 & 0 & 1.0 \end{pmatrix}$$

Below we include a MAD input file corresponding to the beam optics of the gantry's design, for the readers who a familiar with the MAD code, to reproduce the results.

INITIAL:β_x=0.172 m, α_x=1.076, β_y=0.10 m, α_y=0.3904
L1A=0.174533,AA=0.6108653,LT=any_ang,D1=0.1,
LQ=0.15, D2=0.3, D3=0.724265, D4=0.725736
V1=28.0625, V2=-18.7446, V3=13.8274

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B1: SBEND, L=L1A, ANGLE=AA, TILT=LT
S1:DRIFT,L=D1, Q1:QUAD, L=LQ, K1=V1,TILT=LT
S2:DRIFT, L=D2, Q2: QUAD, L=LQ, K1=V2,TILT=LT
S3:DRIFT,L=D3, Q3: QUAD, L=LQ, K1=V3,TILT=LT
S4:DRIFT,L=D4, Q4:QUAD,L=LQ,K1=-2*V3,TILT=LT
S4:DRIFT,L=D4, Q3: QUAD, L=LQ, K1=V3 ,TILT=LT
S3:DRIFT, L=D3, Q2: QUAD, L=LQ, K1=V2,TILT=LT
S2: DRIFT, L=D2, Q1: QUAD, L=LQ, K1=V1,TILT=LT
S1: DRIFT,L=D1
B1: SBEND,L=L1A,ANGLE=-AA,TILT=LT

L1B=0.174533,AB=0.7854,D5=0.1,D6=0.2,D7=0.2,
D8=0.15,D8=0.15,V5=1.0654,V6=19.8557,V7=8.85194,
V8=6.04386

B2: SBEND, L=L1B, ANGLE=AB, TILT=LT
S5:DRIFT,L=D5, Q5:QUAD, L=LQ, K1=V5, TILT=LT
S6:DRIFT,L=D6, Q6: QUAD, L=LQ, K1=V6, TILT=LT
S7:DRIFT,L=D7, Q7:QUAD, L=LQ, K1=V7, TILT=LT
S8:DRIFT,L=D8, Q8:QUAD,L=LQ,K1=V8, TILT=LT
S8:DRIFT,L=D8, Q7: QUAD, L=LQ, K1=V7 , TILT=LT
S7:DRIFT,L=D7, Q6: QUAD, L=LQ, K1=V6, TILT=LT
S6:DRIFT,L=D6, Q5: QUAD, L=LQ, K1=V5, TILT=LT
S5:DRIFT,L=D5
B2: SBEND, L=L1B, ANGLE=AB, TILT=LT
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CONCLUSION

We presented a design of an "uncoupled" and achromatic gantry which can deliver the same beam spot size, independent of the angular orientation of the gantry, on a tumor. The beam size depends on the beam emittance and beam parameters at the entrance of the gantry. We are planning a similar design of a gantry which utilizes combined function magnets which can transport and focus on the tumor beams with rigidity of ~7 T-m.

REFERENCES

[1] N. Tsoupas et. al. "Uncoupled achromatic tilted S-bend" Presented at the 1st Biennial European Particle Accelerator Conference, Genoa, Italy, June 23-27, 2008.
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[4] F. Iselin "MAD" CERN/LEP-TH/88-38.