NEAR REAL-TIME ORM MEASUREMENTS AND SVD MATRIX GENERATION FOR 10 HZ GLOBAL ORBIT FEEDBACK IN RHIC

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Abstract

To reduce the effect of trajectory perturbations (~10 Hz) due to vibrations of the final focusing quadrupoles at RHIC, global orbit feedback was successfully prototyped during run-10. After being upgraded to a system with 36 BPMs and 12 correctors, 10 Hz feedback was tested successfully in Run-11 and is in operational status for the physics program. The test and operation of the system has been performed using transfer functions between the beam position monitors and correctors obtained from the online optics model and a correction algorithm based on singular value decomposition (SVD). One of our goals is to self-calibrate the system using SVD matrices derived from orbit response matrix (ORM) measurements acquired real-time using the new FPGA-based signal processing. Comparisons between measurement matrix and model matrix and the generation of SVD matrix for the feedback operation are presented.

INTRODUCTION

The 10 Hz global orbit feedback system was designed to damp the 10 Hz horizontal beam perturbations in both rings that are suspected to be caused by vibrations of the final focusing quadrupoles (triplets) [1, 2]. Prototype testing was successfully carried out during RHIC Run-10 in store condition with 4 new dipole correctors (with independent power supplies) and 8 stripline beam position monitors (BPMs) per accelerator. The upgraded system [3] for Run-11 consists of 36 BPMs, corresponding to 2 per triplet in each of the 12 triplet locations and two in each of the 6 arcs, and 1 dipole corrector at each triplet location for a total of 12 correctors [4]. With limited machine development time, the new system significantly damps the 10 Hz perturbations to both beams and has been in operational status shortly after the physics program started. Fig. 1 shows the 10 Hz oscillation amplitude with and without 10 Hz feedback engaged.

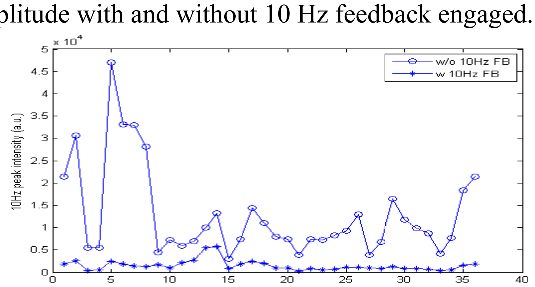

Figure 1: 10 Hz oscillation amplitude in frequency domain at dedicated BPMs for cases of with and without feedback.

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ORBIT RESPONSE MATRIX

The required corrector strength can be obtained based on the beam position measurements and inversion of response matrix using either SVD (singular value decomposition) algorithm or least-square algorithm. To reduce the transition time, python scripts were developed for Run-11 to retrieve Twiss parameters at BPMs and correctors for 10 Hz feedback system from the online model OptiCalc for orbit response matrix calculation; to invert the response matrix with regularization of eigenvalues and to save SVD matrix in a format ready to be used by the controller, ML510.

The algorithm for the response matrix calculation is based on the closed orbit distortion [5] induced by dipole field error, of which the coefficient is

$$R_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 sin\pi\nu} \cos\left(\pi\nu - |\varphi_i - \varphi_j|\right) \quad (1)$$

R_{ij} here represents the response of the ith BPM to the jth corrector; β and φ are the corresponding beta function and phase advance; ν is machine tune. These coefficients constitute a $m \times n$ response matrix R for a feedback system with m BPMs and n correctors.

The matrices generated from online model have been confirmed using an independent model. Difference of only a few percent between these two methods has been observed.

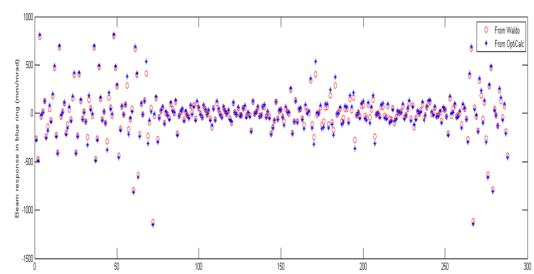

Figure 2: Comparison of the generated model matrices for blue.

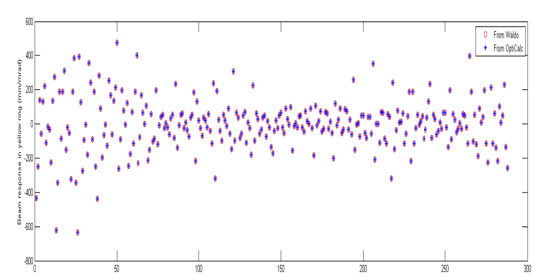

Figure 3: Comparison of the generated model matrices for yellow.

In order to avoid errors in BPM and corrector calibrations and in the optics model, ORM measurements were performed. The closed orbits at dedicated BPMs were logged while exciting the 12 correctors sequentially, each for 40 s with 5 s in between; the correctors were driven at 12 Hz; the BPM measurements were recorded at a sampling rate of 1 kHz. Although the response of orbit at BPMs to strength of correctors are expected to be larger at store than at injection because of beta squeeze, higher corrector current is required at store to produce noticeable driven oscillation due to larger beam rigidity. The current amplitude is set as 6 A at injection (23.8 GeV), and 10 A at store (250 GeV).

Three methods for generating the response matrix from the experimental data have been evaluated [6]. All methods aim to extract the slope Δx/Δθ due to the applied excitation while minimizing contributions due to other sources of perturbation to the beam trajectory such as from the triplet vibrations. The difference observed between these three methods was ~ 2%.

In the following plots, the measured matrix elements generated by the Fast Fourier Transform method are presented to be compared with model matrix. The measured response of BPMs to strength of correctors in IR 6 & 8 region is ~30% less than expected; ~20% less in other regions. The discrepancy justifies the self-calibration of the system and is consistent with recent beta beat measurements [7].

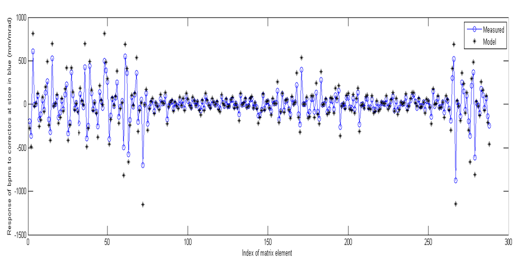

Figure 4: Comparison of the measured matrix with model matrix for blue.

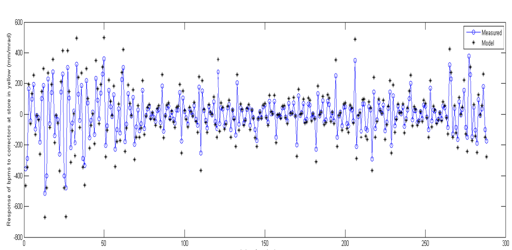

Figure 5: Comparison of the measured matrix with model matrix for yellow.

SVD MATRIX GENERATION

With either model or measured response matrix, an inversion of the matrix determines the response of correctors to BPM measurements in order to damp the orbit oscillation amplitude. Considering the intentional degeneracy (for exception handling) of the response matrix, SVD algorithm [8] is adopted to invert the matrix because of the luxury of eigenvalue manipulation. Using SVD algorithm, the response matrix is decomposed as

$$R = USV^T \tag{2}$$

Where U is a $m \times m$ unitary matrix, V is a $n \times n$ unitary matrix, and S is a $m \times n$ diagonal matrix with nonnegative eigenvalues on the diagonal in descending order.

$$S = \begin{pmatrix} \sigma_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \sigma_2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \sigma_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \sigma_n & 0 \end{pmatrix} \tag{3}$$

After manipulation of eigenvalues which will be discussed afterwards, the inversion of response matrix, called SVD matrix here, can be expressed as

$$R_{SVD} = V(S')^{-1}U^T \tag{4}$$

Where S' is the new diagonal matrix after eigenvalue manipulation; superscript -1 means the pseudo inverse of S'. The SVD matrix R_{SVD} is $n \times m$ by dimension.

The point of eigenvalue cut or any eigenvalue regularization is to cut large corrector strength correspond to small eigenvalues resulted from the degeneracy of the feedback system, which is extremely helpful when the corrector strength are limited. Furthermore, it helps to reduce the impact of BPM measurement noise. The importance of eigenvalue cut was also demonstrated from Run-10 experience, during which keeping 2 of the 4 eigenvalues produced 10 Hz oscillation damping while feedback without eigenvalue cut failed. Considering the necessity of eigenvalue regularization, a code was developed to study the effect of different eigenvalue cut and optimize feedback performance. The basic idea is to generate random BPM measurements (even better with real data), calculate the required corrector strength and residual orbit oscillation at BPMs.

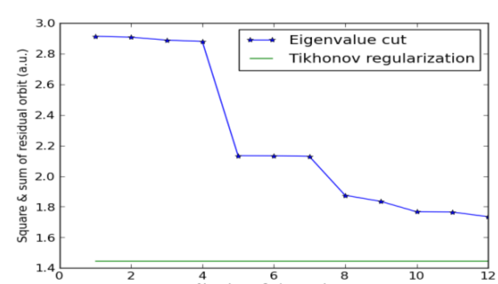

Figure 6: Expected residual oscillation at BPMs with various eigenvalue cut and Tikhonov regularization.

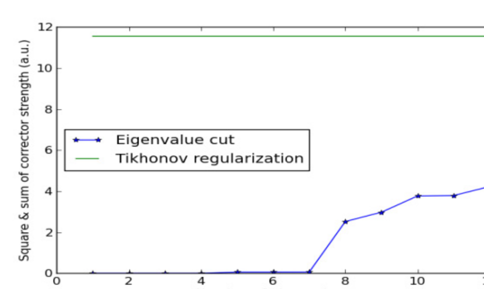

Figure 7: Expected corrector strength with various eigenvalue cut and Tikhonov regularization.

Besides eigenvalue cut, Tikhonov regularization (with $\alpha = 1$) was studied as well; further study of Tikhonov regularization is under way to optimize Tikhonov factor. The inversion of response matrix is more precise with more eigenvalues being kept, therefore, better feedback performance is expected, which however requires stronger corrector strength. One has to balance the expected damping effect against corrector strength to make the feedback system practical. Therefore, 6 of the 12 eigenvalues were kept for the SVD matrix for feedback operation in Run-11, which ensured steady feedback performance and was later on confirmed by the experimental study to be an optimal choice. The new diagonal matrix after eigenvalue cut is

$$S = \begin{pmatrix} \sigma_1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & \sigma_6 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \end{pmatrix} \quad (5)$$

Experimental study of eigenvalue cut has been performed in the APEX (Accelerator Physics Experiment) session. Only 5 data points were recorded because the corrector strength exceeded the limit with 8 eigenvalues which caused erratic feedback behaviour.

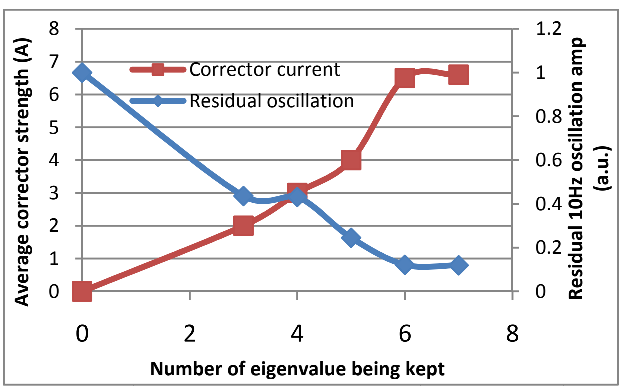

Figure 8: Damping effect and corrector strength with different eigenvalue cut.

The residual 10 Hz oscillation for the secondary vertical axis is the average of the peak amplitude at frequency ~ 10 Hz at BPMs, which is normalized with respect to the average without feedback. The case with no eigenvalue kept means no feedback being applied. The result shown in Fig. 8 proved the simulation result in Fig. 6 & 7. The eigenvalue cut 6 and 7 produced similar results in terms of both corrector strength and residual oscillation.

The following plot shows the residual 10 Hz oscillation at all 36 BPMs of which 24 IR BPMs being used in the feedback loop for both cases. The first two and last two BPMs are at IR6 triplets, followed by 2 arc BPMs and 5 sets of 4 IR BPMs + 2 arc BPMs. The relative large residual oscillation at IR10 BPMs is expected due to the magnet's longitudinal offset [3] to the source of 10 Hz oscillation.

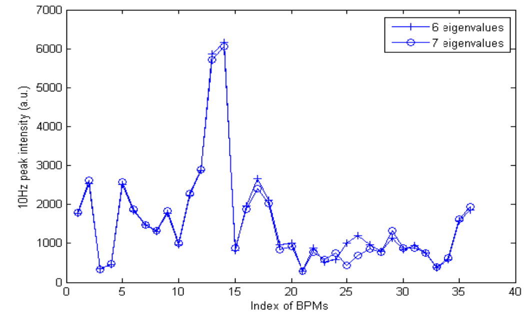

Figure 9: The residual 10 Hz oscillation peak intensity in frequency domain at 36 dedicated BPMs for eigenvalue cut 6 and 7.

CONCLUSION

The orbit response matrix for 10 Hz global orbit feedback system was generated based on the Twiss parameters from the online model. SVD algorithm was adopted for inversion of the response matrix; simulation of eigenvalue cut effect shows optimal feedback performance for 6 eigenvalues being kept with achievable corrector strength. This was later on proved by the experimental study. The success of every respect of the 10 Hz global orbit feedback system, including matrix generation, made it operational shortly after Run-11 started.

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