

# THE EFFECTS OF BETATRON PHASE ADVANCES ON BEAM-BEAM AND ITS COMPENSATION IN RHIC\*

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## Abstract

In this article we perform simulation studies to investigate the effects of betatron phase advances between the beam-beam interaction points on half-integer resonance driving terms, second order chromaticity and dynamic aperture in RHIC. The betatron phase advances are adjusted with artificial matrices inserted in the middle of arcs. The lattices for 2011 polarized proton (p-p) run and 2010 RHIC Au-Au runs are used in this study. We also scan the betatron phase advances between IP8 and the electron lens for the proposed Blue ring lattice with head-on beam-beam compensation.

## INTRODUCTION

According to Ref. [1], the triplet quadrupoles in the interaction regions (IRs) IR6 and IR8 contribute most of the second order chromaticities for the RHIC lattices. The reason is that  $\beta$  functions in the triplets are very big due to the low  $\beta^*$  at the interaction points (IPs) IP6 and IP8.

According to Ref. [2, 3], the second order chromaticities are given by

$$\xi_{x,y}^{(2)} = -\frac{1}{2}\xi_{x,y}^{(1)} + \frac{1}{8\pi} \oint [\mp K_1 \pm K_2 D_x] \frac{\partial \beta_{x,y}}{\partial \delta} ds + \frac{1}{8\pi} \oint \pm K_2 \beta_{x,y} D_x^{(2)} ds, \quad (1)$$

where  $K_1$  and  $K_2$  are the strengths of quadrupoles and sextupoles.  $\xi_{x,y}^{(1)}$  are the first order chromaticities.  $\beta_{x,y}$  are betatron amplitude functions.  $D_x$  and  $D_x^{(2)}$  are the first order and second order horizontal dispersion.  $\delta$  is the relative momentum deviation.

The second term in Eq. (1) is the dominant one which is determined by the off-momentum  $\beta$ -beat  $\frac{\partial \beta_{x,y}}{\partial \delta}$ . The relative off-momentum  $\beta$ -beat [4] is

$$\frac{1}{\beta_{x,y}(s)} \frac{\partial \beta_{x,y}(s)}{\partial \delta} = \pm \frac{1}{2 \sin(2\pi Q_{x,y})} \oint \beta_{x,y}(s') [\mp K_1(s') \pm K_2(s') D_x(s')] \cos(2|\phi_{x,y}(s) - \phi_{x,y}(s')| - 2\pi Q_{x,y}) ds'. \quad (2)$$

We define half integer resonance driving terms as

$$h_{20001} = \sum_i^N [-(K_1 L)_i + (K_2 D_x L)_i] \beta_{x,i} e^{-i2\phi_{x,i}}, \quad (3)$$

$$h_{00201} = \sum_i^N [(K_1 L)_i - (K_2 D_x L)_i] \beta_{y,i} e^{-i2\phi_{y,i}}. \quad (4)$$

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Here  $h_{20001}$  and  $h_{00201}$  are the horizontal and vertical half integer resonance driving terms. Comparing Eqs. (3) and (4) to Eq. (2), the relative off-momentum  $\beta$ -beat is governed by the half integer resonance driving terms.

Therefore, minimizing the contributions from the triplets from IR6 and IR8 to the half integer resonance driving terms will reduce off-momentum  $\beta$ -beat and second order chromaticities. In the following we perform simulation studies to scan the betatron phase advances between IP6 and IP8 to investigate their effects on the second order chromaticities and dynamic aperture.

## PHASE ADVANCE ADJUSTMENT

### RHIC Facts

The RHIC ring lattices have more or less three fold symmetry, with three outer arcs and three inner arcs. Between adjacent arcs are interaction regions. Each arc consists of 11 FODO cells. In RHIC, all the focusing and all the defocusing quadrupole magnets in the arcs are powered by the same power supplies. So far it is not possible to adjust individual arc's phase advances with quadrupoles in FODO cells. Considering the three-fold symmetry of RHIC rings, the betatron phase advances in the 3 outer or in the 3 inner rings are more or less the same.

To adjust the betatron phase advance between IP6 and IP8, we should adjust the betatron phase advances for all the outer and inner rings at same time. To keep the tunes constant, if we increase or decrease the phase advances of all outer rings, we will decrease or increase same amounts of betatron phase advances in all inner rings.

### Phase Adjustment in Simulation

To avoid lengthy real lattice matching, in the simulation we adjust the phase advances by inserting artificial phase shifting matrices in the centers of arcs. The phase shifting matrices will not affect tunes, Twiss parameters and dispersion functions. Again, to keep the tunes constant, if the phase shifting matrices in the outer rings increase or decrease the phase advances, the phase shifting matrices in the inner rings will decrease or increase the same amount.

Comparing the real lattices and the above 'fake' simulation study lattices, the phase advances between IPs and nonlinear field errors in the IRs are the same although the phase advances of FODO cells are different. As we know, the dynamic aperture is mainly determined by the nonlinear field errors and beam-beam interactions for RHIC lattices [5]. Therefore, the results from our simulation study

with artificial phase shifting matrices will reflect most of the physics with the real lattices.

## RESULTS

### Simulation Setup

In this study we adopt the lattices for the 2011 RHIC polarized proton (p-p) run and 2010 RHIC Au-Au runs, and the Blue ring lattice for head-on beam-beam compensation. For the head-on beam-beam compensation lattice, we will study the effects of phase advance between IP8 and e-lens which is located 1.0 m away from IP10.

For each lattice, we will scan the horizontal and vertical phase advances between IPs. For each step, we calculate the absolute values of half integer resonance driving terms only from the triplets in the IR6 and IR8 to check the cancellation between IR6 and IR8. The reference point of resonance driving terms is IP6. We also calculate second order chromaticities and dynamic apertures (DAs). The second order chromaticities change in the scan because of the betatron phase changes between IRs. The phase shifting matrices themselves have no contributions to chromaticities.

DAs are calculated in  $10^6$  turn trackings with code SimTrack [6]. The tunes of zero-amplitude particles are set to (28.675, 29.68) for proton run lattices and (31.23, 32.22) for Au-Au run lattices. These tunes are with beam-beam interaction at IP6 and IP8. The first order chromaticities are set to (1,1). The initial relative momentum deviation is 0.0005 for protons and 0.0017 for Au ions. The DA for Au-Au run is after RF re-bucketing. The proton bunch intensities for the 2011 p-p run lattices is  $1.5 \times 10^{11}$ . The Au ion bunch intensities for the 2010 Au-Au run lattices is  $1.0 \times 10^9$ . The proton bunch intensity for the head-on beam-beam compensation test is  $2.5 \times 10^{11}$ . Half beam-beam compensation is included in this lattice.

### 2011 p-p run Blue Lattice

As an example, here we give the results with the 2011 p-p run Blue ring lattice. Fig. 1 shows the calculated horizontal half integer resonance term  $h_{20001}$  from the triplets in IR6 and IR8 for the 2011 p-p run Blue ring lattice. The default betatron phase advances between IP6 and IP8 are  $(10.65\pi, 8.64\pi)$ . From Fig. 1, the best cancellation of horizontal half integer resonance term occurs at  $\Delta\Phi_x = 10.65\pi$ . And the best cancellation of vertical half integer resonance term occurs at  $\Delta\Phi_y = 8.7\pi$ .

Fig. 2 shows the horizontal second order chromaticity versus the betatron phase advance between IP6 and IP8. The minimum of horizontal second order chromaticity occurs around  $\Delta\Phi_x = 10.4\pi$  and  $\Delta\Phi_x = 10.8\pi$ . The minimum of vertical second order chromaticity occurs around  $\Delta\Phi_x = 8.5\pi$  and  $\Delta\Phi_x = 9.0\pi$ . The minimum of half integer resonance terms and second order chromaticities do not always coincide with each other since the half integer resonance terms are calculated only with triplets in IR6 and

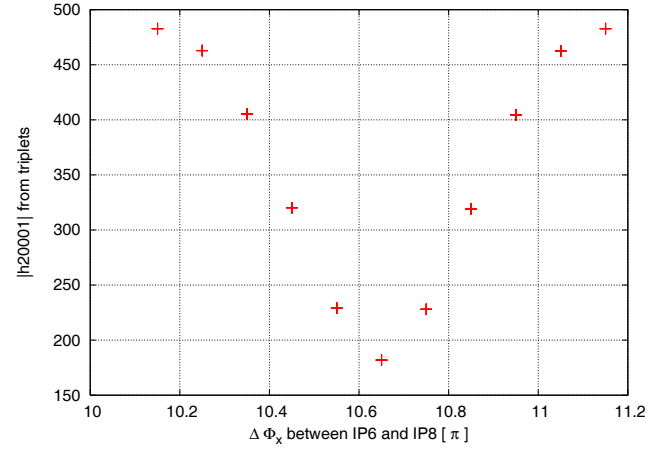


Figure 1: 2011pp-Blue: Horizontal RDT versus phase advances between IP6 and IP8.

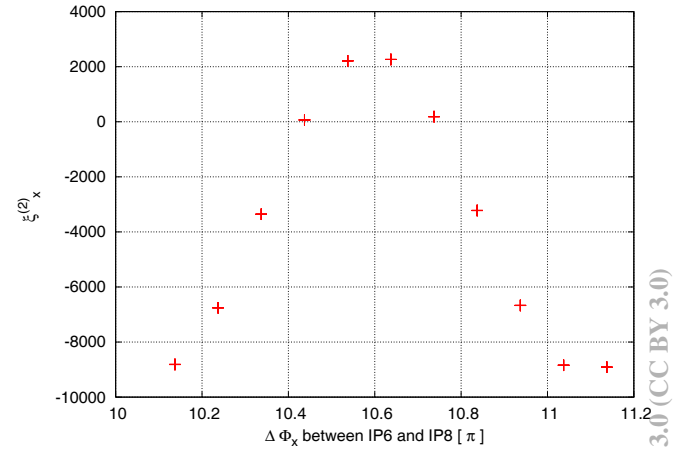


Figure 2: 2011pp-Blue: Horizontal second order chromaticities versus phase advances between IP6 and IP8.

IR8. With the artificial phase shifting matrices, the betatron phase advances in the arcs are not same as the real lattice.

Fig. 3 shows the DAs in the phase advance scan. The beam-beam interactions at IP6 and IP8 are included. From Fig. 3, there are a large area of good DA between  $\Delta\Phi_x = (10.4 - 10.8)\pi$  and  $\Delta\Phi_y = (8.4 - 8.6)\pi$ . The maximum DA happens at  $(\Delta\Phi_x = 10.65\pi, \Delta\Phi_y = 8.5\pi)$  where the second order chromaticities are less than 2000.

### Beam-beam Compensation Lattice

As another example, here we show the calculated DAs in the scan of betatron phase advances between IP8 and the center of e-lens. In this study, half beam-beam compensation is adopted. Half head-on beam-beam compensation will compensate half of the proton-proton beam-beam parameter. In the current design, we would like to have  $k\pi$  phase advances between IP8 and e-lens for better cancellation of beam-beam nonlinearities at IP8 [7]. In the early head-on beam-beam compensation study, we assumed the phase advances between IP8 and e-lens to be  $(9\pi, 11\pi)$ .

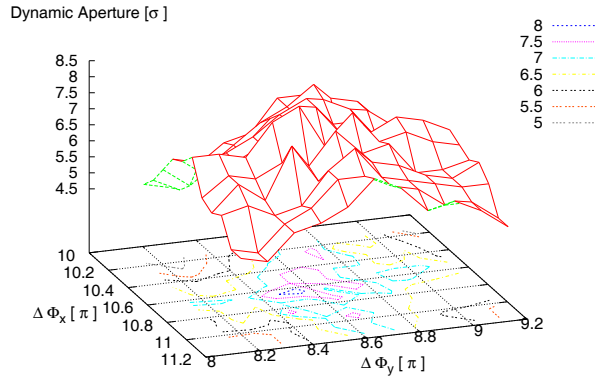


Figure 3: 2011pp-Blue: Dynamic aperture versus phase advances between IP6 and IP8.

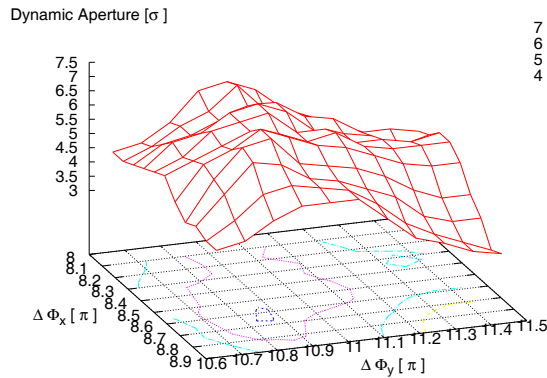


Figure 4: E-lens lattice : Dynamic aperture versus phase advances between IP8 and e-lens.

We calculated the second order chromaticities in the scan of phase advances between IP8 and the center of e-lens. The minimum of horizontal and vertical second order chromaticities happen at  $\Delta\Phi_x = 8.35\pi$  and  $\Delta\Phi_y = 11.07\pi$ .

Fig. 4 plots the DAs in the same phase advance scan between IP8 and the center of e-lens. The maximum DA occurs at  $(\Delta\Phi_x = 8.65\pi, \Delta\Phi_y = 10.85\pi)$ . There are a large good DA region between  $\Delta\Phi_x = (8.1 - 8.9)\pi$  and  $\Delta\Phi_y = (10.7 - 11.1)\pi$ .

The default phase advances between IP8 and the e-lens center is  $(8.5\pi, 11.0\pi)$ . The phase advances in the design are  $(9.0\pi, 11.0\pi)$  which is away from the good DA region shown in Fig. 4.

## DISCUSSION

In the above we calculated half integer resonance driving terms, second order chromaticities and DA in the scan of phase advances between IP6 and IP8. Our goal is to find out the optimum betatron phase advances between IPs which give smaller second order chromaticities and bigger

DAs. Smaller second order chromaticities are very important for the Au-Au run where RF-rebucketing increases the off-momentum deviation.

From the above studies, the phase advances for the minimum half integer resonance driving terms and for the minimum second order chromaticities do not always coincide. This may be explained by the fact that half integer resonance driving terms are calculated only from the triplets in IR6 and IR8. The contributions from other quadrupoles and sextupoles are excluded since we do not have their exact phase advances with artificial phase shifting matrices.

From second order chromaticities and DA calculation, we always find that good DA area in the  $(\Delta\Phi_x, \Delta\Phi_y)$  plane always have small second order chromaticities. For low  $\beta^*$  lattices, it is very important to compensate the large second order chromaticities. From our study, it is possible to adjust the phase advances between IPs to minimize the triplets' contributions to second order chromaticities.

In the above study we fixed the tunes unchanged during the scan of phase advances between IPs. We also need to do tune scan to find out their best settings to maximize the dynamic aperture. The tunes used in this study are close to their values in the RHIC operations.

To confirm the optimum betatron phase advances between IP6 and IP8 found in this study, we need to generate real RHIC lattices and calculate second order chromaticities and DA again. This work will be discussed in the coming RHIC summer shutdown.

## SUMMARY

In this article we performed simulation studies to investigate the effects of betatron phase advances between the beam-beam interaction points on half-integer resonance driving terms, second order chromaticity and dynamic aperture in RHIC. The betatron phase advances are adjusted with artificial matrices inserted in the middle of arcs. The lattices for 2011 polarized proton (p-p) run and 2010 RHIC Au-Au runs are used in this study. We also scan the betatron phase advances between IP8 and the electron lens for the proposed Blue ring lattice with head-on beam-beam compensation.

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