CHROMATIC ANALYSIS AND POSSIBLE LOCAL CHROMATIC CORRECTION IN RHIC*

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Abstract

In this article we will answer the following questions: 1) what is the source of second order chromaticities in RHIC? 2) what is the dependence of second order chromaticity on the on-momentum β-beat? 3) what is the dependence of second order chromaticity on β^* at IP6 and IP8? To answer these questions, we use the perturbation theory to numerically calculate the contributions of each quadrupole and sextupole to the first, second, and third order chromaticities. possible methods to locally reduce chromatic effects in RHIC rings are shortly discussed.

PERTURBATION THEORY

Based on the perturbation theory [1], the horizontal and vertical betatron tune changes due to a small quadrupole error $\Delta k_1(s)$ are

$$\Delta Q_{x,y} = \pm\frac{1}{4\pi}\oint \beta_{x,y}(s)\Delta k_1(s)ds. \quad (1)$$

where $\beta_{x,y}(s)$ are the unperturbed horizontal and vertical amplitude functions.

The off-momentum tune shifts are

$$\Delta Q_{x,y} = \xi^{(1)}_{x,y}\delta + \xi^{(2)}_{x,y}\delta^2 + \xi^{(3)}_{x,y}\delta^3 ... \quad (2)$$

where $\delta = \Delta p/p_0$. The first define the first, second, and third order chromaticities are

$$\xi^{(1)}_{x,y} = \frac{\partial Q_{x,y}}{\partial \delta}, \quad (3)$$

$$\xi^{(2)}_{x,y} = \frac{1}{2}\frac{\partial^2 Q_{x,y}}{\partial \delta^2}, \quad (4)$$

$$\xi^{(3)}_{x,y} = \frac{1}{6}\frac{\partial^3 Q_{x,y}}{\partial \delta^3}. \quad (5)$$

For an off-momentum particle, $\Delta k_1(s)$ in Eq. (1) from quadrupoles and sextupoles is given by

$$\Delta k_1(s)_{x,y} = [\pm K_1(s) \mp K_2(s)D_x(s)](\frac{1}{1+\delta} - 1) \quad (6)$$

where K_1 and K_2 are the nominal strengths of quadrupoles and sextupoles, $D_x(s)$ is the horizontal dispersion.

Plugging Eq. (6) into Eq. (1) and only keep the terms of δ, we obtain the first order chromaticities

$$\xi^{(1)}_{x,y} = \frac{1}{4\pi}\oint \beta_{x,y}(s)[\mp K_1(s) \pm K_2(s)D_x(s)]ds. \quad (7)$$

Take derivative of Eq. (7) with respect to δ and keep the terms of δ^2 in Eq. (6), we obtain the second order chromaticities [2, 3]

$$\begin{aligned}\xi^{(2)}_{x,y} = & -\tfrac{1}{2}\xi^{(1)}_{x,y} + \tfrac{1}{8\pi}\oint[\mp K_1 \pm K_2 D_x]\tfrac{\partial \beta_{x,y}}{\partial \delta}ds \\ & +\tfrac{1}{8\pi}\oint \pm K_2\beta_{x,y}D^{(2)}_x ds.\end{aligned} \quad (8)$$

Similarly, the third order chromaticities [3] are

$$\begin{aligned}\xi^{(3)}_{x,y} = & -\tfrac{1}{3}\xi^{(2)}_{x,y} + \tfrac{1}{24\pi}\oint[\mp K_1 \pm K_2 D_x]\tfrac{\partial^2 \beta_{x,y}}{\partial \delta^2}ds \\ & +\tfrac{1}{24\pi}\oint[\pm K_1 \mp K_2 D_x]\tfrac{\partial \beta_{x,y}}{\partial \delta}ds \\ & +\tfrac{1}{24\pi}\oint[\pm K_2 D^{(2)}_x]\tfrac{\partial \beta_{x,y}}{\partial \delta}ds \\ & +\tfrac{1}{24\pi}\oint[\pm K_2 D^{(3)}_x]\beta_{x,y}ds \\ & +\tfrac{1}{24\pi}\oint[\mp K_2 D^{(2)}_x]\beta_{x,y}ds \\ & +\tfrac{1}{24\pi}\oint \pm K_2 D^{(2)}_x\tfrac{\partial \beta_{x,y}}{\partial \delta}ds.\end{aligned} \quad (9)$$

where we defined $D_x = \frac{\partial x_{co}}{\partial \delta}$, $D^{(2)}_x = \frac{\partial^2 x_{co}}{\partial \delta^2}$, and $D^{(3)}_x = \frac{\partial^3 x_{co}}{\partial \delta^3}$.

SOURCES OF CHROMATICITIES

In this section, we will use the above Eqs. (6)-(8) to search the sources of second order chromaticities. In our studies, the 2009 RHIC 100 GeV polarized proton (p-p) run Blue ring lattice, the 2011 RHIC 250 GeV p-p run Blue ring lattice, and the 2010 RHIC 100 GeV Au-Au run Yellow ring lattice are used. For the 2009 RHIC 100 GeV p-p run Blue ring lattice and the 2010 RHIC 100 GeV Au-Au run Yellow ring lattice, the nominal β^* at IP6 and IP8 is 0.7 m. For the 2011 RHIC 250 GeV p-p run Blue ring lattice, the nominal β^* at IP6 and IP8 is 0.65 m. The simulation code SimTrack [4] is used for this study.

To localize the sources of chromaticities, the first order chromaticities are not corrected. The uncorreced horizontal and vertical chromaticities are (-89.8, 87.1), (-95.2, -94.0), and (-95.6, -101.3) for the 2009 RHIC 100 GeV p-p run Blue ring lattice, the 2011 RHIC 250 GeV p-p run Blue ring lattice, and the 2010 RHIC 100 GeV Au-Au run Yellow ring lattice. We define each arc is between Q(and Q9. Q9s are not included in interaction regions (IRs).

As an example, Fig. 1 shows the contributions from each quadrupole and sextupole to the first order horizontal chromaticity $\xi^{(1)}_x$. Figure 2 plots each section's contributions in percentage. In this example, the 2011 RHIC 250 GeV p-p run Blue ring lattice is used. The first order chromaticities are not corrected. Table 1 summarizes each section's contributions in percentage to the whole ring for the three above lattices. The percentage is defined as the contribution of one section divided by the absolute value of the whole ring's value.

For the above three lattices, IR6 and IR8 contributed about 50% or more to the first order chromaticities and

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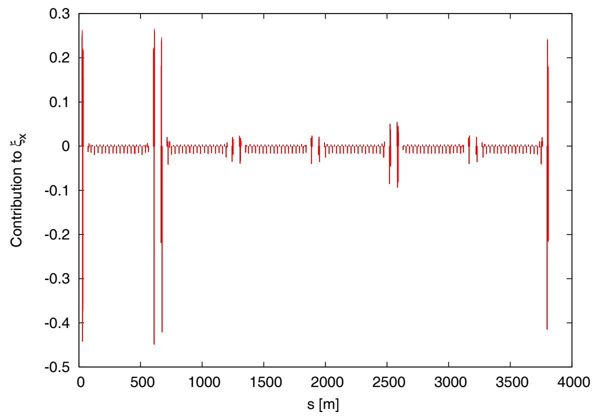

Figure 1: Each element's contribution to $\xi_x^{(1)}$ along the ring for 2011 250 GeV p-p run Blue ring lattice.

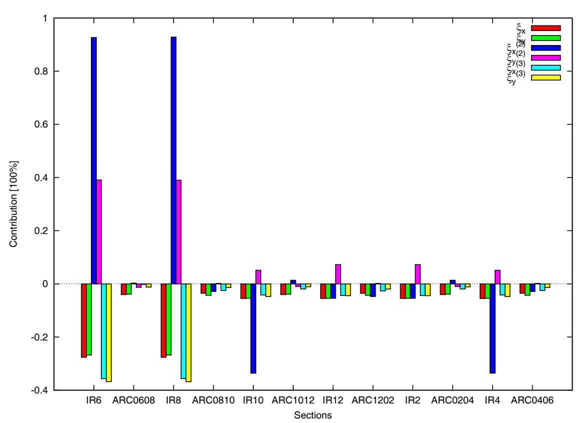

Figure 2: Section contributions to chromatities with 2009 250GeV p-p run Blue ring lattice ($\xi^{(1)}$ uncorr.)

more than 70% to the third order chromaticities. IRs contribute more than 90% to the second order chromaticities. For the p-p run lattices, IR6 and IR8 contribute 185% and 120% of the total horizontal second order chromaticity for 2009 and 2011 run lattices. The non-colliding IRs contribute -77% and -20%. For the 2010 Au-Au run Yellow ring lattice, each IR contribute about 20% to the horizontal second order chromaticity. IR6 and IR8 contributes about 120% to the vertical chromaticity. For the $\xi_{x,y}^{(1)}$ uncorrected lattices, the arcs contribute about 20% to the first order chromaticities. However, their contributions to the second and third order chromaticities are small.

CHROMATICITY DEPENDENCE ON ON-MOMENTUM β-BEAT

In this section, we will investigate the effects of on-momentum β-beat on the second order chromaticities. The on-momentum β-beat is the β change of on-momentum particles due to quadrupole errors. For RHIC store lattice, the on-line measured on-momentum β-beat is about 20%.

In this simulation study, we randomly assign strength errors to all the quadroples to generate on-momentum β-beat. The quadrupole errors are given in percentage. The error percentage are generated with the formula $MaxErr*(rnd(1)-0.5)*2$, where function $rnd(1)$ is the random number generator which produces random numbers with a uniform distribution between (0-1), $MaxErr$ is maximum strength error in percentage.

Table 1: Contributions to the Linear and Nonlinear Chromaticities

Sections	$\xi_x^{(1)}$	$\xi_y^{(1)}$	$\xi_x^{(2)}$	$\xi_y^{(2)}$	$\xi_x^{(3)}$	$\xi_y^{(3)}$
2009-pp-B:						
IR6 and IR8	-0.55	-0.53	1.85	0.78	-0.71	-0.73
Other IRs	-0.21	-0.21	-0.77	0.24	-0.17	-0.18
Arcs	-0.22	-0.24	-0.07	-0.02	-0.11	-0.081
2011-pp-B:						
IR6 and IR8	-0.58	-0.57	1.20	0.84	-0.74	-0.78
Other IRs	-0.20	-0.20	-0.20	0.17	-0.15	-0.16
Arcs	-0.21	-0.22	0.00	-0.02	-0.10	-0.05
2010-Au-Y:						
IR6 and IR8	-0.47	-0.50	0.34	1.15	-0.74	-0.86
Other IRs	-0.26	-0.24	0.71	-0.20	-0.17	-0.03
Arcs	-0.25	-0.25	-0.06	0.04	-0.08	-0.10

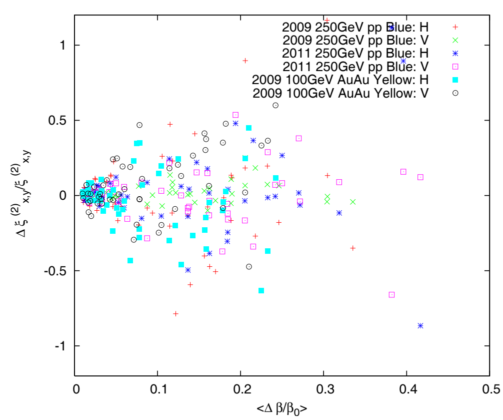

Figure 3: Second order chromaticity changes versus on-momentum β-beat

We focus on the averaged on-momentum β-beat $<\Delta\beta_{x,y}/\beta_{x,y,0}>$ along the ring. In most cases, the horizontal and vertical on-momentum β-beats scale with each other. Therefore, in the following discussion, we will only use the averaged transverse on-momentum β-beat $<\Delta\beta/\beta_0>$. The changes of second and third order chromaticities are defined by $\Delta\xi_{x,y}^{(2)}/\xi_{x,y,0}^{(2)}$ and $\Delta\xi_{x,y}^{(3)}/\xi_{x,y,0}^{(3)}$.

Figure 3 shows the changes in the second order chromaticities due to the on-momentum β-beat for the above three lattices. In this plot, "H" represents the horizontal plane and "V" represents the vertical plane. For the above three lattices, 10% on-momentum β-beat may introduce 30% changes in second order chromaticities which is about 1200 units. When $<\Delta\beta/\beta_0>$ is below 20%, the third order chromaticity changes are less than 50%. According to

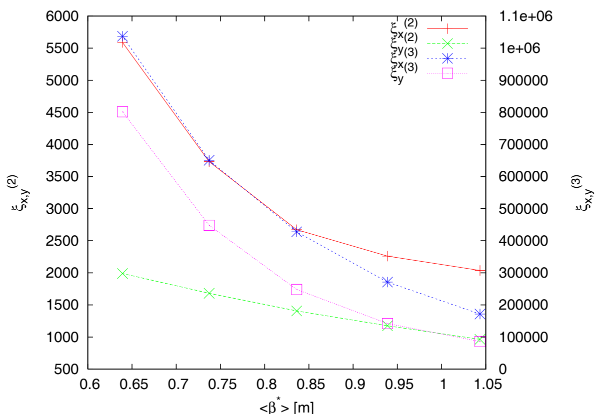

Figure 4: Chromaticities versus β^* for 2010 100 GeV Au-Au run Yellow ring lattices

the RHIC online second order chromaticity correction experience, to correct 1000-2000 units of second order chromaticity shoul not be very difficult. Therefore, below 10% on-momentum β-beat is tolerable for second order chromaticity correction.

CHROMATICITY DEPENDENCE ON β^*

In this section we investigate the nonlinear chromaticity's dependence on β^* at IP6 and IP8. In this study, the 2009 100 GeV p-p run Blue ring lattices and the 2010 100 GeV Au-Au run Yellow ring lattices are used. The first order chromaticities are corrected to (1,1).

As an example, Fig. 4 shows the second and third order chromaticities as functions of β^* for the 2010 100 GeV Au-Au run Yellow ring lattices. The absolute values of second and third order chromaticities increase when the β^*s are decreased. And in most cases, the increase in nonlinear chromaticities with β^* are faster than linear growth. For the 2009 100 GeV p-p run Blue ring lattices, when the β^* is 0.5 m, the vertical second order chromaticity reaches 6000 and the horizontal third order chromaticity reaches 2.5×10^6 . For the 2010 100 GeV Au-Au run Yellow ring lattices, when β^* is 0.65 m, the horizontal second order chromaticity reaches 5500 and the horizontal third order chromaticity reaches 1.0×10^6. For lattices with $\beta^* < 0.6$ m, the second order chromaticities needed to be corrected to keep acceptable store beam lifetime.

CHROMATICITY COMPENSATION

For Eq. (8), the second order chromaticities are mostly determined by off-momentum β-beat and therefore by half-integer resonance driving terms. Half-integer resonance driving terms are defined as [5]

$$h_{20001} = \sum_i^N [-(K_1 L)_i + (K_2 D_x L)_i] \beta_{x,i} e^{-i2\phi_{x,i}}, \quad (10)$$

$$h_{00201} = \sum_i^N [+(K_1 L)_i - (K_2 D_x L)_i] \beta_{y,i} e^{-i2\phi_{y,i}}. \quad (11)$$

Here h_{20001} and h_{00201} are the horizontal and vertical half integer resonance driving terms.

In the operation of RHIC, based on the contributions of the chromatic sextupole families to half-integer resonance driving terms, we sorted the RHIC chromatic sextupoles into 4 pairs to correct the second order chromaticities. This method is called 4-knob method [5]. Its advantage is that it will not change the first order chromaticities and it reduces the unbalance in the correction strengths among the sextupole families, and avoid reverting sextupole polarities. This method was tested and implemented into on line RHIC control system.

Since the triplet quadrupoles in the IR6 and IR8 contribute most of the second order chromaticities for RHIC lattices. This gives a possibility to adjust the betatron phase advances between IP6 and IP8 to cancel their contributions to their half integer resonance driving terms to minimize the global second order chromaticities. Simulation study have shown that this method is effective and increases the dynamic aperture [6, 7]. Adjusting the phase advances between IP6 and IP8 in a real RHIC lattice is under consideration.

Another possible way to correct the second order chromaticities in RHIC is to install extra sextupoles in the interaction regions IR6 and IR8 to locally correct the chromatic effects. The location of local correction sextupoles, their strength requirement, and how to cancel their contributions to the third order resonance driving terms are being studied.

SUMMARY

In this article we performed chromatic analysis with RHIC store lattices. The sources of chromaticities are mostly localized in IR6 and IR8 . Chromaticity dependences on on-momentum β-beat and β^* are studied. The correction methods to second order chromaticities are reviewed. Possible local correction methods, like adjusting phase advances between IPs and installing extra sextupole correctors in IR6 and IR8 are shortly discussed.

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