ANALYSIS OF NSLS-II TOUSCHEK LIFETIME*

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Abstract

As scrapers are adopted for the loss control of NSLS-II storage ring, Touschek lifetime estimations for various cases are required to assure the stable operation. However, to estimate the Touschek lifetime, momentum apertures should be measured all along the ring and, if we want to estimate the lifetime in various situations, it can take extremely long time. Thus, rather than simulating for each case, a semi-analytic methods with the interpolations are used for the measurements of the momentum apertures. In this paper, we described the methods and showed the results.

INTRODUCTION

Having enough Touschek lifetime is important for synchrotron light source for the users to perform experiments with stable beams. In NSLS-II, the scrapers will be installed for the loss control. Especially, the horizontal scrapers will be installed where the dispersions are maximum. Therefore, we need to find the proper scraper gap values which do not reduce the lifetime too much for the stable beam operation. To estimate reliable Touschek lifetime, we should measure the momentum apertures at many positions along the ring. For the rough estimation of the Touschek lifetime, the RF momentum acceptance can be used and for a more detailed estimation, the linear approximation of synchrotron oscillation can be used. However, for the strong focussing synchrotrons, like NSLS-II, the linear approximation is not enough to obtain the reliable momentum apertures, and, in general, particle tracking simulations are used. However, for NSLS-II case, we need to track the particle about 400 turns at each point to make it a full synchrotron oscillation period and to obtain the reliable Touschek lifetime we need measure the momentum apertures at several hundred positions at least. Therefore, it can take quite a long time if we want to have the reasonable resolution for the measurements. Furthermore, at the simulation, if we want to measure the aperture inside a element, we should divide the element and this will make the simulation longer.

Here, we developed a systematic method to calculate the momentum aperture using interpolation tables. Once the tables are prepared, the precise momentum aperture can be calculated instantly at arbitrary position for various situations.

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MOMENTUM ACCEPTANCE

We can make a Hamiltonian for the longitudinal dynamics for phase ψ and $\dot{\psi}$ as [1]

$$H = \frac{1}{2}\dot{\psi}^2 + \frac{\Omega^2}{\cos\psi_s}\left(\cos(\psi_s+\psi) - \cos\psi_s + \psi\sin\psi_s\right). \quad (1)$$

In the linear approximation we can obtain the relation between $\dot{\psi}$ and momentum as

$$\dot{\psi} = -\omega_{rf}\eta_c\delta, \quad (2)$$

where ω_{rf} is angular velocity of RF, $\eta_c = 1/\gamma^2 - \alpha_c$, and α_c is the dilation factor. With Eqs. 1 and 2, we can obtain the synchrotron oscillation in phase space as in Fig. 1.

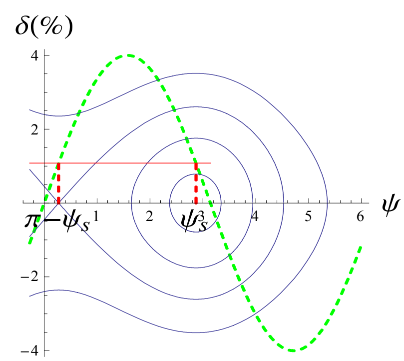

Figure 1: Longitudinal phase space in linear approximation with RF voltage in arbitrary unit.

From Fig. 1, we can find the momentum acceptance from the separatrix where the trajectory passes the point ($\pi-\psi_s$, 0) and it is about 2.6% in this case. On the other hand, with the general path length dilation $\Delta l / L_0$, the relation between $\dot{\psi}$ and δ becomes

$$\dot{\psi} = -\omega_{rf}\left(\frac{\delta}{\gamma^2} - \frac{\Delta l}{L_o}\right). \quad (3)$$

LONGITUDINAL SPACE DYNAMICS AND TOUSCHEK LIFETIME

Path Length and RF Phase Advance

If there is no vertical curvature, the total path length L can be written by

$$L = \oint\sqrt{(1+\kappa x)^2 + x'^2 + y'^2}\,ds, \quad (4)$$

where κ is the horizontal curvature. If we express x and y as

$$x = x_0 + x_\beta$$
$$y = y_0 + y_\beta \quad (5)$$

where x_0, y_0 are closed orbits and x_β, y_β represent betatron oscillations, with the betatron amplitudes J_x and J_y, the path length dilation can be written as

$$\Delta L = \oint \kappa x_0 ds + \frac{J_x}{4}\oint \gamma_x ds + \frac{J_y}{4}\oint \gamma_y ds. \tag{6}$$

For the evaluation of $\oint \kappa x_0 ds$, we should note that closed orbit x_0 can be written as the sum of betatron amplitude independent and dependent closed orbits, $\tilde{x}_0$ and x_{0J}, and the betatron amplitude dependent term comes from the sextupoles. If we denote the strength of a sextupole *j* as m_j, the average kick from the sextupole, in thin lens approximation, can be written as

$$\Delta\theta_j = \frac{1}{2} m_j (J_x \beta_{xj} - J_y \beta_{yj}). \tag{7}$$

Then the closed orbit distortion from all the sextupoles become

$$x_{0J}(s) = \frac{\sqrt{\beta_x(s)}}{2\sin \pi\nu_x} \frac{1}{2} \sum_{j=sext} m_j l_j (J_x\beta_{xj} - J_y\beta_{yj})\sqrt{\beta_{xj}} \cos(|\psi_{xj} - \psi_x(s)| - \pi\nu_x). \tag{8}$$

Using Eq. 8, amplitude dependent contribution in Eq. 6 becomes [2,3,4];

$$\Delta L_J = \oint \kappa x_{0J} ds + \frac{J_x}{4}\oint \gamma_x ds + \frac{J_y}{4}\oint \gamma_y ds = -\pi(\xi_x J_x + \xi_y J_y). \tag{9}$$

where ξ_x and ξ_y are horizontal and vertical chromaticities, respectively.

If we apply Eq. 3 to one turn, we get

$$\Delta\psi = \dot{\psi} T_0 = -2\pi h\left(\frac{\delta}{\gamma^2} - \frac{\Delta l}{L_o}\right), \tag{10}$$

where *h* is the harmonic number. And, summing up all the contributions, Δl can be expressed as

$$\Delta l = \oint \kappa \tilde{x}_0 ds - \pi(\xi_x J_x + \xi_y J_y), \tag{11}$$

where $\tilde{x}_0$ is betatron amplitude independent closed orbit.

To obtain the momentum aperture, we launch a particle from the origin only with the momentum deviation. Therefore, if the particle is launched at a position with non-zero closed orbit, the particle will have the betatron oscillation with amplitude corresponding to the closed orbit, which is the dispersion when we do not consider the machine imperfection. The longitudinal dynamics of such particle is, if we consider at least one synchrotron oscillation period, is equivalent to the particle launched at the position with the same energy and the specific betatron amplitude without the dispersion. For the reliable tracking for the strong focussing synchrotron, like NSLS-II, it was known that the dispersion is required at least up to the 3rd order [5]. However, instead of analytic formulae for higher order dispersion and Twiss functions, we used the simulation code ELEGANT [6] to find the closed orbits for various energies. Also we used the Twiss functions for various energies obtained from ELEGANT simulation.

Radiation

The synchrotron radiation power depends on energy (E) and magnetic field (B) as [7]

$$P_\gamma = \frac{e^2 c^3}{2\pi} C_\gamma E^2 B^2, \tag{12}$$

where C_γ is a constant,

$$C_\gamma \approx 8.85\times 10^{-5} mGeV^{-3}. \tag{13}$$

If the power of Eq. 12 is integrated along the time when the particle travels the curvature, we can obtain the radiated energy. From Eqs. 9, the path length in the curvature should be expressed as,

$$\Delta l_{\kappa>0} = \oint_{\kappa>0} \kappa \tilde{x}_0 ds - \pi(\xi_x J_x + \xi_y J_y) - \frac{J_x}{4}\oint \gamma_x ds - \frac{J_y}{4}\oint \gamma_y ds. \tag{14}$$

Tracking

Now, using Eqs. 11 and 14, we can track the particle turn by turn in the longitudinal space, assuming the particle energy is constant for one turn and J_y=0. For the damping wigglers, we consider only their radiations neglecting the effects on the path length. Here, we used the energy dependent chromaticity and beta functions from ELEGANT instead of analytic formula [8].

Using the data described above, we obtained the momentum apertures at various horizontal amplitudes (J_x) as described in Fig. 4.

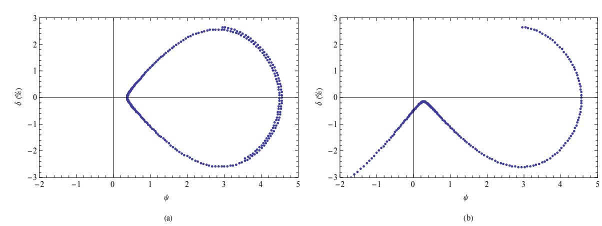

Figure 2: The particle trackings with J_x=0 starting at (δ, ψ) = (2.65%, ψ_s=2.87) (a) and (δ, ψ) = (2.66%, ψ_s=2.87) (b), respectively. We can see that the positive momentum aperture is between 2.65% and 2.66%

The betatron amplitude can be found using the Twiss functions as the following equation.

$$J_x = \gamma_x(\delta,s)[\tilde{x}_0(\delta,s)]^2 + 2\alpha_x(\delta,s)\tilde{x}_0(\delta,s)\tilde{x}_0'(\delta,s) + \beta_x(\delta,s)[\tilde{x}_0'(\delta,s)]^2 \tag{15}$$

And using the interpolation, the momentum aperture for the given betatron amplitude (J_x) is obtained as Fig. 3.

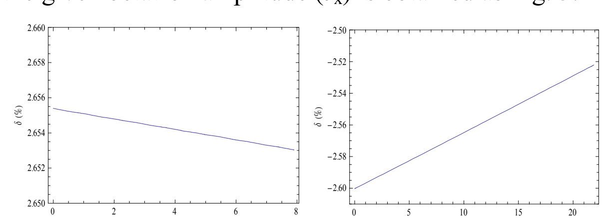

Figure 3: The positive and negative momentum apertures for horizontal betatron amplitudes.

Now, the momentum aperture at the given position can be obtained from the Fig. 3 and Eq. 15. And momentum apertures along the ring show agreement with the ELEGANT simulations. Using the negative momentum apertures, which are smaller because of non-linear dispersion, we can obtain the Touschek lifetime for NSLS-II as 3.64 hours.

TOUSCHEK LIFETIME WITH HORIZONTAL SCRAPERS

In this section we studied the effect of the physical aperture on the Touschek lifetime. Fig. 4 shows the simplified horizontal physical aperture for one super cell. The bars of upper side show the photon absorbers and the two bars at the lower side are scrapers.

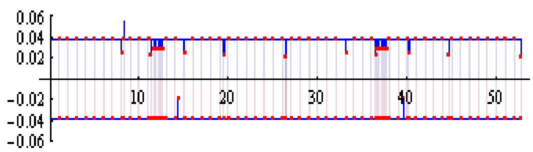

Figure 4: Horizontal physical aperture with photon absorbers and scrapers.

Since the negative dispersion is larger than the positive one, we found that the momentum apertures are limited only by the scrapers, as shown in Fig. 5.

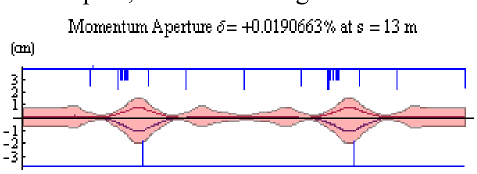

Figure 5: Beam envelope with scrapers at -19 mm defining the positive momentum aperture at s=13 m.

By tracking the particle in longitudinal space, with energies inside the momentum apertures obtained in the previous section, we can obtain the momentum apertures along the ring with scrapers of various insertion positions. Fig. 6 shows one of the results in the negative direction, which decides the Touschek lifetime, together with the apertures obtained from the ELEGANT simulation when the scraper insertion position is -19 mm. In this figure, the blue line is the ELEGANT result and the slight disagreement is supposed to be coming from the uneven positions of measurements as well as the finite number of turns in the simulations.

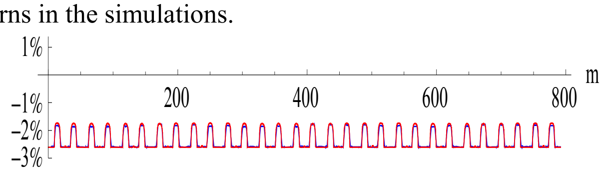

Figure 6: The momentum aperture when the scraper position is -19 mm. The red line is from the longitudinal beam dynamics and the blue line is from ELEGANT simulation.

Using the momentum, we obtained the Touschek lifetime variation depending on the scraper gap as in Fig. 7, together with the ELEGANT result.

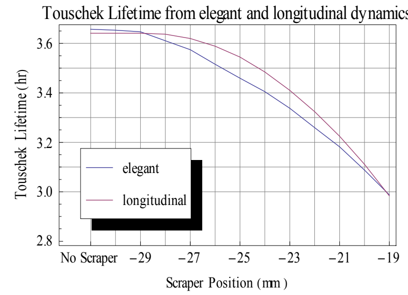

Figure 7: Touschek lifetimes for the scraper gaps, from longitudinal dynamics and from ELEGANT simulation.

SUMMARY AND DISCUSSION

By tracking the particle in the longitudinal space, we obtained reliable momentum apertures and corresponding Touschek lifetimes. Even though it also takes some time to obtain the basic parameter tables using interpolation, the applications to the various situations are almost instant. That is, for the given lattice, we can obtain the momentum aperture at arbitrary point quickly and we can also apply these tables to the cases with various physical apertures, such as scrapers. Furthermore, we can change the measurement positions at any time and need not sacrifice the resolution of the momentum aperture to obtain the results in the reasonable time as in simulation.

Without scrapers, the momentum apertures are given by the longitudinal dynamics only. However, if RF voltage is high enough to accommodate the large betatron oscillation amplitude which, through couplings, can cause the beam loss from the horizontal and vertical spaces not from the longitudinal dynamics, the apertures from longitudinal space are providing only the outmost boundaries and the momentum apertures will be decided by the nonlinear dynamics from the large betatron amplitude or by physical apertures. Especially, when the ring has the narrow-gap undulators, as in NSLS-II, the momentum aperture can be decided by the gap heights. The momentum apertures in these cases can be obtained by extending the method with the scrapers, considering the dynamic apertures too.

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