STUDY OF SINGLE AND COUPLED-BUNCH INSTABILITIES FOR NSLS-II *

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Abstract

We study single and coupled-bunch instabilities for the NSLS-II storage ring with a recently developed parallel tracking code. For accurate modelling of the coupled-bunch instability, we investigate improvements to current point-bunch models to take into account finite bunch-size effects.

INTRODUCTION

Accurate modelling of single and coupled-bunch instabilities is of crucial importance for the machine performance of light sources such as NSLS-II, that provide high current beams [1]. The complexity of the model consists in the accurate calculation of the impedance of the various components of the ring that can excite single and multi-bunch instabilities via short and long range wakefields. Moreover, a full account of the coupling between transverse and longitudinal dynamics must be taken into account to study effects such as chromaticity and Landau cavity effects. A parallel algorithm for the study of single and coupled-bunch instabilities has been implemented in a particle tracking code. The theoretical framework for single bunch instabilities is the same used in the code TRANFT [2], while for coupled-bunch instabilities a self-consistent algorithm has been implemented to allow the study of finite bunch-size effects and multibunch effects in arbitrary filling modes. In this paper we present numerical studies of the microwave instability for NSLS-II and discuss the self-consistent algorithm for simulation of coupled-bunch instabilities.

PHYSICAL MODEL FOR SINGLE BUNCH INSTABILITIES

For the study of single bunch instabilities we use the same physical model implemented in the particle tracking code TRANFT [2]. Model considers only one transverse variable and couples the synchrotron and betatron motion.

The one turn map ($s \to s+C_0$, where s is path length and C_0 the ring circumference) for the longitudinal dynamics is

$$\begin{aligned}\bar{\epsilon} &= \epsilon + \frac{q}{mc^2}(V_0(\tau) - V_n(\tau)) + \delta\epsilon - \frac{T_0}{T_r}\epsilon + V_s(\tau;s)\\ \bar{\tau} &= \tau + \frac{T_0\eta}{\beta^2\gamma_0}\bar{\epsilon}\end{aligned} \quad (1)$$

where

$$\begin{aligned}V_1(\tau) &= \sin(\omega_{rf}\tau + \phi_s) - \sin\phi_s\\ V_n(\tau) &= \frac{1}{n^2}\sin\phi_s(\cos n\omega_{rf}\tau - 1) + \frac{1}{n}\cos\phi_s \sin n\omega_{rf}\tau\\ V_s(\tau;s) &= -\int_{-\infty}^{\tau} W_s(\tau-\tau')\lambda(\tau';s)d\tau'.\end{aligned}$$

Here ϵ is the energy deviation from the reference particle, τ is arrival time, $\delta\epsilon$ is a quantum excitation random kick, T_0 is the revolution time, T_r is the synchrotron radiation damping time, η is the frequency slip factor and $\beta = v_0/c$ for a reference particle with velocity v_0 and Lorentz factor γ_0. V_1 is the RF voltage of the fundamental cavity operating at $\omega_{rf} = h\omega_0$ where $\omega_0 = 2\pi/T_0$ and V_n is RF voltage of the Landau cavity operating at the n^{th} harmonic of ω_{rf}. For the operation of NSLS-II a third harmonic Landau cavity will be used to increase the bunch length without increasing the energy spread. V_s is the longitudinal voltage originated by the longitudinal bunch density $\lambda(\tau, s)$.

The one turn map for the transverse dynamics is composed by the map

$$\begin{aligned}\bar{x} &= x\cos\psi(\epsilon) + p\sin\psi(\epsilon)\\ \bar{p} &= -x\sin\psi(\epsilon) + p\cos\psi(\epsilon) + xV_d(\tau;s) + V_x(\tau;s)\\ \psi(\epsilon) &= \psi_0 + \frac{2\pi\xi}{\beta^2\gamma_0}\epsilon,\end{aligned} \quad (2)$$

where

$$\begin{aligned}V_d(\tau;s) &= \int_{-\infty}^{\tau} W_d(\tau-\tau')\lambda(\tau';s)d\tau',\\ V_x(\tau;s) &= \int_{-\infty}^{\tau} W_x(\tau-\tau')D_x(\tau';s)d\tau'\end{aligned}$$

and by a kick due to radiation damping and diffusion

$$\bar{x} = x - \frac{T_0}{T_x}x + \delta x, \qquad \bar{p} = p - \frac{T_0}{T_x}p + \delta p. \quad (3)$$

Here ϕ_0 is the on-momentum phase advance, ξ is the chromaticity, T_x is the transverse radiation damping time and δx and δp are quantum random excitations. The collective force term V_d is driven by the transverse detuning (or quadrupolar) wake W_d and the collective force term V_x is driven by the transverse wake potential W_x. $D_x(\tau;s) = \int x f(\epsilon,\tau,x,p;s)d\epsilon dx dp$ is the instantaneous transverse dipole density, where f is the phase space density.

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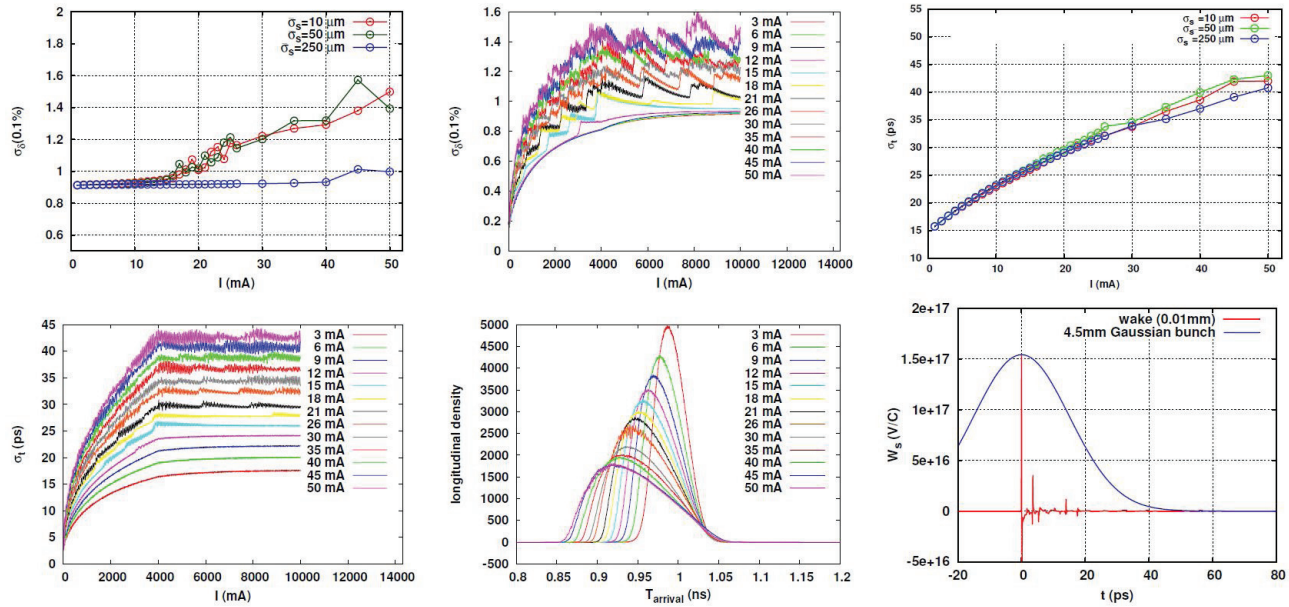

Figure 1: Top left: comparison of rms energy spread vs current for values of the wakepotential driven by a bunch length of $\bar{\sigma}_s = 0.01$mm, $\bar{\sigma}_s = 0.05$mm and $\bar{\sigma}_s = 0.25$mm. Top center: rms energy spread vs number of turns for $\bar{\sigma}_s = 0.01$mm. Top right: the same as top left for bunch length vs current. Bottom left: bunch length vs number of turns for $\bar{\sigma}_s = 0.01$mm. Bottom center: longitudinal density for different values of current for $\bar{\sigma}_s = 0.01$mm. Bottom right: wakepotential calculated with the code ECHO for $\bar{\sigma}_s = 0.01$mm.

Algorithm for Particle Smoothing

In the particle tracking code TRANFT an algorithm based on fast Fourier transforms is used for smoothing. Here we propose a density estimation technique based on a Fourier expansion (see [4] for a detailed discussion). The longitudinal densities are expanded in a finite Fourier series and the Fourier coefficients estimated via a Monte-Carlo integration. This gives a representation of the densities of class C^∞. This Fourier expansion is very efficient for parallelization since the computation of the Fourier coefficients can be distributed between the different processors and done without slave-to-slave communications. The simulations for microwave instability discussed in the next section have been done using 15M particles on 1000 processors at NERSC with a CPU time of approximately 20 minutes.

MICROWAVE INSTABILITY SIMULATIONS

Microwave instability simulations for NSLS-II have been done in [3]. The effect of pseudo-Green's functions for the calculation of the longitudinal wakepotential on the instability threshold has been studied for a minimum driving bunch length of $\bar{\sigma}_s = 0.05$mm, not to be confused with the bunch length σ_s of the particle distribution of the ring (the unperturbed nominal bunch length is $\sigma_s = 4.5$mm, as shown in Figure 1 (right frame)). For a good approximation of the Green's function used in the computation of the wake $\bar{\sigma}_s$ must be chosen small enough to give an accurate representation of the wakepotential. The pseudo-Green's functions have been calculated with the code ECHO. The microwave instability threshold was estimated to occur at an average single-bunch current greater than I=5mA. In this paper we calculate the instability threshold for a wakepotential calculated from $\bar{\sigma}_s = 0.01$mm as shown in Figure 1 (bottom right). To check converge in the results we calculate the wakepotentials corresponding to $\bar{\sigma}_s = 0.05$mm and $\bar{\sigma}_s = 0.25$mm convolving the pseudo-Green's function from $\bar{\sigma}_s = 0.01$mm with a Gaussian with $\sigma = 0.05$m and $\sigma = 0.25$m respectively. We used the same NSLS-II ring parameters as described in [3]. Here we recall that the number of particles used in the simulations shown in Figure 1 is 15M. It is found that the instability thresholds for $\bar{\sigma}_s = 0.01$mm and $\bar{\sigma}_s = 0.05$mm are roughly the same, as plotted in Figure 1 (top left), therefore showing that a driving bunch of $\bar{\sigma}_s = 0.05$mm gives a good approximation to the wakepotential for microwave instability simulations. In Figure 1 (top center) we plot the rms energy spread vs number of turns for $\bar{\sigma}_s = 0.01$mm. A microwave instability starts to develop at 3000 turns for $I = 12$mA. We conclude that the microwave instability threshold is ≈ 10mA. In (top right) we compare the bunch lengthening as a function of current for the different wakepotentials. The bunch lengthening is not so sensitive to the difference in $\bar{\sigma}_s$. In (bottom left) and (bottom center) we show bunch lengthening vs number of turns and longitudinal densities for different current values for $\bar{\sigma}_s = 0.01$mm respectively.

SELF CONSISTENT MODELLING OF COUPLED BUNCH INSTABILITIES

The model implemented in TRANFT for the study of coupled bunch instabilities (CBI) consists in tracking one bunch according to a kick produced by all the other bunches assumed to be distributed around the ring in a given configuration. This allows a fast calculation of CBI thresholds. Studies of CBI driven by resistive wall impedance for NSLS-II has been done in [1]. The model is not self-consistent and raises a question about accuracy and reliability. Moreover, it does not allow the study of CBI in arbitrary filling modes. For self-consistent modelling of coupled bunch instabilities, we developed a parallel algorithm where M bunches are distributed to M processors. The "history" of each bunch is stored in the master processor and broadcasted to the slave processors for the calculation of the coupled bunch kick. Let us consider the case of M bunches filling uniformly the ring and interacting via a transverse dipole wakefield. We let the bunches circulate around the ring for $\hat{n}$ turns and then turn on the coupled bunch interaction. We assume that bunch j ($j = 0, .., M-1$) receives a coupled bunch kick at locations $s_j = s_0 + jC_0/M$

$$V_{CB}^{j}(\tau, s_j) = \sum_{m=0}^{M-1}\sum_{k} c_{mk}\int_{-\infty}^{\tau} W_x(\tau-\tau') \times D_x^m(\tau', s_j - kC_0)d\tau', \quad (4)$$

where $c_{mk} = (1 - \delta_{0m}\delta_{0k})$ and D_x^m is the instantaneous transverse dipole density of bunch m.

Finite Bunch Length Effects

In many applications, such as coupled bunch instabilities driven by higher-order-modes in RF cavities, the long range wakefields vary over the bunch length of consecutive bunches, therefore finite bunch length effects must be taken into account.

In the case of the transverse dipole wakefield , if the long range part varies over the support of $D_x^m(\tau, s_j - kC_0)$, we calculate V_{CB} taking advantage of the Fourier expansion mentioned above. Specifically, from

$$\hat{D}_x^m(z, s) = \sum_{j=0}^{J} c_j^m(s)\phi_j(z),$$

where $\{\phi_j\}$ is the orthonormal basis $\phi_0(z) = 1$ and $\phi_j(z) = \sqrt{2}\cos(j\pi z)$ for $j \geq 1$, $z \in [0, 1]$, it follows

$$D_x^m(\tau, s) = \frac{1}{2L}\sum_{j=0}^{J} c_j^m(s)\cos\left[\frac{\pi j}{2}\left(\frac{\tau}{L}+1\right)\right], \quad (5)$$

where $\tau \in [-L, L]$, i.e. we assume that the distribution in arrival time is zero outside the interval $[-L, L]$, $L = 5\sigma_\tau$. Therefore, the contribution to the coupled bunch kick of bunch 1 on bunch 0 at the present turn is

$$V_{CB}^{0\leftarrow 1}(\tau, s) = \int_{-\infty}^{\tau} W_x(\tau-\tau')D_x^1(\tau', s)d\tau' \quad (6)$$
$$= \frac{1}{2L}\sum_{j=0}^{J} c_j^1(s)\int_{-\infty}^{\tau}\cos\left[\frac{\pi j}{2}\left(\frac{\tau'}{L}+1\right)\right]W_x(\tau-\tau')d\tau'.$$

This representation allows a fast calculation of CBI effects since in typical applications one is interested in calculating only few Fourier coefficients $c_j^m(s)$. Notice that the $c_j^m(s)$ are the only dynamical quantities to be determined for the coupled bunch kick and that the integral in (4) can be calculated upfront before to start the particle tracking. In case the CBI is driven by the transverse resistive wall wakefield

$$W_x(\tau) = H(\tau)\frac{cL_x}{\pi b^3}\sqrt{\frac{Z_0\rho}{\pi c\tau}}\ (\tau \gg \bar{s}/c), \quad \bar{s} = \left(\frac{2b^2\rho_c}{Z_0}\right)^{1/3}$$

where H is the Heaviside step function, we notice that since the separation between bunches is $T_b = T_0/M$ and the bunch length $\sigma_\tau << T_b$, assuming $W_x(\tau) \approx W_x(T_b)$ for $\tau \in [T_b - 5\sigma_\tau, T_b + 5\sigma_\tau]$ it follows

$$V_{CB}^{0\leftarrow 1}(\tau, s_0) = V_{CB}^{0\leftarrow 1}(s_0) = W_x(T_b)X^1(s_0).$$

where $X(s) = \int D_x(\tau; s)d\tau$, thus recovering the standard formula for point bunches

$$V_{CB}^{j}(s_j) = \sum_{m=0}^{M-1}\sum_{k} c_{mk}W_x(a_{mk}^j)X_k^m(s_j - kC_0),$$
$$\text{where } a_{mk}^j = (m-j)T_b + c_{mk}kT_0 + H(j-m)T_0.$$

CONCLUSION

In this paper we discussed the microwave instability for the NSLS-II ring in the limit of very short driving bunches for the calculation of the longitudinal wakepotentials, showing that $\bar{\sigma}_s = 0.05$mm gives a good representation of the longitudinal wakepotentials. We discussed a self-consistent model of coupled bunch instabilities and how the model takes into account of finite bunch length effects. We are planning to apply the self-consistent model to study coupled bunch instabilities for NSLS-II. Specifically, we plan to study chromaticity effects and Landau cavity effects in arbitrary filling mode. Based on these studies, a model of a transverse bunch-by-bunch feedback system will be included in the simulations to damp the instabilities.

REFERENCES

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