

# SC UNDULATOR WITH THE POSSIBILITY TO CHANGE ITS STRENGTH AND POLARIZATION BY FEEDING CURRENT

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## Abstract

We describe design of optimized undulator with SC windings able to generate magnetic field of opposite helicities, including the elliptic and the linear ones oriented as desired. For the undulator period 25mm and aperture 8mm, K factor could be changed from zero up to 1.5 by changing the feeding current. Polarization changed by changing the currents in additional helical windings.

## OVERVIEW

Undulators are planned for installation in all SR facilities. Unique properties of Undulator Radiation (UR), make it being a powerful tool for exploration in wide sciences. Important properties of UR are monochromaticity and polarization [1].

One unique peculiarity of installation such undulators in linear accelerators (ERL, FEL, ILC, etc) is that the aperture of undulator could be small; no necessity to make undulator with varying aperture there, as it sometimes required in cyclic machines for injection of the beam.

The ability to change polarization and K factor are crucial factors for many experiments. Meanwhile the undulator which is able to generate the radiation with variable polarization suggested a long time ago, [2]. Possibility of realization of such type of undulator with  $K \sim 1$  became open with usage of SC winding now.

The undulator considered there has two SC windings with opposite helicities, but with the same period. By changing the current in windings it is possible to arrange any polarization –from linear to a circular one and vice versa. Third winding having the same helicity as one of already existing, but shifted in longitudinal direction by half period, could be added for change the orientation of linear polarization.

## RADIATION

The total number of photons in all spectrum radiated by a single particle in an undulator having M periods and undulatority factor  $K \leq 1$  goes to be

$$N_\gamma \cong \frac{4\pi}{3} \alpha \cdot M \cdot \frac{K^2}{1+K^2} \quad (1)$$

where  $K = \frac{eH_0\lambda_u}{2\pi mc^2} (\cong 93.4 \cdot B[T] \cdot \lambda_u[m])$ ,  $H_0$  stands for the amplitude of magnetic field,  $\lambda_u$  is period of undulator. Cut-off energy (frequency) of first harmonics is a Doppler-shifted spatial frequency of undulator field

$$E_{\gamma_{\max}} \cong \hbar\omega_c = \frac{\hbar\Omega}{1-\vec{n}\vec{v}} \cong \frac{2.48 \cdot (\gamma/10^5)^2}{\lambda_u[cm](1+K^2)} [MeV], \quad (2)$$

where  $\Omega = 2\pi c / \lambda_u$ ,  $\vec{n}$  stands for the unit vector in direction to the observer,  $\vec{v}$  is an instant particle's velocity. As the period of undulator fixed by mechanical constrains, the only possibility to change the energy of the photon radiated in the fixed direction remains by changing the energy of particle, changing K factor, more precisely—the amplitude of magnetic field in an undulator. Other possibility—to change observed photon energy by varying observation angle inevitably changes polarization [1].

Really for  $K \leq 1$ , the Stokes parameters could be presented as [6]

$$\xi_{21} = \xi_{22} = \frac{1-\gamma^4 g^4}{1+\gamma^4 g^4}, \quad (3)$$

So for the observation angles  $g \cong 1/\gamma$  polarization comes to zero. This means, that mechanism of changing polarization should work independently from the observation angle.

As we mentioned, the photon energy could be changed by changing the beam energy. Changing energy of beam in ERL type machines requires keeping a RF balance of powers arranged for some specific energy and the current running in the machine. All these parameters restrained by the coupling of RF structure with the external RF generator. Basically ERL is tuned to fixed energy, although, some energy change is possible even here.

The only instance when the length (the number of periods M) might play important role could be described as the following. During each period of oscillation with spatial frequency  $\Omega$ , the front of radiation goes ahead of particle to the distance  $\cong \lambda_u \cdot (1+K^2) / 2\gamma^2$ . So if the observer would like to see the interference of radiation between the head and tail particles in a finite-size bunch having the length  $l_b \cong c\tau$ , the number of periods should be at least

$$M \geq \frac{2\gamma^2 l_b}{\lambda_u \cdot (1+K^2)} \cong \frac{2\gamma^2 c\tau}{\lambda_u \cdot (1+K^2)}. \quad (4)$$

Substitute here for estimation the time duty of a bunch  $\tau \cong 1ps$ ,  $\lambda_u \cong 2.5cm$ ,  $\gamma \cong 10^4$ ,  $K \sim 0$  (weak undulator), one can obtain  $M \geq 2.4 \cdot 10^6$ , i.e. the length  $L = \lambda_u M \geq 6 \cdot 10^6 cm$ . For 1 femtosecond bunch-length the length of undulator goes to  $L \geq 6 \cdot 10^3 cm = 60m$

minimum. It is clearly seen that this type of observations are problematic.

The average distance between particles in the bunch as it is seen by the observer in a Lab system is

$$\delta \cong (V/N)^{1/3} \cong (\sigma_x \sigma_y l_b / N)^{1/3}, \quad (5)$$

where  $N$  stands for the bunch population,  $\sigma_x, \sigma_y$  -are the transverse bunch sizes for corresponding directions. Substitute here for estimations

$$\sigma_x \cong \sigma_y \cong l_b \approx 1 \mu m (\tau \approx l_b / c \cong 3 \cdot 10^{-15} \text{ sec} \cong 3 \text{ fs}),$$

$N \cong 10^8$ , one can obtain  $\delta \cong 2.15 \cdot 10^{-7} \text{ cm}$ , meanwhile the wavelength of radiation for the first harmonic is  $\lambda_u / 2\gamma^2 \cong 2.5 / 2 \cdot 10^8 \approx 1.25 \cdot 10^{-8} \text{ cm} \cong 1.25 \text{ \AA}$ . One can see, that radiation, emitted by arbitrary chosen electron, will touch neighboring electron after  $\cong \delta / (\lambda_u / 2\gamma^2) \sim 20$  periods. This “touch” will yield some splashes of coherence in radiation. So the length of undulator must be 20 periods or 50cm minimum for 5fs long bunch (for 5ps the length comes to 5m minimum). The upper limitation defined by engineering only; we could see, that undulator, where radiation interference of radiation of neighboring electrons in bunch of 2 picoseconds, occurs if undulator is at least 10m long.

Variation of  $K$ -factor allows control of spectrum produced by undulator for coherence purposes. This means that for formation of radiation spectrum all undulator length plays role. Thus, the situation may happen, when energy radiated is so low, that it is not enough to build up quanta. In this case radiation is going statistically so in average, the energy radiated is the one calculated by classic formula. This effect might be important in process of Optical Stochastic Cooling at low intensity of radiation [8]. So one can imaging experiment at ERL, where beam power is high, the passage through weak undulator,  $K \rightarrow 0$ , could generate radiation which will manifest extreme statistical properties. That might be the situations, when passage of some bunches will generate much smaller power than the others.

### NUMERICAL MODEL

We investigated technological possibilities for such undulator here. As helical undulator is pure 3D system, we used appropriate codes able to carry 3D calculations of helical structure. We made a model for calculation with 2D and 3D code MERMAID [4]. 2D calculation with MERMAID is going by substitution of  $x, y, z$  coordinates by  $z$ -dependent rotating system which angle is a function of  $z$

$$B_\phi(\rho, \phi, z) \propto B(\rho) \cos[\phi - \phi_0 - 2\pi z / \lambda_u]. \quad (6)$$

Usage of Iron yoke increases axial field ~2 times. Few examples of different polarization could be generated with these three windings are presented in Figures below.

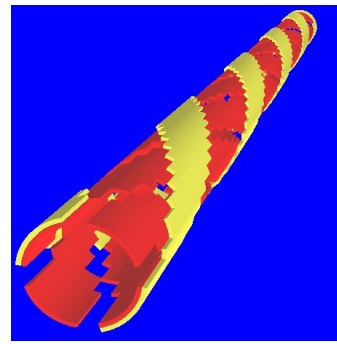


Figure 1: Scheme of windings as they appear in 3D MERMAID model. Two upper layer (yellow) windings have left-handed helicity inner windings are right handed ones.

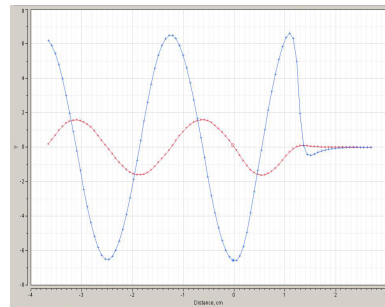


Figure 2: Elliptic polarization. Each component represented by it's individual plot. Field values are represented in kilo-Gauss units.

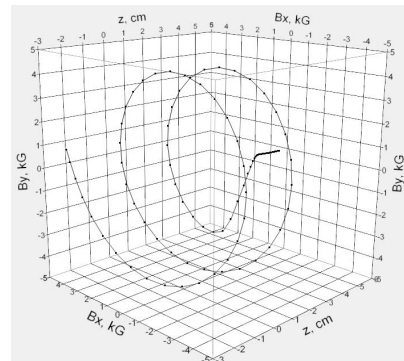


Figure 3: Circular polarization. The dots represent magnetic field values  $B_x, B_y$  as functions of longitudinal coordinate.

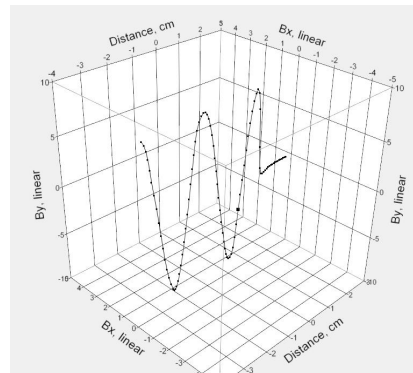


Figure 4: Linear polarization of magnetic field.

### PRACTICAL MODEL

We erected model of such undulator using Oxygen free Copper thin-wall tube of 8 mm-inner diameter and period of ~25mm, having the length ~30 cm. This gives for 5 GeV beam ( $\gamma \cong 10^4$ ) the cut-off energy for  $K=1$   $E_{\gamma_{max}} \approx 5keV$ . By changing  $K$  to a smaller value range, one can shift frequency by octave higher. This is going at fixed energy. Ability to change energy in some margins adds to the possibility for tuning ( $\sim \gamma^2$ ).

We used ribbon-type flat 6 wire strands stick together with Formvar for winding. Bare wire diameter is 0.3 mm, insulated-0.33 mm, ratio of SC to Copper is 1.2:1. First layer has 48 wires total; the outer layer has 2x12 wires.

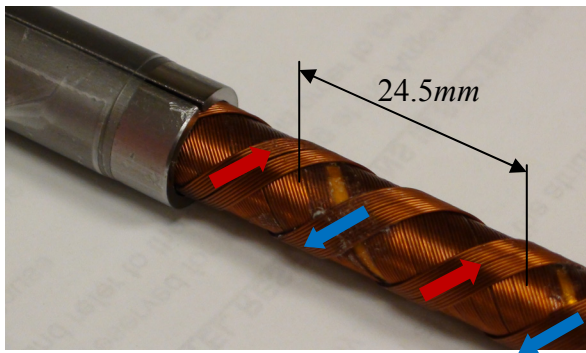


Figure 5: Coils wounded above Copper tube with six wire strands stick together in flat SC cable. Period 24.5 mm. Outer diameter is 10 mm, inner diameter clear for the beam is  $\varnothing_{inner}=8$  mm. Direction of current in the corresponding outer-layer coil is shown by the arrow.

Example of installation of this kind of cold mass in a cryostat one can find in [7]. Undulator, fabricated with technology tested could have the length up to 10 m as a single piece. Vacuum is not a problem here due to cryogenic pumping of cold inner surface of vacuum chamber.

As the vacuum chamber looking to the beam (made from Oxygen free Copper) is cooled down to Helium temperature, the vacuum is not a problem due to cryopumping. Additional pumping stations will be located between sections of undulator, unified with collimators.

Heating of vacuum chamber by imaginary current reduced here, as the resistance  $\rho$  of Copper at Helium temperature becomes lower by RRR factor and resistance  $\sim 1/\sqrt{RRR}$ . Conservative estimation for the losses by the normal skin effect can be as big as  $P[W/m] \cong I^2 R / \tau / f_0 \cong 1.1$  for 100mA average current (ERL). For such short pulses,  $\tau \sim 2ps$ , at low temperature the process is well under anomalous skin effect phenomenon, however. The mean free pass length ratio to the conductivity is not a function of temperature as

$\frac{1}{\rho} \cong \frac{ne^2 l_{free}}{p_F} \cong \frac{ne^2 l_{free}}{\hbar n^{1/3}}$ , where  $p_F$  is Fermi momentum,  $n$  is the electron density,  $l_{free}$  is a free electron pass length [8]. Calculations show that the free electron path is

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bigger than the normal skin depth~10 times. As  $\delta_{an}^3 \cong \delta_n^2 \cdot l_{free} = \delta_n^3 \cdot (l_{free} / \delta_n)$ , where  $\delta_n$  stands for the normal skin depth,  $\delta_{an}$  stands for anomalous one [8], the losses due to the anomalous skin effect will be less in a ratio  $10^{1/3} \cong 2.1$  times coming to  $P[W/m] \cong 0.55$ . So the losses are below 1 Watt/m and could be absorbed.

For operation of undulator three set of windings and three power supplies required. One of them serves for rotation of plane of polarization in arbitrary direction.

### SUMMARY

The type of undulator described allows easy manipulation by polarization and  $K$  factor independently. Ability to change  $K$  factor within 0-1.5 allows fine tuning of radiated spectrum. For mostly experiments this ability might be crucial.

Undulator is inexpensive, having small transverse outer size of cryostat (~3”) and fits well in SC RF systems planned worldwide. This type of undulator could be recommended for ERL in first place. Large aperture (8 mm inner diameter), period 25 mm, single piece length up to 10 m, varying  $K$  factor and polarization allows wide range of experiments to be carried with this device. Ratio of aperture diameter to the length of undulator could reach  $0.8cm/1000cm=8 \cdot 10^{-4}$ , what is the same as successfully operated pulsed undulator fabricated at Cornell for SLAC experiment E-166 (0.8mm/1000mm).

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