TRACKING PARTICLES THROUGH A GENERAL MAGNETIC FIELD*

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Abstract

A method that tracks particles directly through a general magnetic field described in a 3D field table was added to the code `elegant` recently. It was realized by converting an arbitrary particle's motion to a combination of free-drift motion and centripetal motion through the coordinate system rotation and using a general linear interpolation tool developed at the Advanced Photon Source (APS). This method has been tested by tracking particles through conventional magnetic elements (dipole, sextupole, etc.) to verify reference coordinate system conversions, tracking accuracy, and long-term tracking stability. Results show a very good agreement between this new method and the traditional method. This method is not designed to replace mature traditional methods that have been used in most tracking codes. Rather, it is useful for magnets with complicated field profiles or for studying edge effects.

INTRODUCTION

Magnets are one of the most common elements in accelerators, so tracking particles through a magnetic field is an important issue. In most of the widely used tracking codes [1, 2, 3] the magnets are divided into different groups based on their field profile, and different algorithms must be used. In addition, a real magnet always has a fringe-field, and to simulate its effect, some approximations must be made so that the equation of motion can be solved.

On the other hand, magnet design tools have developed rapidly and can provide a precise 3D field. A general linear interpolation module [4] based on the SDDS protocol [5] was recently developed at the APS. Using this tool, a new element called FTABLE was added to `elegant`, which provides a direct method of tracking particles through a general magnetic field described in a 3D field table.

In this paper, we first describe the method, i.e., how a particle is tracked through a general magnetic field. In the second section, we explain how the fringe-field is included and the treatment of a reference orbit with curvature. Finally, we present an example of tracking particles through the APS storage ring, with one of its sextupole families replaced by the FTABLE element.

EQUATION OF MOTION

Figure 1(a) illustrates a particle with momentum $\vec{p}$ traveling through a magnetic field $\vec{B}$ for a distance ds, with dz (the user-specified step size) being the projection of ds on the z-axis. In this normal coordinate system, the equation of motion is not a simple one. To find a general equation of motion, we rotated the coordinate system from (x, y, z) to a new coordinate system (u, v, w) using

$$\begin{aligned} \vec{u} &= \frac{\vec{B}\times\vec{p}}{|\vec{B}\times\vec{p}|} \\ \vec{v} &= \frac{\vec{B}}{|\vec{B}|} \\ \vec{w} &= \vec{u}\times\vec{v}. \end{aligned} \quad (1)$$

The particle's momentum $\vec{p}(u, v, w)$ and the magnetic field $\vec{B}(u, v, w)$ after rotation are shown in Fig. 1(b).

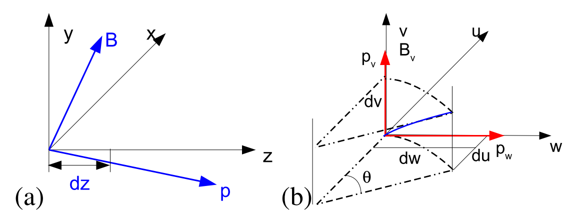

Figure 1: Magnetic field and particle momentum: (a) before coordinate system rotation; (b) after coordinate system rotation.

From Eq. (1), it's clear that in the (u, v, w) coordinate system, $p_u = B_u = B_w = 0$. The particle's motion can be decomposed into two parts: a free motion in direction v and a centripetal motion in plane $u - w$. If $\vec{B}$ is constant, the equation of motion can be solved exactly: the bending radius ρ in the u-w plane is given by $\rho = (B\rho)_0/|B|$, where $(B\rho)_0$ is the magnetic rigidity. The change of u, v, w is given by:

$$\begin{aligned} \Delta u &= -\rho(1-\cos\theta) \\ \Delta v &= \frac{p_v}{p_w}\rho\theta \\ \Delta w &= \rho\sin\theta, \end{aligned} \quad (2)$$

where θ is the bending angle and can be determined by transfering $(\Delta u, \Delta v, \Delta w)$ back to the (x, y, z) system and making dz equal to the given tracking step size

$$[\ \Delta x \quad \Delta y \quad \Delta z\ ]^T = R^T[\ \Delta u \quad \Delta v \quad \Delta w\ ]^T, \quad (3)$$

where R is the transfer matrix from the (x, y, z) system to the (u, v, w) system. Once θ is calculated, Δx and Δy are calculated using the same equation. At the end of a tracking

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step, we have:

$$\begin{aligned} p_{u,1} &= -p_{w,0}\sin\theta \\ p_{v,1} &= p_{v,0} \\ p_{w,1} &= p_{w,0}\cos\theta. \end{aligned} \quad (4)$$

Rotating the coordinate system back, we obtain the momentum $\vec{p}_1$ at the end of the tracking step. The path length change $\Delta l = \sqrt{(\rho\theta)^2 + dv^2}$, the length of the spiral line.

The next question is how to calculate $\vec{B}$ for each step. For a particle with initial coordinates $(x_0, x'_0, y_0, y'_0, p)$, the coordinates after step Δz are approximated by a simple drift. The field $\vec{B}$ is calculated at the midpoint of its path (i.e., a drift of $\Delta z/2$) through a linear interpolation of the known magnetic field given at the eight surrounding grid points.

The possible errors in this method are: using linear interpolation to calculate magnetic field instead of a method that reflects Maxwell's equations; using a constant $\vec{B}$ for each tracking step; and using a straight-line drift approximation to find the interpolation point. Because of these approximations, the method presented here is not a symplectic one. In order to determine how serious these errors might be, a group of particles that form a tiny phase space area are tracked through a sextupole magnet with $K_2L = 200m^{-2}$. The Hamiltonian of the system is

$$H = -\sqrt{1 - q_x^2 - q_y^2} + \frac{K_2}{6}(x^3 - 3xy^2), \quad (5)$$

with $q_x = x'/(1 + x'^2 + y'^2)$ and $q_y = y'/(1 + x'^2 + y'^2)$. The value of H is calculated at the entrance (H_0) and at the exit (H_1). Ideally, $\Delta H/H_0$ should be equal to zero. Figure 2 shows the tracking results for the same element but described by FTABLE or KSEXT, which uses a symplectic integrator. From the results, we see that the new method provides fairly good accuracy.

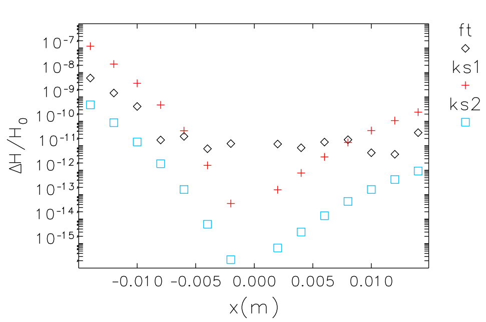

Figure 2: Change of H vs. initial launch position. (ft - use of FTABLE element; ks1 - use of KSEXT element with N_KICKS= 100; ks2 - use of KSEXT element with N_KICKS= 400.)

FTABLE ELEMENT

To use the FTABLE element for a general magnetic field, which is able to include the fringe-field region and work with the reference orbit change, we choose the reference frame as shown in Figure 3. It has the same definition of

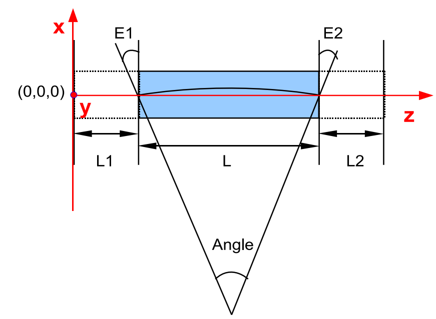

Figure 3: Reference frame of FTABLE element.

the CSBEND in elegant, except the length L here is the designed straight length instead of the arc length, and two more lengths $L1$ and $L2$ are added to each side of the element to include the fringe-field region. For example:

```
CB: ftable, input_file ="CB.input", &
n_kicks=100, l=0.5, l1=0.2, l2=0.2, &
angle=0.06, e1=0.03, e2=0.03
```

where, the file "CB.input" is a three-page SDDS file that contains the 3D field data (Bx, By, and Bz, each on a separate page). FTABLE also supports misalignments. Usage details, including file organization, are available in the online elegant manual.

Tracking particles through such an element proceeds in three steps, as illustrated in Figure 4). First, back-track particles through a "variable" drift to line 1, so the tracking will start from $z = 0$ of the FTABLE element. This is necessary because the particles begin at the hard-edge reference plane for the nominal element, which may be inside the fringe-field region. Second, track particles through the FTABLE element as described in the previous section. Third, back-track particles through a "variable" drift to line 2, so all particles will start from $z = 0$ of the next element (i.e., at the ideal reference plane). Figure 5 shows the initial

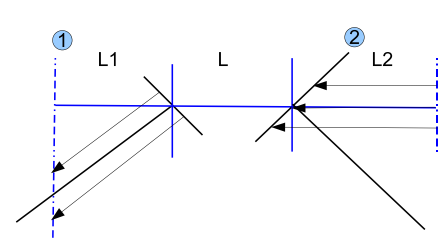

Figure 4: Reference frame converting.

coordinates and final coordinates when tracking through a hard edge bending magnet (no edge effects). These agree very well, indicating that the conversion of the reference frames is performed correctly.

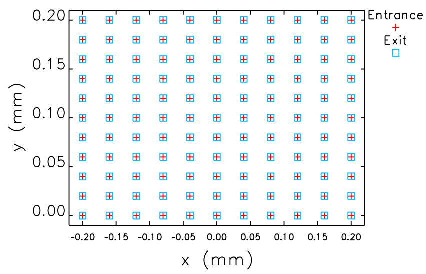

Figure 5: Particles' coordinates at the entrance (red) and exit (blue) of a bending magnet (excludes the edge effect.)

EXAMPLE

To check how the FTABLE element works with long-term tracking, we replaced one sextupole family (40 magnets in total) in the APS lattice with a corresponding FTABLE element. Particles with different initial amplitudes are tracked for 1000 turns. Their coordinates at the beginning of each turn were saved and are plotted in Figure 6. The same particles were tracked through the normal APS lattice (no FTABLE element). The phase space plot for this case is shown in Figure 7. In this example, we see no difference between the methods, which indicates that tracking for time scales appropriate for a dynamic acceptance or momentum acceptance determination is reliable.

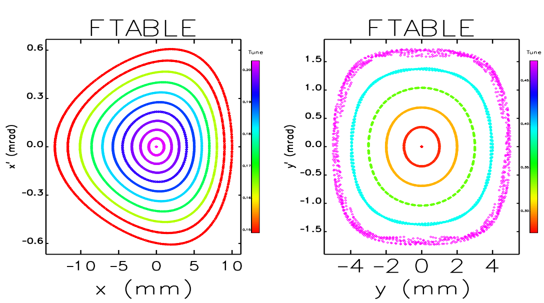

Figure 6: Phase space plot - tracking with FTABLE element for 1000 turns.

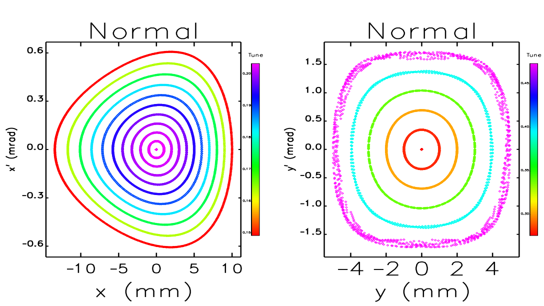

Figure 7: Phase space plot - tracking with KSEXT element for 1000 turns.

FUTURE DEVELOPMENT

Although the method described here is useful and reliable, we plan to add more features to improve both the physics and ease of use. Addition of classical and quantum aspects of synchrotron radiation should be straightforward using the code already in elegant.

The ability to specify the location of the vertex point (the intersection of the ideal incoming and outgoing trajectories) will be useful in translating measured or simulated magnetic field data into the proper relative coordinates compared to the upstream and downstream elements. Similarly, we will add the ability to scale the magnetic field to ensure that the bending angle, including fringe effects, matches the ideal value.

SUMMARY

By rotating the coordinate system, particle motion inside a general magnetic field is decomposed into two simple components: a free drift and a centripetal motion. The equation of motion can be solved easily, and its form is exactly a spiral line. The magnetic field is calculated by linear interpolation of a known 3D field table. Even though the assumptions made don't guarantee that Maxwell's equations are satisfied, meaning that the method is not symplectic, our testing results show the accuracy and the stability are still very good. Thus, the method can be used for studying electron storage rings with fringe-field effects or magnets with complicated profiles. In such a cases, finding a suitable symplectic integrator is very hard, and our tools provide a reasonable alternative. This method was added to elegant as the FTABLE element. It is also parallel capable.

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